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COMMENTS ON CONTRIBUTORS _____

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EDITORIAL

Upcoming issues of *delta-K* will be on three themes:

- Manipulating and Activating Mathematics
- Teaching Ideas
- Continuity, Integration and Instruction in Mathematics

We have already received some submissions for the issue on Manipulating and Activating Mathematics, but more are always welcome! The Teaching Ideas issue will focus on good, interesting and innovative ways to address topics in mathematics. For this issue, we would like the articles to be brief descriptions of ready-to-use activities, and we need your help to produce this issue, so please start planning your contribution now! The third issue deals with attempts to integrate mathematics with other subjects, and attain continuity in instruction in mathematics. Thus, for this issue, we would really appreciate contributions from those educators who are currently employing techniques of, or researching topics in, continuity and integration.

This issue of *delta-K* focuses on Technology in the Classroom.

- George Cathcart presents eight ways of using the computer in instruction, with Logo as the learning environment.
- Marlow Ediger presents a philosophical overview of the uses of computers in mathematics, and offers suggestions on the choice of software.
- Dale Burnett uses the Logo environment to bring “play” to problem solving.
- Sol Sigurdson describes research into the use of calculators in test preparation, and the use of calculators by students during test writing.
- Judy Barnes takes a qualitative look at how teachers and students feel about studying mathematics through computer-assisted instruction.
- In the Problem Corner, John Percevault suggests that simply solving a particular problem does not necessarily involve students in the problem-solving process.
- In the Teaching Ideas section, Craig Loewen introduces a computer program and problems that are useful in teaching motion geometry.
- Finally, John Heuver comments on the new Mathematics 10 program. (The editors invite commentaries from other readers.)

It is an ambitious undertaking to publish regularly a journal such as *delta-K*, and we need your support and assistance if it is to continue and remain viable. *delta-K*, published according to the mandate of the Mathematics Council of The Alberta Teachers' Association, is a forum in which Alberta educators share ideas and learn about recent developments in mathematics education in Alberta. We depend on our readers for their contributions, and we believe there are as many good ideas in Alberta as there are good teachers. Please take the time to contribute your ideas.

A. Craig Loewen
John B. Percevault

Eight Ideas for Using the Computer in Elementary Mathematics

W. George Cathcart

Computer use has been categorized in a variety of ways. For example, Taylor (1980) has compiled a set of readings that describe the computer as a tutor, tool and tutee. A more extensive taxonomy might include at the least the following uses.

Computer-Assisted Instruction

This category includes all practice courseware such as Math Blaster (Davidson & Associates) and Addition Logician (MECC). Tutorial software as well as simulations and educational games could also be classified as computer-assisted instruction, together with some problem-solving courseware. Using the computer for demonstration or to generate materials for children may also be considered computer-assisted instruction.

Computer-Managed Instruction

In computer-managed instruction, the computer helps organize and manage instruction, for example, by keeping records of students' progress (including whether specified objectives have been achieved), and generating reports for teachers, parents and other interested parties. Testing could also be classified in this category.

Productivity Applications

Wordprocessing, spreadsheets, data base management, drawing, painting and desktop publishing are often referred to as productivity uses of the computer. More teachers are using these tools for personal and in-class productivity.

Eight Quick and Easy (Spontaneous) Uses

Many commercial courseware packages are pedagogically sound and have some "bells and whistles"

that appeal to children. These packages should be used when they mesh with the mathematics program in the classroom, but often the match of courseware to a program is not as close as one would like. A use developed spontaneously by the teacher then becomes the better way to go.

The purpose of this article is to describe a few, simple ideas for using the computer in the mathematics program—the kind of things a teacher could incorporate into a lesson plan for the next day. These ideas would be classed as computer-assisted instruction. The sample procedures have been written in Apple (LCSI) Logo but could be done in other Logo dialects or in BASIC. First, here are a few utility procedures.

Utility Procedures

The following procedures are used in the examples. They need to be loaded into the workspace before the other procedures will run. More experienced Logo users may want to bury these procedures in a **STARTUP** file.

```
TO SETTEXT          TO PRESSKEY
TEXTSCREEN CLEARTEXT SETCURSOR [5 23]
SETCURSOR [0 0]     TYPE [PRESS ANY KEY TO
END                  CONTINUE.]
                     MAKE "INPUT READCHAR
                     END
TO BLANKLINE :HOWMANY
REPEAT :HOWMANY
[PRINT[]]
END
TO TAB :NUMBER
REPEAT :NUMBER [TYPE
CHAR 32]
END
```

Scenario 1: Counting

Teachers of young children are creative in developing activities with counters of various kinds (chips,

buttons, beads, popsicle sticks). Calculators with a built-in constant can provide additional variety in the kind of counting experiences children have, and the computer can also be used for this purpose.

Groups of students could be engaged in counting activities with counters, calculators, and using computers as follows. COUNTUP begins a counting sequence with the first four numbers. After that, when a key other than "S" is pressed, the next counting number appears. Pressing "S" clears the screen and stops execution.

```

TO COUNTUP
INSTRUCTIONS
SETTEXT
BLANKLINE 5
COUNTFOUR 1 0 1
END

TO INSTRUCTIONS
SETTEXT
BLANKLINE 3
PRINT [TOUCH ANY KEY TO COUNT.]
BLANKLINE 1
PRINT [WHEN YOU ARE READY TO STOP COUNTING.]
BLANKLINE 1
PRINT [TOUCH THE "S" KEY.]
PRESSKEY
END

TO COUNTFOUR :S :CT :INC
IF :CT > 3 [COUNTREST :S :INC] [TYPE :S SPACE 1]
COUNTFOUR :S +INC :CT + 1 :INC
END

TO COUNTREST :S :INC
MAKE "K READCHAR
IF :K = "S [SETTEXT THROW "TOPLEVEL] [TYPE :S
SPACE 1]
COUNTREST :S + :INC :INC
END

```

INSTRUCTIONS may be omitted because of the reading level. Some children may be capable of counting on from some number other than 1. COUNTUP2 enables children to specify the starting number as input to the procedure. Otherwise, it works the same as COUNTUP. The subprocedure COUNTFOUR was written for the general case so it can be used for both these settings as well as for the next one, skip counting.

```

TO COUNTUP2 :START
INSTRUCTIONS
SETTEXT
BLANKLINE 5
COUNTFOUR :S 0 1
END

```

Scenario 2: Skip Counting

```

TO SKIPCOUNT
INSTRUCTIONS
SETTEXT
PRINT [BEGIN AT?]
BLANKLINE 1
MAKE "BEGIN FIRST READLIST
BLANKLINE 1
PRINT [COUNT BY?]
BLANKLINE 1
MAKE "INC FIRST READLIST
BLANKLINE 1
COUNTFOUR :BEGIN 0 :INC
END

```

Skip counting is often used as a lead-in activity to multiplication. For instance, when children count by 3s (3, 6, 9, 12 . . .), they are *teased* into thinking about the three times table. Children can gain experience in many ways with skip counting, including using the calculator and the computer. One group of children might use the Logo procedure SKIP-COUNT. An interactive procedure, this asks for the starting number and the increment. It then counts as before using the same counting subprocedure COUNTFOUR. For example, if 7 and 3 are entered as the beginning number and the increment respectively, then the starting screen display will be 7 10 13 16.

Scenario 3: Addition Table

After children have had substantial experience with the basic facts (using concrete materials in real-world settings), they are expected to master the facts to the point of instant recall. Games, puzzles, riddles and many other techniques can help children master the basic facts. Much of the available commercial courseware is designed to provide practice with the four basic operations. An alternative is to use the computer to generate an addition table as the students need it to check work or as a reference while they do a problem. The following procedures allow children to produce only the portion of the addition table they need at the moment.

```

TO ADD :N
SETTEXT
BLANKLINE 3
(PRINT [ADDITION TABLE FOR] :N)
BLANKLINE 1
ADDTABLE :N 0
END

TO ADDTABLE :N :C
IF (OR :N < 1 :N > 9 :C > 9) [STOP]
(PRINT :N "+" :C "=" :C + :N)
ADDTABLE :N :C + 1
END

```


Scenario 4: Multiplication Table

The same comments apply as for the addition table. The following analogous procedures could be used to generate the multiplication table for a designated multiplier.

```

TO MULTIPLY :N
SETTEXT
BLANKLINE 3
(PRINT [MULTIPLICATION
TABLE FOR] :N)
BLANKLINE 1
MULTABLE :N 0
END

TO MULTABLE :N :C
IF (OR :N < 1 :N > 9
:C > 9) [STOP]
(PRINT :N "X" :C "= :C*:"N)
MULTABLE :N :C + 1
END
    
```

Scenario 5: Numeric Exploration

Elementary school students need a variety of experiences to help them build a concept of the magnitude of one million. Most do not have a good feel for this. Estimating how big a container would be needed to store a million bottle caps, or how many pages of "Xs" would give a million "Xs" are examples of activities that might be done in the classroom. The last idea could be performed on the computer.

Another computer experience that might contribute to a child's appreciation of the magnitude of a

million could make use of these Logo procedures. Ask the children to guess how long it will take the computer to count to 1,000,000. Execute **MILLION**. Press a key after about 5 seconds and note the number displayed. This is how far the computer counted in the time allowed. Execute **MILLION** again for about 20 seconds before pressing a key. Observe how far the computer counted and ask the children if they want to adjust their original guess. Check a few of these guesses by timing the computer as you do other class work.

```

TO MILLION
MILLIONC 1
END







TO MILLIONC :ST
IF KEYP [PRINT :ST STOP]
MILLIONC :ST + 1
END
    
```

Scenario 6: 2-D Pattern Exploration

Logo is an excellent medium for exploring the properties of geometric figures. For example, children could complete the activity shown in Figure 1, working in pairs.

TURTLE SHAPES

Find numbers for the blanks in each Logo statement that will produce the figure. The square has been completed for you.

	REPEAT _____ [FD 55 RT _____]	Describe as many patterns as you can in the numbers you used.
	REPEAT <u>4</u> [FD 55 RT <u>90</u>]	
	REPEAT _____ [FD 55 RT _____]	
	REPEAT _____ [FD 55 RT _____]	
	REPEAT _____ [FD 55 RT _____]	
	REPEAT _____ [FD 55 RT _____]	

If you wanted to make a figure with 18 sides, what Logo statement could you use?

Figure 1. A Logo Activity to Observe Patterns in 2-D Figures

Scenario 7: Geometric Constructions



Children can also learn about the typical line constructions using Logo. In pairs, children can explore parallel lines, perpendicular lines, perpendicular bisectors, angle bisectors, obtuse angles, oblique parallel lines with a perpendicular

bisector, and many other possibilities. Children can work in immediate mode using only **PENUP**, **PENDOWN**, **FORWARD**, **BACK**, **RIGHT** and **LEFT** to position the turtle, and then use this **LINE** procedure to draw a line where :N is the length of line desired. Children with more Logo experience might try writing procedures for some of their constructions.

```
TO LINE :N
FD :N BK :N
END
```

Scenario 8: Flips

Logo is also an excellent medium for helping children understand the concept of a flip (reflection or mirror image). A study of flips should begin with the use of concrete devices such as the geoboard or mira. Once children understand what a flip does, Logo could be used to highlight one property of a flip.

Demonstrate for children **VFLINE** and **HFLINE**. These procedures simply draw a line, either vertically (**VFLINE**) or horizontally (**HFLINE**), through the centre of the screen. This line will be the *flip line*. Children should work in pairs and agree on either a horizontal or vertical flip line and execute the appropriate procedure. The first student will then create any kind of a design on one side of the line, recording on paper each Logo command in sequence, with the last two commands being **PENUP** and **HOME** to return the turtle to its starting position. The second student then uses the listing of commands on paper to create a flip image of the design. He or she may want to change the color to make the image more distinct from the object¹.

```
TO VFLINE
CS
FD 80 BK 160 FD 80
END

TO HFLINE
CS
RT 90 FD 120 BK 240 FD 120
LT 90
END
```

After each student has had at least one turn in each role, ask them to write a statement about how they created flip images. They should be able to observe that to create a flip image, all **RTs** become **LTs**, all **LTs** become **RTs**, and all inputs to **RT**, **LT**, **FD** and **BK** remain unchanged. Figures 2 and 3 provide a sample of possible inputs by students.

¹Students may need to be told, particularly when using **HFLINE**, that part of their design may not show. When the design is complete, they should hold down the **CONTROL** key and touch "L" and the full screen will be visible. **CONTROL-S** returns them to split screen.

Student 1

```
VFLINE
PU RT 90 FD 20 LT 90 PD
FD 55 RT 45
FD 50 RT 115
FD 77 RT 66
BK 22 LT 22
FD 66 RT 95
FD 60
PU HOME
```

Student 2

```
SETPC4
PU LT 90 FD 20 RT 90 PD
FD 55 LT 45
FD 50 LT 115
FD 77 LT 66
BK 22 RT 22
FD 66 LT 95
FD 60
PU HOME
```

Figure 2. Sets of Logo Commands to Produce an Object and its Flip Image

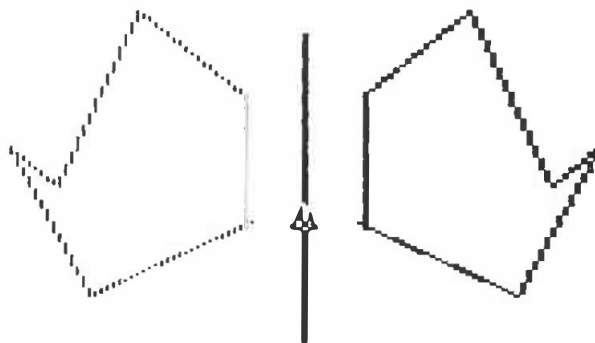


Figure 3. Object and Flip Image Produced by Commands in Figure 2

Conclusion

My purpose in writing this article is to point out that there are many classroom settings where computer activities could be created quickly and simply to enhance the mathematics lesson. I describe eight such settings, only a few of the many that are possible.

To make the most of these opportunities, teachers should be able to design simple procedures in Logo or some other language. With the exception of those used to generate the addition and multiplication tables, the procedures in this article require minimal programming expertise. Many settings do not require writing procedures at all, as children can explore many ideas and patterns in immediate mode. The computer is a tool that can enhance many mathematics lessons. Let's use it!

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Philosophy, Computers and Mathematics

Marlow Ediger

The mathematics curriculum ought to emphasize the use of computers. Software packages should guide learners to achieve objectives, stressing the acquisition of subject matter in sequence so that students can make the best possible progress. Sequential learning relates new subject matter to that previously acquired, and knowledge that is related is retained longer by students than fragmented knowledge. Thus, good software in mathematics should provide content that is correlated and integrated with subject matter previously studied.

Software content that captures students' interest and attention and permits the establishment of sets will motivate students to achieve appropriate objectives in mathematics. Students also need to understand the purpose of achieving vital facts, concepts and generalizations in mathematics, and mathematics teachers should encourage students, inductively or deductively, to accept reasons for learning. Once they have understood the purpose, students must become actively involved in comprehending vital subject matter presented by the software.

Philosophy of Using Software in Mathematics Education

The mathematics teacher needs to study and analyze diverse schools of thought on the subject of computer software curriculum. Each philosophy makes its own recommendations for teaching-learning situations so the philosophy chosen should fit the personal learning style of the student.

The Problem-Solving Strategy

To implement a problem-solving strategy in lessons and units, the first step is to identify the mathematical problem carefully. This problem should be selected by students with the guidance of the teacher

in a contextual situation. Each problem should have to do with real life and society, making clear the societal relevance of the mathematics curriculum to the student.

The next step requires gathering information to solve the problem, some of which may be obtained from relevant software, as well as other reference sources such as mathematics textbooks and workbooks, and audiovisual materials. A hypothesis, tentative and subject to modification as a result of testing, is then generated to help answer the problem.

Problem-solving strategies in mathematics do not

- have predetermined objectives for student attainment;
- possess a logical sequence in which the mathematics teacher/educator selects sequential learning activities for students;
- emphasize a behavioral model of teaching. Behaviorism stresses objective, measurable results in each step of student learning;
- advocate paper-and-pencil tests of student achievement, but rather measure it in terms of ability to define and solve real problems in mathematics;
- stress a highly structured sequence in the curriculum. Problem solving emphasizes openness, creativity and flexibility in problem identification, information gathering, hypothesizing and testing, and revising the hypothesis.

Good software should assist with information gathering and testing hypotheses.

On problem solving, Grossnickle et al. (1983, 177) write:

Problem solving is a process by which the choice of an appropriate strategy enables a pupil to proceed from what is given in a problem to its solution. Often the answer is the least important part of the problem-solving process; few of the answers

children obtain in school mathematics will have much value in their lives. The ideas used in the process are much more valuable than the answer. Thus it is important for teachers to determine whether an incorrect answer is due to an error in process or in computation. Do not, however, infer from this discussion that errors in computation are acceptable; rather, keep in mind that overemphasis on answers may impede the pupil's understanding of the process. A pupil with poor computational ability who understands the process can use a calculator to get the answer. A pupil who can compute rapidly and accurately but does not understand the process is lost.

The Decision-Making Strategy

A second school of philosophical thought concerns the decision-making strategy. Here, the student selects learning activities from among alternatives. In mathematics, an adequate number of software packages are available for students to choose from, and they select those that have purpose, meaning and interest. Besides software programs, activities with textbooks, workbooks and audiovisual materials are available.

This strategy may be implemented using a learning centres approach. With this method, each learning centre is equipped with software related directly to the mathematics unit being taught. The content appearing on the monitor guides the student interactively until vital objectives are mastered. Students may choose their own objectives—preferably, the most challenging ones—and the kinds of software by which these objectives can be attained.

A contract system may also be useful in a decision-making strategy. With the teacher's guidance, the student selects the computer programs and writes them up in contract form with a specified due date, and the student and teacher both sign it. The student then selects the sequence of programs to complete. Further, the student may select objectives to achieve, with student and teacher cooperating to determine the learning opportunities and the computer or other activities to be undertaken. Appraisal procedures are also cooperatively developed.

Decision-making strategies do not emphasize

- a predetermined mathematics curriculum. Instead, students must invest time and effort into selecting their learning objectives and opportunities and their appraisal procedures within the framework of the mathematics software/computer curriculum;

- a subject-centred curriculum only. It is equally important to emphasize attitudes and the affective dimension when using computer software in the mathematics curriculum;
- an essential or basic curriculum for all students. Rather, students choose among alternatives what software activities and experiences they will pursue;
- a logical curriculum whereby the mathematics teacher or objectives set by the government determine the sequence of what students are to learn. The students determine themselves what they are to learn as part of a "psychological curriculum";
- a formal mathematics curriculum. Instead, the students' ability to choose what software they will use makes for flexibility and open-endedness.

Abelson (1982, ix) writes on students' use of Logo:

Logo is the name for a philosophy of education and for a continually evolving family of computer languages that aid its realization. Its learning environments articulate the principle that giving people personal control over powerful computational resources can enable them to establish intimate contact with profound ideas from science, from mathematics, and from the art of intellectual model building. Its computer languages are designed to transform computers into flexible tools to aid in learning, in playing, and in exploring.

We try to make it possible for even young children to control the computer in self-directed ways, even at their very first exposure to Logo. At the same time, we believe Logo should be a general-purpose programming system of considerable power and wealth of expression. . . . More than 10 years of experience at MIT and elsewhere have demonstrated that people across the whole range of "mathematical aptitude" enjoy using Logo to create original and sophisticated programs. Logo has been successfully and productively used by preschool, elementary, junior high, senior high, and college students, and by their instructors.

Measurement-Driven Instruction (MDI)

This third philosophy of instruction requires predetermined objectives of student attainment to be stated in precise, measurable terms before computers are used. The objectives may be mandated by the government, the school district, or the mathematics teacher. Good-quality software programs provide the subject matter in the appropriate sequence students need to attain the objectives. To ensure validity of content, appraisal procedures should also accord with

the objectives. Reliability in measurement procedures must be established, using test-retest, split-half or alternate forms.

A carefully constructed sequence of objectives, learning opportunities involving computer software use, and appraisal procedures is required. Each of these components is related to the others. Thus, designing the curriculum with a measurement-driven instruction model might help students succeed better in the computer software mathematics curriculum.

Measurement-driven instruction does not emphasize

- an open-ended mathematics curriculum whereby students choose their own software programs;
- a psychological sequence in which students direct their own learning experiences. Instead, mathematics educators select the objectives, activities and appraisal procedures;
- decision-making by the students as to the scope and sequence of the curriculum;
- problem-solving activities that relate society to school mathematics within the computer software curriculum;
- general objectives in mathematics.

Instead, precise, measurably-stated ends are emphasized.

Conclusion

In summary, software programs in mathematics might emphasize

- problem-solving strategies. Real-life problems in mathematics need to be identified and solved;

- decision-making strategies. Students can select from among alternative objectives, learning activities and appraisal procedures;
- measurement-driven instruction. With precise, predetermined objectives, students are clearly seen to achieve or fail to achieve these ends.

However, I recommend that emphasis continue to be placed on problem solving. In the real world, personal and social problems exist that need solution, and people need to be able to solve them.

Software programs in mathematics can emphasize

- drill and practice—to help review what students have learned;
- tutorials—to guide students in acquiring new content in sequence;
- simulations—in which students play roles to solve problems;
- games—in which students individually or in small groups compete against others in an atmosphere of respect and appreciation.

These four kinds of software should help students learn in the mathematics curriculum.

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Logo and Problem Posing

J. Dale Burnett

Every tool carries with it the spirit by which it has been created.

(Heisenberg 1958, 27)

Educators continue to struggle with Logo. Where does it fit in? I would like to quote from two books about Logo, to set the tone, and then provide examples of problem posing taken from different domains.

This is a book about exploring mathematics . . .
(Abelson & diSessa 1981, xix)

You probably won't read the book cover to cover; instead you'll get excited by a particular project and spend a month or two exploring it. . . . You're learning the discipline of serious thinking and of taking pride in your work. . . . You're supposed to be interested enough already to explore on your own. . . . I think it's better to encourage your creativity by letting you invent your own exercises.

(Harvey 1985, x-xiv)

Problem solving was the agenda for action in the 1980s. I hope that problem posing will be on the agenda for the 1990s.

Educators are beginning to realize that a sub-skill approach is relatively ineffective in teaching students to read or write. Mathematics educators have yet to realize that this applies also to mathematics education. A significant proportion of the present mathematics curriculum is predicated on the assumption that practice in computational and algebraic skills will result in an understanding of mathematical ideas, despite substantial evidence that this is incorrect. We have adopted the maxim that if something isn't working, then more of it will. Mathematics has yet to adopt an integrated approach in the sense that the underlying purpose is comprehension and understanding of relationships and ideas.

Even the goal of fostering problem solving has degenerated into rote application of tried-and-true

procedures to type problems. The spirit of problem solving has disappeared. How many different ways can you approach a problem? What are the features of each? Where do these approaches come from? The writings of Brown and Walter (1983) and Mason (1985) exemplify such an approach, as do many books on Logo (Abelson & diSessa 1981; Harvey 1985; Clayson 1988).

Playing with "Forward"

FD 50. This may be the first phrase that a novice turtle trainer speaks. The result is straightforward—a line about an inch long. For many, this is also the end of **FD**. They quickly move on to other commands, failing to take advantage of an opportunity to explore. How does one explore a command? How does one explore an idea like addition? Let me leave the latter question, not because I don't have suggestions about it, but because I shouldn't deprive you of the chance to have your own ideas first. However, I would now like to explore the possibilities of **FORWARD**.

We can approach the task from many angles. Let's begin with syntax. A few thoughts come to mind. One pertains to abbreviations. **FD 50** works. Are other abbreviations possible? What about the first two letters? This is okay for California, but not for Montana! We might try **FO 50**. Another possibility is **FWD 50**. I have seen young children try **DF 50**. And what about upper and lower case? **Fd 50** seems promising. And **fd 50**? Will **fd 50** generate a line or an error message? Then there is the space between the command and the number. Let's experiment systematically: start with zero spaces, then try one space, two spaces, and so on. Is there an upper limit? In pursuing such questions, the student becomes familiar with the command, its nuances, when it behaves and

when it does not. Now, what happens if the number precedes the command, as in `50 FD`?

Now let us focus on the number following the command. What kinds of values work here? `FD 50` works, but what happens when you type `FD 100`? The line is longer. It appears to be about twice as long. How could you be sure? Will `FD 1000` work, or `FD 1000000`? Can you put commas in the number as we do when we write 1,000,000? Is there a maximum number that is valid for `FD`? Do the answers to any of these questions depend on where the turtle is on the screen? Or on how long the computer has been responding to Logo commands? Besides using a comma, what about a period? More than one period? Thinking in terms of the number line, we have been playing numbers to the right, larger and larger numbers. What happens when we think of the left—negative numbers? Exploring with negative numbers can be interesting in itself. The real number aficionados may have noted that we jumped over another interesting number: zero. `FD 0` should do nothing, shouldn't it? Does it do nothing? There are also zero's near neighbors—the very small numbers. Is there a minimum number? Do you have some other favorites? Some people like pi.

We can extend our questions about the numbers that follow the `FD`. For example, can there be more than one number to the right of the command? Try `FD 2 + 2`, or `FD 2 - 1`. We can explore the use of parentheses. With some experience with Logo, we may use variables or have the expression to the right of the `FD` a procedure that acts as an operation or

a command. There is a popular saying to the effect that if we say a word 10 times it is ours. What should we say about Logo primitives?

Playing with Lists

Many people introduced to Logo remain within the subset of primitives associated with turtle geometry. However, Logo, in a fundamental sense, is an example of a list-processing language. Clarification of some of the concepts that are central to list processing can lead to a deeper appreciation of the overall structure of the language. Concepts such as word, sentence and list form the heart of the language. Logo primitives like `FIRST`, `BUTFIRST`, `LAST` and `BUTLAST` provide an introduction to the types of operations that are possible. Character strings may consist of letters, numbers, special characters and spaces. In a very real sense, anything goes. The task facing the student is to bring meaning to this situation. Simple receptive learning, where the student is told, or reads, about the distinctions is usually not sufficient. One needs some practice with the possibilities. In order to delimit the problem, at least initially, one may wish to focus on strings that begin with

- a letter
- a “
- a [and end with a]
- a :

as each of these cases represents a special situation in the language.

FIRST TURTLE

I don't know how to TURTLE

doesn't work

FIRST "TURTLE

You don't say what to do with T

try a double-quoted string

it works, but acts like an operation

PRINT FIRST "TURTLE

T

okay

PRINT FIRST "TURTLE TRIP

T

I don't know how to TRIP

try two words

nope

PRINT FIRST "TURTLE "TRIP

T

You don't say what to do with TRIP

try two words again

still nope

PRINT FIRST "123

1

try numbers

okay

PRINT FIRST 123

1

PRINT BUTFIRST "TURTLE
URTLE

PRINT FIRST FIRST "TURTLE
T

PRINT FIRST BUTFIRST "TURTLE
U

PRINT LAST BUTLAST "TURTLE
L

try again
wow!

try another command
okay

try combining primitives
okay, but I was expecting a U

try again
now I have my U

can I get an L?
yes

One of the problems with an extended protocol like this is that it only reflects the sequence of mental experiments by the author. It represents a two-minute sequence of conjectures. Within half-an-hour or so it should be possible to make substantial progress on how these primitives work together.

Pedagogical questions arise now. For example, (1) is this an efficient way to learn these ideas about list processing, and (2) can students actually do this? Regarding the first question I suggest that a small fact sheet be given to students to help focus the inquiry:

Types of Data

- words
 - unadorned box
 - quoted "box
 - dotted :box
- numbers 423
- lists [box]

Logo Primitives

FIRST
BUTFIRST
LAST
BUTLAST

With respect to the second question, I'm not sure. However, I would like to see education move toward a goal where the answer is a clear yes. One difficulty at the moment is that many students have had little experience with such activities—no wonder they feel a little lost at the beginning. The same applies at the teaching and learning levels for most teachers. However, we are now touching on what I see as the real educational value of Logo. It is not the syntactic rules, nor even how to program, but rather the

sense to which it focuses debate on the nature of deep understanding and intelligent behavior. In this context, intelligence is viewed as an ability to adapt to new situations, not as a score on an IQ test.

Playing with Time

A number of people have suggested that a slower turtle might be advantageous when young children are playing with Logo. The question can be a springboard for a substantial research agenda. However, I would like to play with a different board. Terms like "slow" and "fast" give rise to other ideas like speed and distance and time. Is it possible to play with Logo-time? What could a term like Logo-time even mean? We seem to have adapted to the notion of a turtle-step. What about a turtle-second?

Here are three ways to move a turtle 200 units.

FD 200

REPEAT 200 [FD 1]

REPEAT 100 [FD 2 WAIT 5]

Each approach is noticeably slower than the preceding one. Now what?

A tentative definition of a turtle-second is the time it takes a turtle to move forward 1 turtle-step. A second approach is to define it in terms of the **WAIT** primitive. Once a particular approach is adopted the questions begin to flow.

How would one create time-conversion procedures so one could move between systems? Analogies to fast-forward and slow-motion come to mind.

If turtles are walking in Europe (metric system) or the United States (imperial system) or Canada (both systems), how can one compare stories of their journeys?

Imagine a turtle walking along the perimeter of a right-angled triangle. Could one prove Pythagoras' theorem using time? Similarly, what about the relation of the radius of a circle to its circumference?

Consider problems of falling bodies. If one threw a barometer off the top of a building, how could one determine the height of the building?

Consider distance problems involving two vehicles. For example, if John (a turtle) leaves A, travelling east at . . . ?

Then there are races, such as those of the tortoise and the hare, Mario Andretti, the America's Cup; hockey—passing a puck from one turtle to another; projectile problems; relativity theory; instant Logo.

This is back where we started.

Playing with Recursion

Books have been written on this topic. Obviously, there is more here than meets the eye. But what? How does one explore the idea(s) of recursion? New branches of mathematics are evolving—witness fractal geometry and chaos theory. Conway's Game of Life and its variants are also relevant (Poundstone, 1985). Harvey (1985) has four chapter fives exploring the idea from different perspectives. Abelson (1982) provided a classic example distinguishing tail recursion from the more general notion. The idea of recursion can be considered within a graphic setting (e.g., fractals), a numeric setting (e.g., Fibonacci numbers), or an alphabetic setting (e.g., palindromes).

Let's begin with a simple example of a program that calls itself.

```
TO BOX
  BACK 50
  LEFT 90
  BOX
END
```

How might we (a) modify, and (b) extend this program? Suppose we have:

```
TO BOX
  BACK 50
  BOX
  LEFT 90
END
```

The first procedure draws an (endless) box, whereas the second does nothing of the kind.

A simple extension is to examine programs that utilize inputs.

```
TO BOX :SIDE
  IF :SIDE > 100 [STOP]
  BACK :SIDE
  LEFT 90
  BOX :SIDE + 2
END
```

Playing with negative numbers can also be interesting:

```
TO BOX :SIDE
  IF :SIDE < -100 [STOP]
  BACK :SIDE
  LEFT 90
  BOX :SIDE - 2
END
```

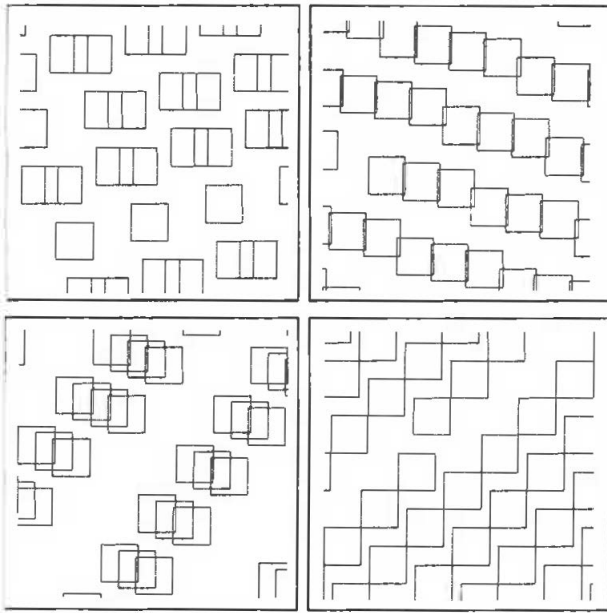
Abelson and diSessa (1981) provide a provoking example right at the beginning of their book when they experiment with different values for spiral procedures (pp. 17-20). A simple three-line procedure can provide hours of thoughtful exploration. But who is to say how to explore?

Playing with Patterns

One can make a career of playing with patterns. Mathematicians tend to think of patterns in terms of the shapes that are to be combined. Topics like tessellation and symmetry arise. Artists tend to focus more on the space between the shapes. People such as M.C. Escher bridge both approaches.

I would like to begin with the simple square. It is a relatively easy task to write a Logo procedure that tessellates these squares to form a perfect checkerboard. While tessellations are pleasing in the sense that they come out right, they tend to bore most people. Clayson (1988) has shown that Logo can be used as a vehicle for visual thinking. He approaches squares in a less inhibited manner, splashing them on the screen to see if he can produce some mathematically messy but aesthetically pleasing patterns. He admonishes us to "get in the habit of tinkering" (p. 5). See the page opposite for four examples.

It is tempting to imagine what could be done next with a powerful color facility, on the screen and on paper. We have just begun to play. After squares come other shapes—I suspect triangles would be particularly interesting. Burnett (1985) played with Islamic art patterns, which turned out to be a fascinating trip to other regions of the mind. One version of Logo permits the drawing of three-dimensional



figures. If this could be combined with animation and color, the resulting product would be dazzling to contemplate. And some people think Logo is only for kids! Others complain, "What do you do after you have drawn a house?"

Perhaps the difficulties lie with our conceptions of the curriculum (Burnett 1988). A curriculum that has trouble finding a place for imagination and extensive thought is a curriculum in trouble.

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Test Preparation and the Use of Calculators

Sol E. Sigurdson

A recent decision by Alberta Education to use calculators in their Grade 9 Mathematics Achievement Test poses the question of whether students who have been trained to use calculators in tests would be at an advantage. Calculators are here to stay. It remains for us to determine their wise use. Teachers have major concerns over their role in instruction, but their role in testing is also cause for concern (Heid 1988). Although the curriculum calls for the general use of calculators in junior high school mathematics and students should know how to use calculators, the issue in testing is whether students who have had special preparation in the use of calculators in tests would have an unfair advantage. Evaluation experts in Alberta Education have expressed an interest in the question. With their cooperation, a study was undertaken to examine the issue of preparation for the use of calculators in the multiple-choice provincial Grade 9 Mathematics Achievement Test 1987.

The use of calculators is part of the larger issue of general preparation for tests. What do teachers normally do in preparing for tests? Tests are meant to determine a student's level of mathematical knowledge. Is special preparation of any kind unjustified then? Teachers have long believed that preparation for tests is important. When the author was studying Grade 12 mathematics, 35 years ago, the course was complete by Easter and the remaining two-and-a-half months were spent going over old examinations. This seems a bit extreme—but it got good results on provincial examinations.

The Research Question

The general question is: In preparing students for the Provincial Achievement Test in Mathematics, what type of preparation would be most effective? To formulate this as a research question, four test

preparation treatments were devised. The first addresses the question of knowledge of mathematics, the remaining three the matter of knowing about tests and testing procedures:

1. *General Review*, using questions from the textbook to ensure students have a full understanding of the mathematics in the particular units
2. *Specific Structured Review*, using question sheets containing typical multiple-choice questions
3. *Multiple-Choice Preparation*, using the Specific Structured Review questions but attending to the multiple-choice format and how it could be advantageous in developing alternative solutions to the questions
4. *Calculator-Use Preparation*, using a focus on multiple-choice format, with special emphasis on how the calculator could be used to advantage in answering multiple-choice questions

Each treatment builds upon the previous one so that the fourth treatment included the general review, the specific review, a focus on multiple-choice format, and calculator-use preparation.

Development of Treatments

An initial survey of teachers who were interested in participating in the study showed a considerable range of emphasis on test preparation. Some teachers claimed to give students more than 200 specific test review questions. Others spent two or three days reviewing. In developing our treatments, because some of them were to include a lot of activities, we opted for every treatment to be of 11 days' duration. The researcher felt that time spent on review would make a difference to test scores, but was interested in whether what the teacher did during review time made a difference. The daily topics for all groups were:

- Day
1. Ratio and proportion
 2. Number systems, operations
 3. Exponents
 4. Pythagorean theorem
 5. Surface area, shaded area
 6. Measurement and graphing
 7. Algebra—solving equations
 8. Algebra—operations, polynomials
 9. Algebra—factoring
 10. Algebra—problem solving
 11. General test, with all types of questions

General Review. This treatment consisted of reviewing, using only the textbook, the *mathematics* skills and concepts in each topic. The content was divided into 10 sections covering the Grade 9 course, leaving the last day for an overview. Students could use calculators during this review and be familiarized with multiple-choice questions.

Specific Structured Review. For this treatment 11 single-page handouts containing multiple-choice questions were used, one being assigned each day. Furthermore, teachers were to structure the class period as follows:

- 10 minutes: review previous day's work
- 15 minutes: teacher presentations of skills and concepts relating to the topic
- 15 minutes: allow students to work on a new handout
- Homework: any work not done in class must be done at home

Students were told that the test was multiple choice and that calculators could be used.

Multiple-Choice Preparation. In addition to following the format of the Specific Structured Review, teachers were asked to emphasize the guidelines in *Guidelines for Answering Multiple-Choice Test Items* (see appendix) in this treatment. Teachers were asked to spend 10 to 15 minutes each day showing students these typical multiple-choice questions and encouraging them to use these ideas as they did the sample questions on their handouts.

Calculator-Use Preparation. Teachers in this treatment were asked to stress the points in *Guidelines for Using Calculators on Multiple-Choice Tests* as they went over the typical test questions, and to make use of the ideas suggested in the *Guidelines for Answering Multiple-Choice Test Items*.

Design of the Study

Twelve volunteer teachers, each with two Grade 9 classes, were recruited for the study. Each teacher

was inserviced in two of the treatments. They were randomly assigned to offer one treatment to one class and the second to the other class. Thus three teachers offered the General Review and the Specific Structured Review to two different classes. Four comparisons were made, each involving three teachers (and six classes):

1. General Review vs. Specific Structured Review
2. Specific Structured Review vs. Multiple-Choice Preparation
3. Multiple-Choice Preparation vs. Calculator Use Preparation
4. General Review vs. Calculator Use Preparation

For these analyses, the student's mark on the provincial Grade 9 Mathematics Achievement Test was used as the dependent variable and the third report card mark of the student was used as the covariate. The advantage of this design was that comparisons were made only between classes that had the same teacher.

Results of the Analysis

For several reasons, including that some data were not available for the covariate (the student's third report card mark), four teachers had to be omitted from the data analysis, which meant that the numbers in each comparison were lower than was desirable. An analysis of covariance, using the student's *raw score* out of 75 and the student as a unit, yielded the means (adjusted by the covariate), given in the four tables following. Examining these tables reveals no statistical differences between any of the treatment comparisons. This means that the different test preparations made no difference and, in particular, that preparing students to use the calculator on multiple-choice tests does not lead to higher test scores. However, the great differences between classes in different comparisons (from 63.7 to 46.2 [raw score]), that is, between classes with different teachers, is noteworthy.

This latter observation could signify that the *teacher* is the major factor in differences between comparisons, caused by some teachers taking the Achievement Test seriously and others not. This interpretation justifies the design of the study (comparing two classes of the same teacher), and helps explain why no differences resulted, because, regardless of which treatments they had been assigned, some teachers made sure that both classes were well prepared for the test. This interpretation suggests that

Table 1. Comparison between General Review and Specific Structured Review Groups on Adjusted Means of Provincial Mathematics Achievement Tests

	Number of Students	Raw Score out of 75 Adjusted Means	Statistical Significance
General Review	73	52.9	
Specific Structured Review	66	51.6	No Difference

Table 2. Comparison between Specific Structured Review and A Focus on Multiple-Choice Format on Adjusted Means of Provincial Mathematics Achievement Tests

	Number of Students	Raw Score out of 75 Adjusted Means	Statistical Significance
Specific Structured Review	28	63.7	
Multiple-Choice Format	28	62.4	No Difference

Table 3. Comparison between Multiple-Choice Preparation and Calculator-Use Preparation on Adjusted Means of Provincial Mathematics Achievement Tests

	Number of Students	Raw Score out of 75 Adjusted Means	Statistical Significance
Multiple-Choice Preparation	43	46.2	
Calculator-Use Preparation	44	46.9	No Difference

Table 4. Comparison between General Review and Calculator-Use Preparation on Adjusted Means of Provincial Mathematics Achievement Tests

	Number of Students	Raw Score out of 75 Adjusted Means	Statistical Significance
General Review	41	50.0	
Calculator-Use Preparation	66	52.5	No Difference

the general preparedness of the class, not only mathematically but also psychologically, could be a more important factor in determining test scores than any particular test preparation. The choice of covariate, the third report card mark, is of little help in sorting out the teacher effects because it is dependent upon the teacher. This interpretation, which the present study was not designed to assess, sees the teacher playing the role of the coach getting his players "up for the game." It might warrant further study.

Discussion

What have we learned from this study? First of all, basing the study on the Provincial Achievement Tests was problematic in that these tests are taken *very seriously* by some teachers making any form of experimentation impossible. An informal survey taken at the beginning of the study showed that some teachers had planned extensive reviews (test preparations). In fact, during the course of the study, it became clear that several teachers participated in the study precisely because they viewed test preparation as an important matter. In the study, some teachers thought that spending only 11 days on the review was inadequate.

Second, the analysis of the multiple-choice format and the use of calculators on tests was probably too sophisticated for Grade 9 students, especially as they were to use this knowledge in a test situation. That is, it would take a fairly confident and sophisticated student who, in the middle of this important test, would consider the various options to answering a particular test item. For Calculator-Use Preparation, students were given some 20 rules over the 11-day period. Pressure on the student is another reason why using the Provincial Achievement Test as the dependent variable in this study was a poor decision. In any case, if test preparation, including the use of calculators, is to be advantageous, it probably has to be carefully planned and take place throughout the year.

Third, although this study did not show it, one has to believe that test preparation in general is helpful.

Students writing multiple-choice tests should be aware of the guidelines for answering multiple-choice tests and using calculators on multiple-choice tests developed in this study. Other factors such as the enthusiasm of the teacher for the test, the importance the teacher places on the test, and indeed the mathematical knowledge of the student, are also important.

Recommendations

Although the study shows no achievement differences resulting from different types of test preparation, the informal survey taken at the beginning of the study suggests that there are enormous differences between teachers' approaches to preparing for tests. In the study, teacher effects overruled the effects of the test preparations. This suggests that if the test scores are to represent students' knowledge of mathematics, rather than a knowledge of how to take tests, some guidelines be given to teachers on what constitutes reasonable preparation for the Provincial Achievement Tests. If comparable results are required on these tests, Alberta Education should provide these guidelines. As it stands, many teachers are ill-informed about possible alternatives in test preparation, and many are unsure about the ethics of specific test preparations. A starting point for guidelines could be those developed in this study. For example, an 11-day review seems reasonable. In the meantime, although this study was seriously flawed, it does suggest that specific test preparation does not dramatically raise test scores. However, reviewing some of the principles developed in this study and practicing the expected type of test questions should be helpful to all students writing multiple-choice mathematics tests.

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Appendix

Guidelines for Answering Multiple-Choice Test Items

1. *Alternative answers* should be read as part of the question and the best alternative should be selected.

e.g., 2^4 can be written as

- a) 2^{-4} b) 8 c) $\frac{1}{16}$ d) 16

2. Begin by eliminating (marking with x) those that are clearly wrong.

e.g., The equivalent form of x^{-2} is

- a) $-x^2$ b) $\frac{1}{x^2}$ c) x^2 d) $\frac{1}{x}$

3. If alternative choices are similar (additive inverses or reciprocals) be careful about answering. Alternatives always represent common mistakes. They are trying to fool you.

e.g., $-9 + +43 =$

- a) -52 b) -34 c) $+34$ d) $+52$

4. Alternative choices that include multiple answers require each alternative to be worked out. You must keep track of your answers.

e.g., The polygons with equal area are

- a) A, B and C
b) A, B and D
c) A, C and D
d) B, C and D

5. Students should be taught to recognize common stems, such as evaluate, the value of, simplify, sum, product, round off, the TRUE statement, standard form, scientific notation, decimal numeral, express as a percent, typical diagrams (dot paper, shaded areas, 3-D drawings).

6. Although the questions ask for a certain operation, the answer may be achieved by using alternative choices and revising the process.

e.g., The decimal numeral 0.00000281, written in scientific notation, is

- a) 2.81×10^{-3} b) 2.81×10^{-5} c) 2.81×10^{-6} d) 2.81×10^{-8}

e.g., Express 4.3 as a fraction

- a) $4\frac{3}{10}$ b) $4\frac{1}{3}$ c) $4\frac{33}{100}$ d) $4\frac{2}{3}$

e.g., Solve for $6x + 9 = 15$, $x =$

- a) 1 b) 2 c) 3 d) 4

7. In looking at multiple-choice questions, see what the common mistakes are. Common mistakes are given as alternative choices.

e.g., $2^3 + 2^2 =$

- a) 10 b) 12 c) 32 d) 64

Guidelines for Using Calculators on Multiple-Choice Tests

1. Use the calculator for all activities for the 11 days of review.
2. Use a method of calculating with which you are least likely to make a mistake. Do not use memory functions and avoid use of the negative sign in calculations. Use pencil and paper to determine the correct sign. Use pencil and paper to record intermediate results. Do not use the “%” key, divide by 100 instead.
3. Avoid using calculators for simple numbers. Calculators should not be used for manipulating numerals in algebraic expressions such as

$$\frac{8x^4y^3}{2xy^6} =$$

4. Most calculators do not handle:

$$5.6 \times 10^9 =$$

5. Following are examples where calculators are useful:

1. $(0.1)^3 =$

- a) .3 b) 0.001 c) 0.003 d) 3.001

2. Evaluate 4^3

- a) 4 b) 12 c) 64 d) 81

3. $4^3 + 4^2 =$

- a) 20 b) 80 c) 1024 d) 4096

4. Which is the prime number?

- a) 39 b) 4 c) 49 d) 53

5. $3\frac{2}{5}$ is the same as

- a) 34 b) 0.34 c) 3.4 d) 52.3

6. $0.00523 \times 1000 =$

- a) 0.0523 b) 0.5230 c) 5.23 d) 52.3

7. 0.35 divided by 7000 expressed as scientific notation is

- a) 5×10^{-1} b) 5×10^{-5} c) $\frac{1}{20\,000}$ d) 0.5

8. $\frac{7}{20}$ is equivalent to

- a) 0.35 b) 0.72 c) 7.7 d) 20.7

6. Ratio problems can be solved by the unit method with a calculator.

e.g., A store is selling 5 bags of candy for 89 cents. If the regular price is 3 bags for 66 cents, how much is saved per bag by buying at the sale price?

- a) 23 cents b) 17.8 cents c) 4.2 cents d) 4.6 cents

7. For all problems, first write down the equation before you use your calculator to calculate the answer.

e.g., A merchant buys a product for \$200 and adds 25% to cover overhead and profit. The selling price is

- a) \$150 b) \$225 c) \$250 d) \$300

e.g., Bill's commission for selling a \$1200 car was \$60. His rate of commission was

- a) 5% b) 7.2% c) 12% d) 20%

8. Practice using the calculator in evaluating formulas.

e.g., $V = \pi r^2 h$

$r = 3 \text{ cm}, h = 7 \text{ cm}$

$V = l \times w \times h$

$l = 17 \text{ cm}, w = 5 \text{ cm}, h = 34 \text{ cm}$

$V = \frac{4}{3} \pi r^3$

$r = 1.4 \text{ m}$ (Note: Divide by 3 at the end of the calculation.)

9. The effect of altering elements in a formula can be tested by calculating.

e.g., $V = \frac{1}{3} \pi r^2 h$. If the radius is doubled and the height is halved, its volume will

- a) remain the same c) be four times as large
b) be doubled d) be eight times as large

10. In evaluating algebraic expressions, great care must be taken with negatives.

e.g., $x^2 - 5x + 4 =$ when $x = 3$

- a) -5 b) -2 c) 2 d) 5

11. In using a calculator always try to estimate your answers.

12. The calculator can be used to check answers arrived at in other ways.

e.g., Solve $3x + 5 = 14$

(Solve as usual, then test your answer with a calculator.)

Mathematics CAI in the Classroom

Judy M. Barnes

The purpose of this study was to evaluate computer-assisted instruction (CAI) materials developed to teach concepts of polynomials to Grade 12 students. These materials were designed (under contract from Alberta Education) by staff of the Apple Innovation Support Center (AISC)¹, at the University of Alberta, to meet the objectives of the Mathematics 30 curriculum (advanced diploma Grade 12 course) in use in Alberta. The polynomials unit from the CAI materials (about 10 percent of the 125-hour mathematics course) was studied.

Two high schools in a county in central Alberta (Gauss High School and Hilbert High School) were identified as having Macintosh computer laboratories (containing at least 25 Macintosh computers) with networks,² and were invited to participate in the study. The study involved observing the students and their mathematics teachers using the CAI materials for one-and-a-half weeks, and discussions with these participants.

Students at Gauss High School

Four of the 30 students from the class in Gauss High School were interviewed by the researcher. These students were selected by the researcher because they had asked questions of the researcher and shown a willingness to discuss the CAI materials while using them. The four students interviewed were Barb, Jan, Martin and Donald.

¹AISC is housed in the Faculty of Education, University of Alberta. AISC is directed by Dr. M. Petruk, professor with the Department of Adult, Career & Technology Education and executive director of Apple Canada Education Foundation.

²A local-area network, or network, is used to connect machines to a file server that contains information used by those connected to the network.

Barb had taken Mathematics 20 (the Grade 11 advanced diploma course) about one year before entering Mathematics 30, and indicated she sometimes found it difficult to remember the mathematics from the previous year. She was frustrated because the teacher usually reviewed the material from previous years, but the computer materials did not necessarily review material where she needed it. She stated that she was a good mathematics student although it was no longer her strongest school subject. She had never used a computer-teaching program before. However, she said she was comfortable with computers, and using the CAI materials did not affect her feelings toward computers.

Jan had taken Mathematics 33 (the Grade 12 general diploma course) in the previous semester. She stated that mathematics "is my worst subject. I don't catch on to it easy. I have to be taught it over and over again and have lots of practice." Research observations indicated she had difficulty using the computer, which seemed to be partly due to a weak mathematics background. However, she stated in her interview that she was comfortable using a computer, and thus using the CAI materials did not affect her feelings toward computers.

Martin had completed Mathematics 20 the previous year. Mathematics was a subject with which he felt comfortable. He was also comfortable with computers, although initially not with the Macintosh because he had not used one before. His initial reaction to the computer experience seemed one of frustration as he had difficulty accessing the CAI materials on the network. However, after using the Macintosh computer, he stated he was comfortable with it. Martin said that the use of the CAI materials did not affect his feelings toward computers in general but made him feel more positive about using Macintosh computers.

Donald had taken Mathematics 30 before and was repeating the course this term in the hope of improving his grade. Mathematics was one of the subjects Donald once felt comfortable with, but his marks had been "going downhill" since Grade 10. Donald was comfortable using computers and stated that "all a computer really is, is just a tool. It depends on what's in it, the program." He stated that the use of the CAI materials did not affect his feelings toward computers.

Reaction to the Materials

All students were asked whether or not they enjoyed using the CAI materials. Barb responded, "Not really," adding that she found the material "hard to understand. I had to go home at night and reread everything in that little book³ and take notes myself." She implied that she would have preferred the teacher's lessons and the teacher's notes.

When asked if the CAI materials affected her feelings toward mathematics, Barb responded "Yes. I was getting frustrated." She added, "I just need somebody to show me step-by-step. . . . I just can't relate to a computer teaching me." When asked if she thought a computer was a good way to teach and learn Mathematics 30, she answered "No." In her opinion, changes to the computer-assisted materials should include the provision of step-by-step movement through the concepts, with an explanation of each step as the material is presented.

Would Barb recommend the use of the CAI materials to friends who wanted to complete Mathematics 30? She said that if her friends really wanted to use the CAI materials, then she would recommend it, but not if they were struggling with mathematics. Barb indicated that she struggled with the CAI materials. This occurred despite the fact she is a strong mathematics student. She suggested that she may have struggled because she had not taken mathematics for one-and-a-half years and may have forgotten a great deal. Barb thought she needed to review notes from her previous mathematics course and the CAI materials did not provide enough review for her, especially in factoring and expanding polynomials. She felt the CAI materials "jumped right into the mathematics material," and she was left saying, "Wow! Where am I?"

³By "little book," she refers to the print materials provided to students containing most of the same information as the CAI materials.

Jan reacted differently from Barb to the CAI materials. She thought they were a good way to teach and learn Mathematics 30, as long as help was available from the mathematics teachers or other students. She would recommend them to friends wanting to complete Mathematics 30 only if they were good at mathematics. She said, "It can be fun if you are good at math." On the first day, she found the CAI materials difficult, although "kind of neat." The graphs were enjoyable, but after a while she "got kind of sick" of the CAI materials. She thought they were "good" because she "could get help," but there was a lot to work through, so she had to take notes. She also thought that help from other students was not always sufficient because students don't always know the answers; teacher help was deemed necessary. She would have preferred regular classroom instruction over the computer CAI materials because she finds mathematics difficult and needs someone to help her learn mathematics. She explained: "The computer can't sit there and try to explain it to you in different terminology. It just has this one explanation. That's why a teacher can help."

Martin thought that using the computer was a "good way to review [material] and [to receive] help but . . . it's not a substitute for a teacher." He did not feel that the computer was a better way to teach and learn Mathematics 30 than the regular classroom. However, he would recommend the use of these CAI materials to friends wanting to complete Mathematics 30, for home study, for continuing education, or for review. Martin did not enjoy using the CAI materials. He said he "felt tempted to just whiz through it and then you have to work harder to try to understand." He said he "didn't really enjoy mathematics that much this week." He said he usually enjoys mathematics more than he did while using the CAI materials. He missed the one-to-one relationships developed in a classroom and also had some difficulty visualizing the concepts. When asked if the CAI materials could be used instead of a class in Mathematics 30, he replied, "It would need a better textbook to go with it." Rereading one explanation over and over again was not what was needed when one didn't understand. Alternative explanations, or the same ones reworded, were needed.

Donald thought the CAI materials were "okay," except for some programming errors that he mentioned several times. He said he "never really did like mathematics but [the CAI materials were] a change of pace instead of sitting there and listening to a teacher. . . . It was a refreshing change." He

thought the computer was a good way to teach and learn Mathematics 30 "as long as there was a teacher there who knew how to use the computer and who could help you out in trouble spots."

Because Donald had taken Mathematics 30 previously, he was asked to compare the regular classroom experience with the computer experience. He thought the CAI materials were easier for him because he'd taken Mathematics 30 before, and he knew basically what [he] was supposed to do. He wasn't sure it would have been easier if he had used Mathematics 30 CAI materials after only taking Mathematics 20 (the Grade 11 advanced diploma course). He would recommend the use of the CAI materials to friends for "extra help." He also said, "You could use it instead of a teacher as long as there was a teacher who knew what was supposed to be going on and could help you out with questions. . . . I'd like it." He also thought that it could be used alone if a user's manual or reference guide were provided to "tell you what you are supposed to be doing."

It appears that this group of students did not enjoy the CAI materials much and thought that a teacher's assistance was essential for them to use them effectively. The need for a more individualized program of study and better resource materials was also indicated in students' comments. Despite these needed changes, the students recommended the use of the materials to others, provided teacher assistance was available and they were motivated and capable mathematics students.

Mr. Brown thought that using the CAI materials to teach his classes was a good and positive experience. However, he thought the CAI materials could not be used to the exclusion of classroom teaching. He said, "It's a nice supplement." From his discussions with students, he felt students "didn't seem that keen on it. They actually prefer, by their voice count, traditional spoon-feeding to having to read it off the screen. . . . They didn't like having to do the work on their own. They said they had to rush through it too fast." Despite students' reactions, Mr. Brown would use the unit again with his class but in a modified form. He feels he would "pull pieces" of the CAI materials together with his teaching and use them "for a couple of periods or half-periods." Mr. Brown also commented about his students:

I would say, in this group, maybe one student in five is a self-motivated and disciplined student who is probably at least as well off and maybe better

off to do it on the computer. He can take it in his own time frame. He can take it by himself and he can work through things at his own speed. It does cater to a person that has those kind of motivations and abilities but for the student that doesn't really accept the responsibility, doesn't want the challenges or basically needs the verbal feedback to any questions he has, the computer doesn't offer it.

The students and teacher were asked what they liked the most about using the mathematics CAI materials. Jan said she found it fun sometimes and liked doing the graphing. Barb also found it fun and "different from the classroom work." Martin and Donald did not answer this question. Mr. Brown "liked it" and thought the CAI unit was

a good attempt to take the polynomials unit in Math 30 and put it into a sequenced, self-directed learning [package] available to students. I think it's a good opening but it definitely needs some work yet. . . . For those kids who . . . work through the program, for whatever reason, on their own . . . it's a nice option available to them. . . . I don't know how useful it's going to be in a regular classroom [but for] a school where you have totally individualized instruction time it may be more suitable.

The four students interviewed thought they had learned enough from the researcher's introductory demonstration to use the CAI materials. However, Martin had some trouble using the network on subsequent days and suggested that some notes be provided, possibly in the form of a guide, that students could follow to accomplish the start-up procedure. Mr. Brown, however, did not make much use of the user's guide provided because "so much of what's in the guide is actually available on the computer so you don't really need it." However, because the students seemed to need a guide, and the teacher did make some use of the guide, the researcher developed a version of the user's guide to use with future students. This was used at Hilbert High School.

The concerns and problems with the CAI materials that were reported to the researcher by Mr. Brown and his students were addressed and/or corrected by the researcher in the two weeks between use of the CAI materials by Gauss High School students and their use in Hilbert High School.

Students at Hilbert High School

The class at Hilbert High School had completed two units of mathematics guided by the teacher

before beginning use of the CAI materials so they were eager for a change in routine. From this class, four students were interviewed: Carol, Paul, Mark and Cindy.

Carol was the only student who had not used a computer before this experience, and she said she was not initially comfortable using the computer. However, after using the CAI materials she became comfortable using it and her feelings toward computers were positively affected. The other students were all comfortable with computers before using the CAI materials and observed no effect on their feelings toward computers as a result of using these materials.

Carol was a good student in mathematics, but she found the CAI materials difficult to understand and "get through." She had completed Mathematics 30 the previous semester, but, as a result of trauma, had lost much of her memory of previous mathematics courses. She was enrolled in Mathematics 31 (the introductory calculus course), and through this course and a second try at Mathematics 30, she was recovering much of what she had lost.

Paul said he was comfortable using computers and used them as much as he could. Mathematics was not, however, a subject he felt comfortable with. Paul had taken Mathematics 33 (the general diploma Grade 12 course) before entering Mathematics 30.

Mark was comfortable using computers as his family had computers at home. Using CAI materials did not affect his feelings toward computers. Mathematics was a subject with which Mark felt comfortable, but he indicated he was not a strong mathematics student. He had taken Mathematics 33 last semester for extra credits, and it had helped his progress in Mathematics 30.

Cindy, the fourth student in Hilbert High School, entered Mathematics 30 after completing Mathematics 20 (the advanced diploma Grade 11 course) in the previous year. She said that she was a strong student in mathematics and other subjects. She also said that she enjoyed using computers, so using the CAI materials had no effect on her feelings toward computers. She had never used a computer-teaching program before but liked learning mathematics this way.

Reactions to the Materials

Carol enjoyed using the CAI materials and thought "it was fun, I could understand it." This resulted in a more positive reaction to mathematics which she

says is "usually boring." She thought that the computer was a good way to teach and learn Mathematics 30 "as long as a teacher [was] there to help in understanding." Carol thought that using only the computer to complete Mathematics 30 might also get boring. Doing both, that is, using a computer and being in a regular classroom, might be the best arrangement. She recommended using CAI materials to friends. She said, "It's so different. It gives you a new perspective and a new way of thinking about doing it."

Paul thought the CAI materials were "all right." He said, "The computer way [of teaching and learning Mathematics 30] is pretty good. With computers, it's pretty well self-explanatory." He preferred using the CAI materials to the regular classroom instruction because he could "go back and review" material which would be less possible in a classroom. Because of this preference, Paul would recommend using CAI materials to other students.

Mark thought the CAI materials were "pretty good." He felt he paid attention when using the computer but was often distracted by other students in a regular classroom. The CAI materials made "it a bit more fun to come to the math class." He thought the computer was a good way to teach and learn Mathematics 30, and he would like to do another unit by CAI materials rather than in a regular classroom. He thought he would probably do better with the CAI materials because he would pay attention.

Cindy liked using the CAI materials to learn mathematics. She liked working at her "own speed." She said, "I like to have a teacher around to help but I liked doing most of it on my own." She liked having help when she needed it because she "might have gotten frustrated" if she had to do it all at home. She said that the method of computer presentation "might be a better way for students who like to work on their own, but for students who need constant teacher supervision they might not like it." She also said, "I think it would be neat to possibly do the whole course [this way]." She would recommend the CAI materials to friends wanting to complete Mathematics 30, especially if they were taking it by correspondence.

The reaction of students in Hilbert High School was positive toward the CAI materials. All four students would recommend the use of the computer to friends, and all would like to do more Mathematics 30 using the CAI materials. Despite differences in students' mathematics background and perceptions of computers, these students were positive about the experience.

Mr. Carson thought that using the CAI materials to teach his classes was a good and positive experience. However, he thought the CAI materials could be used to the exclusion of classroom teaching. He thought that a "percentage of the time" in Mathematics 30 could be used for computer presentation of information. He said he "could be uncomfortable with an entire five-month block to do the entire course [by computer]." He added:

The computer was as good a way to teach and learn Mathematics 30, if not a better way, given a fairly structured topic. I think some interaction, doing some sections in the classroom time, would help and then maybe doing more [by computer]. I base that on some of the questions I am getting. These students have gone through sections of the program and obviously have missed a primary or secondary point and yet are still working on questions and doing reasonably well. In class we could catch this. . . . I think trying to get the best of both worlds should be the focus. . . . For some instances, for some individuals, [using the computer] would be better [than regular classroom instruction] but not for all. For kids who like to work at their own pace and are sufficiently literate [this would be better]. . . . This is always a concern with any materials of this kind. The reading ability determines the success.

Mr. Carson said he would use the CAI unit again in whatever form seemed to fit his course because he thought that his students reacted positively to it. Some students did express concerns and were frustrated using the CAI materials, but they were among those identified by Mr. Carson as being frustrated in his regular classes.

In reaction to a question of what he liked about the CAI materials, Mark stated the materials "broke the monotony of sitting in class and writing notes [copied] off the blackboard. It made it more exciting to go to math class. We were doing something nobody else was doing. It was fun." Carol thought it was different. She was not used to using a computer but found it enjoyable. She liked some of the graphics. Cindy liked working at her own pace and doing mathematics individually but also emphasized the need for the teacher's help when she "got stuck." She wouldn't want to use the CAI materials without teacher assistance. Paul thought mathematics was easier to do on the computer using the CAI materials and liked the fact that it was "up to me to get the information and make sure I learn."

According to Mr. Carson:

[I] liked the students' ability to move at their own pace and to be forced, for the bottom third [of the students when grouped by ability], into doing and following sequentially through a thought to get through a particular idea or topic as opposed to just going through exercises without being concerned about what they were learning or supposed to be doing. Even though it seemed that there was a fair bit of teacher input as far as individual questions [were concerned], students did have the opportunity to have their questions answered directly by the program—whether moving backward or forward or having to sit there and think for a few minutes to answer their question. That is the strength of CAI in general.

Students were asked what they disliked about the CAI materials. Paul couldn't think of anything. He thought the CAI materials were "really good" and he enjoyed the experience. He would be willing to do another unit by computer if asked to do so.

Cindy disliked the fact that the CAI materials "gave a lot of information that you didn't really absorb. . . . I found that I was just beginning to get the hang of it and we went on to a new topic or more examples. I could have used more practice." She also would complete another unit by computer if asked to do so because she "liked this unit."

Mark disliked the fact that the computer was slow but felt he may have been comparing the speed of the CAI materials on this network (with 25 users) to that of a computer at home (which he alone used). However, despite the slowness of the CAI materials, he would recommend them to friends doing Mathematics 30 and would definitely be willing to do another unit of the CAI materials.

Carol also criticized the CAI materials for being a "little too slow. . . . The computer takes so long for your reaction, for your answers to be in. You just sit around waiting and then it goes on." However, she would be willing to do another unit by computer. She also would recommend the CAI materials to others in Mathematics 30 because she says, "Students in class would benefit from it." She based this on the fact that "students in the other class didn't know some of the stuff and it was surprising I knew all this stuff. They asked me questions and I knew the answers. It's a good way to learn."

Mr. Carson said, "I don't have any major dislikes. The program works quite well. Nothing really stands out. There are a few errors, rough edges, I guess,

but nothing serious.” However, he made recommendations for using the CAI materials:

. . . For individual students, it would depend a lot on the student. Some would make out better than others and get more out of it. I would certainly think that if I have the opportunity to do a portion of the course on computer that I would use it.

He would recommend the CAI unit to a student who could not be scheduled into the course as opposed to recommending Alberta Education’s correspondence materials. He thought it was better than correspondence materials “without question!”:

Any student that was working individually for any reason, [for example, if] they were sick for a month or were trying to fit in a course that wasn’t available to them, this would be far superior to correspondence lessons.

He felt that, if available, it would be appropriate to use this unit with students who had been away from school, even if the class wasn’t doing this unit on the computer: “Given the chance, I would make sure that all students in the course were comfortable with the program so that they could use it” and had the necessary skills to use the CAI materials independently. He would then use it for students who needed the extra material:

It would be useful for remedial, as well as first-time, exposure. Its [the CAI unit’s] major strength in comparison to other things that are available is that it’s matched to the curriculum very well. There are other things out there, but by the time you

figure out where they fit in and try and convince the kids this is the same topic that they are talking about, only discussed from a different vantage point, the point is lost. This will be very useful.

After using the unit during the research project, Mr. Carson went on to acquire another unit of the CAI materials and used it with his class because he wanted to experiment further. He continues to use the CAI materials used in the research and the other CAI units with his classes.

Conclusions

Three main points are apparent from these discussions with students and teachers.

1. Students are not comfortable using the CAI materials without the assistance of a teacher. They need a teacher to help them understand the mathematics.
2. The students interviewed see the use of the CAI materials as valuable for enhancing their learning of mathematics and motivation to learn mathematics. They even used the word *fun* when describing some of their activities using the CAI materials.
3. The teachers view the CAI materials as useful for students unable to attend mathematics class, for students who need mathematics review, and for classes of students to learn concepts in mathematics. However, the teachers do not feel that the CAI materials can be used alone but that teacher intervention in the process enhances the learning of mathematics.

Variations on a Theme

John Percevault

Problem solving is a thinking process, and both critical and creative thinking can be involved. Critical thinking involves analysis of conditions and often depends on the mastery of vocabulary, the technical words and symbols of mathematics. Creative thinking occurs when students develop problems or are given an opportunity to describe conditions.

Altering a Problem

A problem is altered in at least three different ways:

1. The conditions of the problem are changed.
2. The variable or variables are changed.
3. The information sought is changed.

Consider the following example:

Basic Problem

1. I'm thinking of a number (a counting number).
2. The number is less than 50 (< 50).
3. It's also greater than 20 (> 20). ($20 < n < 50$)
4. The number is odd ($2n + 1$).
5. The number is a multiple of 5. ($5x = n$) (5 is a factor.)
6. The sum of its two digits is 8. (7 and 9 are other alternatives.)

Too often, once the solution to a particular problem has been found, the problem is forgotten. In reality, the stage for effective problem-solving process has been set. There is a difference between solving a problem and teaching the problem-solving process. An analysis of possible process follows.

Analyzing the Problem-Solving Process

Condition I

Grade 3 students will attempt to guess the number the teacher has in mind. Instead of moving to the second condition, explore such questions as:

You guessed 33. Why did you choose that number?
How many numbers are there?

Do you think you could ever guess the number I have chosen?

What other information do you need?

Condition II (< 50)

The choice of 50 is completely arbitrary. The condition of the upper limit may be varied.

How many ways may the number 50 be stated? Each poses the upper limit in a different mathematical form.

What does "less than" or its symbol mean to you? Developing synonyms in the students' language broadens and refines the meaning of the word and the ideograph $<$.

How many counting numbers are there that are less than 50? The usual answer is 50.

Could you trap me now?

What would you do to trap me?

Condition III ($n > 20$, $20 < n$)

Again, the definition of 20 or any other number the teacher chooses may be varied. Synonyms should be developed and the range defined in words or in symbols ($20 < n < 50$).

What would you do to find my number?

Where would you start?

Why?

Condition IV (Odd Number)

What does odd mean?

Can you list?

Before listing, determine how many odd numbers there are between 20 and 50 ($20 < \text{odd} < 50$)? Some

Questions noted in previous conditions may be appropriate. (At this point, allow students to develop final conditions for a number of their choice.)

Condition V (Multiple of 5)

How many odd numbers that are multiples of 5 are there between 20 and 50?

Condition VI (Final Clue)

The final clue the teacher chooses will depend on the grade and the mathematics ability of the students in the class. If the number chosen was 35 (the other possible answers are 25 and 45), the following types of clues would be appropriate:

- The number is between 34 and 36.
- The number is also a multiple of 7.
- Seven is a factor of the number.
- The number is one less than a perfect square.
- The difference between the two digits is 2.

The number is the difference of the squares of two numbers.

The number is the product of 8 and 4 increased by 3.

Condition V may be altered. Consider the following:

Clue 1: The number is prime (e.g., 41).

Clue 2: The number is the sum of two consecutive numbers, and the first is the product of 4 and 5.

Clue 3: The number is the sum of two consecutive counting numbers.

Change Condition VI by stating the numbers.

Ask the students to provide the final clue.

Summary

Students' creative thinking may be fostered by having students develop similar problems. The problem-solving process is consistent with the Alberta curriculum which focuses on thinking, continuity and integration. Additionally, it is challenging and fun for the students and the teacher.

Motion Geometry with the Apple IIe

A. Craig Loewen

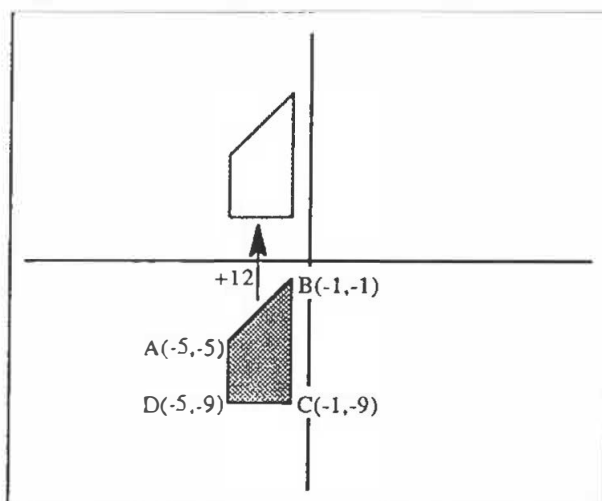
The computer can be used effectively as an environment in which mathematics may be discussed and learned; however, the key to such a use of the computer is the teacher's or programmer's ability to pose problems that challenge students' conceptualizations of mathematical ideas. The teacher-made program presented below is an attempt to construct a computer-learning environment to facilitate the instruction of transformations.

In this application of the computer, transformations are represented as the manipulation of a two-dimensional shape within a Cartesian coordinate system. The program functions by accepting as input the vertices of a closed figure, and these vertices are translated, reflected and rotated. For example, figure ABCD can be translated upward 12 units simply by adding 12 to the ordinate of each vertex of the figure (Figure 1). Thus point A (-5,-5) would translate to point A' (-5,7). Likewise, this constant value (12) would be added to each of the ordinates of the points B, C and D.

It may be worth pausing at this point to consider the following question. What manipulation(s) should be performed on either the abscissa or ordinate (or both) to achieve the following translations:

- Translate figure ABCD downward 2 units (*subtract 2 from ordinates*)?
- Translate figure ABCD 6 units to the right (*add 6 to abscissas*)?
- Translate figure ABCD 1 unit left and 2 units up (*decrease abscissas by 1, increase ordinates by 2*)?
- Reflect figure ABCD about the x-axis (*multiply ordinates by -1*)?
- Reflect figure ABCD about the y-axis (*multiply abscissas by -1*)?

Figure 1. A Basic Figure Translated Upward 12 Units



It is important that questions like these are presented to, and discussed with, students at some point during this exercise. Such discussions encourage generalization of the mathematical processes carried out by the computer.

Getting Started

Motion Geometry is written in Apple BASIC and is intended for the familiar Apple IIe computer. The program is listed in Figure 2 and must be typed in to begin the activity. Once the program is entered into the computer, be sure to save it so that you will not have to type it again the next time you want to use it.

To begin the program, you simply type RUN and press return. This will clear the screen and prepare you to input your vertices. The x- and y-axes of a Cartesian coordinate system are drawn at the top of the screen, and instructions are given at the bottom of the screen.

Figure 2. Motion Geometry Program

```

10 REM WRITTEN BY A. CRAIG LOEWEN
20 REM NOVEMBER 17, 1986
30 REM MOTION GEOMETRY
40 DIM A(20,2),B(20,2)
50 HGR
60 HCOLOR= 2
70 HOME
80 HFLOT 140,0 TO 140,160
90 HFLOT 0,80 TO 279,80
100 HCOLOR= 6
105 FOR X = 1 TO 20:A(X,1) = 0:A(X,2) = 0:B(X,1) = 0:B(X,2) = 0: NEXT
110 VTAB 21
120 PRINT "INPUT CORNER POINTS IN CLOCKWISE ORDER":K = 2
130 INPUT "X,Y: ":R,S
140 A(1,1) = R:A(1,2) = S
145 HFLOT R * 6 + 140,S * ( - 4) + 80
150 INPUT "X,Y: ":A(K,1),A(K,2)
160 HFLOT TO A(K,1) * 6 + 140,A(K,2) * ( - 4) + 80
170 IF A(K,1) = R AND A(K,2) = S THEN 200
180 K = K + 1: IF K > 20 THEN K = 20
190 GOTO 150
200 FOR X = 1 TO K:B(X,1) = A(X,1):B(X,2) = A(X,2): NEXT X
205 HOME
210 VTAB 21: PRINT "OPTIONS: ";; INVERSE : PRINT "Q":; NORMAL : PRINT "
UIT ":
211 INVERSE : PRINT "N":; NORMAL : PRINT "EW ";
212 INVERSE : PRINT "C":; NORMAL : PRINT "ONTINUE"
220 GET A#: IF A# = "" THEN 220
230 IF A# = "Q" THEN TEXT : HOME : END
240 IF A# = "N" THEN RUN
250 IF A# < > "C" THEN PRINT CHR# (7):; GOTO 220
270 VTAB 21: PRINT "OPTIONS: ADD":; INVERSE : PRINT "X":; NORMAL : PRINT
" ADD":
271 INVERSE : PRINT "Y":; NORMAL : PRINT " S";
272 INVERSE : PRINT "I":; NORMAL : PRINT "GNX SI":
273 INVERSE : PRINT "G":; NORMAL : PRINT "NY ";
274 INVERSE : PRINT "S":; NORMAL : PRINT "WITCH"
280 PRINT " ";; INVERSE : PRINT "D":; NORMAL : PRINT "RAW ":
281 INVERSE : PRINT "Q":; NORMAL : PRINT "UIT ";
282 INVERSE : PRINT "C":; NORMAL : PRINT "LEAR ";
283 INVERSE : PRINT "N":; NORMAL : PRINT "EW"
285 GET A#: IF A# = "" THEN 285
290 IF A# < > "X" THEN 340
300 VTAB 23: HTAB 1
310 INPUT "CONSTANT VALUE: ":V
315 VTAB 23: HTAB 1: PRINT " ";; VTAB
23: HTAB 1

```

```

320 FOR X = 1 TO K:B(X,1) = B(X,1) + V: NEXT X
330 GOTO 285
340 IF A# < > "Y" THEN 390
350 VTAB 23: HTAB 1
360 INPUT "CONSTANT VALUE: ":V
370 FOR X = 1 TO K:B(X,2) = B(X,2) + V: NEXT X
375 VTAB 23: HTAB 1: PRINT "                " :: VTAB
23: HTAB 1
380 GOTO 285
390 IF A# < > "C" THEN 480
400 HGR : HCOLOR= 2
410 HFLOT 140,0 TO 140,160
420 HFLOT 0,80 TO 279,80
430 HCOLOR= 6
440 HFLOT A(1,1) * 6 + 140,A(1,2) * (- 4) + 80
450 FOR X = 2 TO K
460 HFLOT TO A(X,1) * 6 + 140,A(X,2) * (- 4) + 80: NEXT X
465 FOR X = 1 TO K:B(X,1) = A(X,1):B(X,2) = A(X,2): NEXT X
470 GOTO 285
480 IF A# < > "D" THEN 530
490 HFLOT B(1,1) * 6 + 140,B(1,2) * (- 4) + 80
500 FOR X = 2 TO K
510 HFLOT TO B(X,1) * 6 + 140,B(X,2) * (- 4) + 80: NEXT X
520 GOTO 285
530 IF A# < > "N" THEN 560
540 RUN
560 IF A# = "Q" THEN TEXT : HOME : END
570 IF A# < > "I" THEN 620
580 FOR X = 1 TO K
590 B(X,1) = B(X,1) * (- 1)
600 NEXT X
610 GOTO 285
620 IF A# < > "G" THEN 670
630 FOR X = 1 TO K
640 B(X,2) = B(X,2) * (- 1)
650 NEXT X
660 GOTO 285
670 IF A# < > "S" THEN PRINT CHR# (7) :: GOTO 260
680 FOR X = 1 TO K
690 T = B(X,1):B(X,1) = B(X,2):B(X,2) = T
700 NEXT X
710 GOTO 285

```

The program is now waiting for you to input the vertices of your two-dimensional figure. These vertices should be entered in a clockwise order. Vertices are entered simply by typing the value of the abscissa and then of the ordinate, separated by a comma. Be sure you hit return only after entering the value of the ordinate. You are allowed to enter a maximum of 20 points, but the last point you enter should close the figure. This is done by ending with the first vertex you input. To enter figure ABCD (Figure 1), you would type:

-5,-5 (*hit the return key*)
-1,-1 (*return*)
-1,-9 (*return*)
-5,-9 (*return*)
-5,-5 (*return*)

Notice that the first and last entry are the same; this closes the figure. The computer will draw each edge of the figure on the top part of the screen as the vertices are entered.

When you have entered a closed figure, you will be given three options to choose from: QUIT, NEW or CONTINUE. You select your option simply by pressing the key highlighted on the screen (Q for quit, N for new and C for continue). If you choose to QUIT, the screen will be cleared and the program ended. To restart the program, you must type RUN and begin again from the start. If you select NEW, the screen will be cleared, and you will be asked to enter new vertices for your figure (you would select NEW if you made an error in entering the vertices). If you select CONTINUE, you will be admitted to the second part of the program where you will manipulate your figure through the various transformations.

Transforming Your Figure

At this point in the program, you will be asked to select commands that will complete the various transformations on your figure. Two sets of commands are needed. The first set of commands facilitates the transformations, and include ADDX, ADDY, SIGNX, SIGNY and SWITCH. The second set is used to control the screen and exit the program and includes DRAW, NEW, CLEAR and QUIT. To select a command, simply type the highlighted character. Each command is explained below.

ADDX: This command is used to add a constant to the abscissa of each vertex of your figure. After selecting this option, you will be asked to input the

integer value of the constant. You may wish to experiment and determine the domain of the coordinate system displayed on your screen.

ADDY: This command is used to add a constant to the ordinate of each vertex in your figure. Again, you will be asked to input the value of the constant. More experimentation may be necessary to determine the range of the system shown on your screen.

SIGNX: This command simply multiplies the value of the abscissa of each vertex by -1. The effect of the SIGNX command is to complete a reflection about the y-axis. For example, point B (-1,-1) becomes B' (1,-1).

SIGNY: This command multiplies the value of the ordinate of each vertex by -1. The effect of SIGNY is a reflection about the x-axis. For example, point C (-1,-9) becomes C' (-1,9).

SWITCH: This command switches the abscissa and ordinate in each vertex. For example, point D (-5,-9) becomes D' (-9,-5). The effect of this move is left for the reader to determine by experimentation.

The reader may have noticed that nothing has yet changed on the screen regardless of what command has been tried. This is deliberate. To view the transformed figure, the DRAW option must be selected. This delay has several advantages: (1) it allows the joining of a series of manipulations before the image is shown; (2) it challenges the predictive capabilities of the students; and (3) it facilitates the development of visualization skills. The remaining commands are discussed below.

DRAW: This command simply sketches the image of the figure according to the transformations selected. None of the other figures previously drawn are erased; the image is simply added to those already shown on the screen.

NEW: When selected, this command clears the screen and takes the student back to the first part of the program where the vertices of the figure were entered. It is used when the student wishes to start over with a new figure.

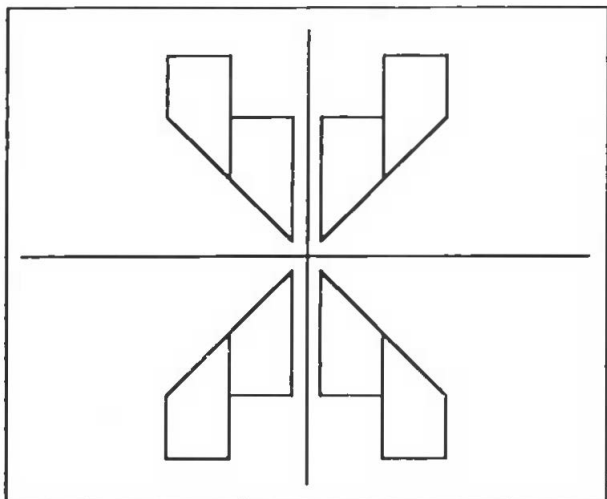
CLEAR: This command simply clears the screen and redraws the original figure and is ready to accept new transformations. It would be used when the screen becomes cluttered.

QUIT: When selected, this command clears the screen and exits the program. To restart, the student types RUN.

An Example

Problem: Using the basic shape and coordinates found in Figure 1, create the diagram shown in Figure 3.

Figure 3. A Design Built from ABCD in Figure 1



The first series of moves will be a series of reflections. If we reflect the figure about the y-axis, we can get the shape in the fourth quadrant closest to the origin. This reflection is achieved by selecting SIGNX. Remember to select DRAW to view the translated shape. Now, we need to reflect the new shape about the x-axis. This can be achieved by selecting SIGNY followed by DRAW. To get the

fourth shape, we complete another reflection about the y-axis and so select SIGNX and DRAW. We now have all four of the shapes closest to the origin.

The last shape we drew was in quadrant two. We must now translate this shape left four units and up four units. This move may be accomplished by selecting ADDX with a constant value of -4. We do not draw the figure yet, as it is not fully positioned. Next, we ADDY with a constant value of 4. We are now ready to DRAW.

To finish our design, we need to make one final set of reflections. First, we reflect the figure about the y-axis using SIGNX and DRAW. Next, we reflect about the x-axis using SIGNY and DRAW, and finally, we reflect about the y-axis, again using SIGNX and DRAW. Our design is now complete.

This simple example can be easily extended to create a richer problem-solving environment by asking questions such as: Is there an easier or faster way? Can this task be completed in fewer steps? Does the order of the steps matter? The ability to answer these questions requires a genuine understanding of the processes involved in the translation of figures about a Cartesian coordinate system.

Problems and Questions

The first set of problems is found in Figure 4. The coordinates for the shaded figure are provided. Using only translations, reflections and rotations, create each of the patterns.

Figure 4. Motion Geometry Problems

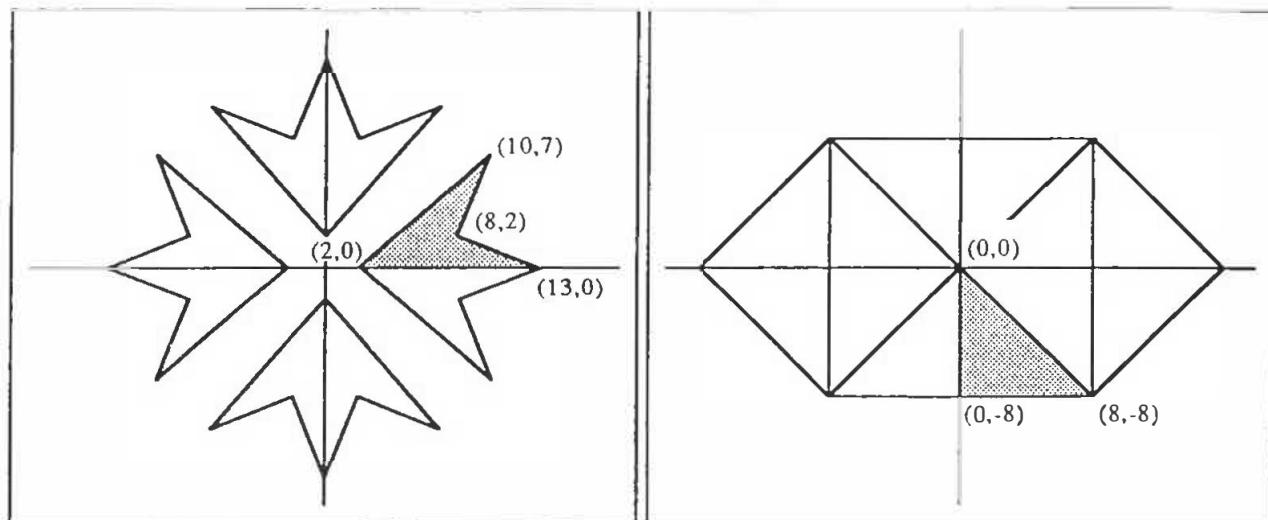
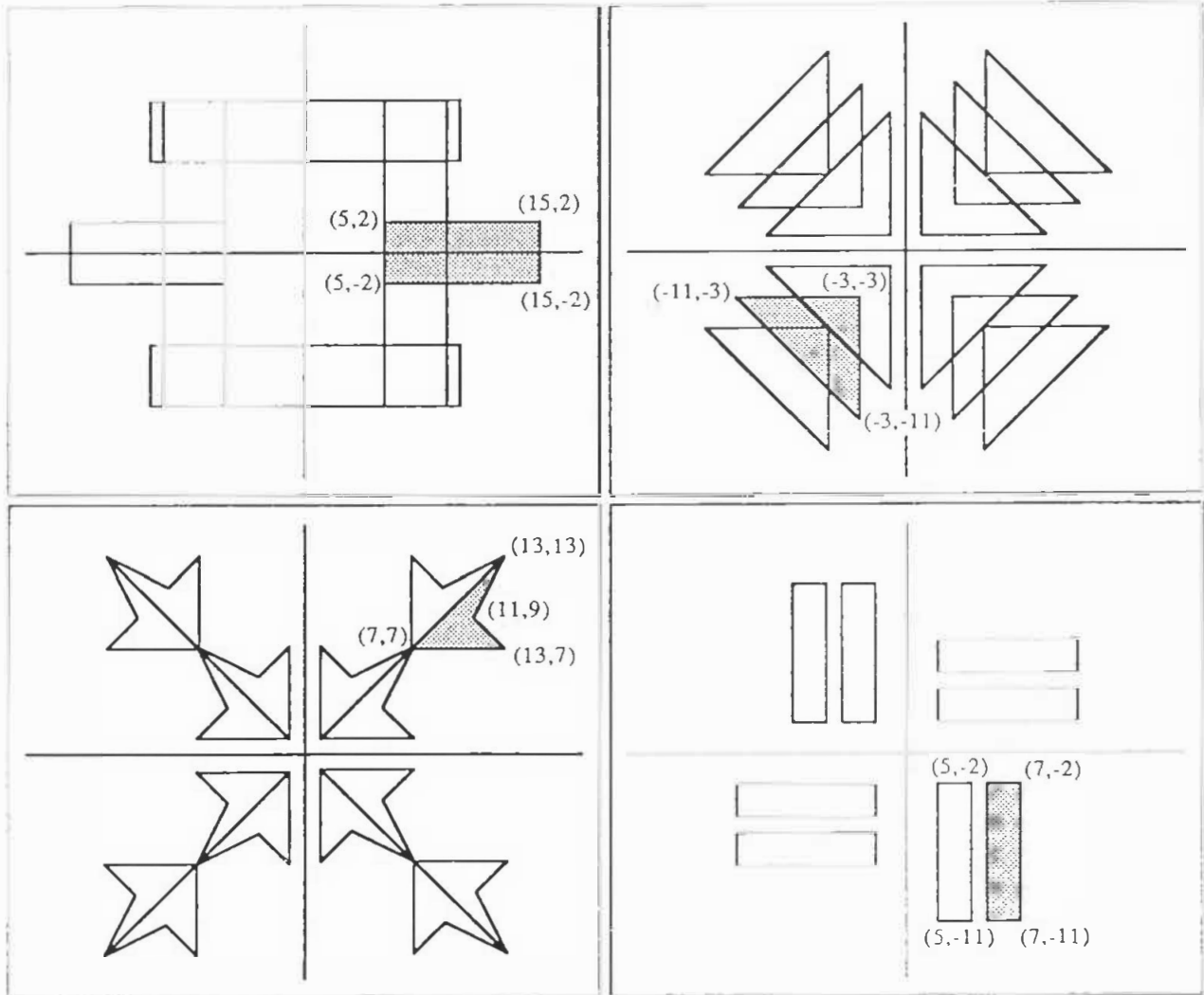


Figure 4. (continued)



Once you are familiar with the manner in which shapes and figures are manipulated, see if you can answer the questions below:

- How is a quarter turn clockwise different from a quarter turn counterclockwise? What is the fastest way to get a half turn?
- How can you reflect a figure about a horizontal or vertical line that is not an axis? For example, try reflecting figure ABCD shown in Figure 1 about the line $x = 5$.

- How can you rotate a figure one-quarter turn counterclockwise about a point that is not the origin? For example, try rotating figure ABCD in Figure 1 about the point $(4,2)$.

A Little Hint

Beginning with triangle ABC with points A $(2,1)$, B $(10,1)$ and C $(10,5)$, what is the effect of the following commands used together in order: SWITCH SIGNX DRAW?

The New Grade 10 Mathematics Curriculum

John Heuver

The new Grade 10 mathematics curriculum was introduced in the 1989-90 school year, adding fresh problems to those that existed in the former program of studies. Students entering Grade 10 have virtually no knowledge of algebra. In the past, it was possible to review briefly with the abler students the elementary algebraic operations and continue with a more in-depth exposition of the old material before expanding on it because they had been exposed to algebra in junior high school. This is no longer the case.

Important drawbacks exist in introducing algebra to students as late as Grade 10. It takes a considerable amount of time before a student becomes familiar with the elementary manipulations of algebra and develops an understanding of such things as why letters are used instead of numbers. As a teacher this year, I had the feeling of being almost a whole year behind at the beginning of the new program. This might be slightly overstating the case but it took almost six weeks or 35 percent of the semester to familiarize students with algebraic skills. Meanwhile, weaker students had lower chances of surviving a course that was completely new to them because of the amount of material that had to be covered in a given time.

When the new Grade 10 course was field-tested, the old junior high curriculum was still in place, so some of these problems were not evident at that time. Beyond the math classroom, Grade 10 students who take sciences need mathematical skills unless they study science as an appreciation course and have no need for calculations. Science teachers will find themselves in the unenviable position of having to teach a lot of remedial mathematics, especially in the physics program. The new science program will have to take into account the mathematical weaknesses of students.

For the less academically inclined students, the problems have become even more difficult. The number of high school credits in academic subjects has been increased to the point where it is almost impossible for these students to take courses in vocational subjects. Such students are unmotivated to study a subject like mathematics if there is not some clear reason why they should do so, and are not moved by the argument that they might need mathematics in the distant future for a training program they might wish to take. The vocational program was supported and funded in the past chiefly by the Federal Government, and Alberta Education has never had its heart in it. This is no doubt one reason why the current curriculum review of vocational programs in high school has been so poorly done.

Some key problems in course content seem related to the review process itself. Courses were planned in a fragmented way. Committees were struck and decided what topics should be covered in a specific course. This may look superficially to be a clever strategy. However, when it comes to selecting a textbook, it is virtually impossible to find a unifying thread that gives coherence to the text from a mathematical as well as a pedagogical viewpoint because the content has been predetermined along other lines. A committee that is bent on designing its own course content is in trouble from the start. From the publishers' perspective, the method works well because, to satisfy the needs of the different provinces, they just have to paste an extra chapter in the textbook and make a few small additions here and there. This makes economic sense to a certain extent. On the other hand, it means students and teachers have to carry textbooks that weigh like bricks—many of their pages are irrelevant—and are unnecessarily expensive. As well, they are poorly organized, with units that are in the prescribed curriculum scattered

throughout the text. Even the curriculum guides are written like manuals for a badly-designed computer program and are hard to obtain because of the cost.

A number of topics have been overlooked in the fragmented approach to building the curriculum. For example, simple equations like $4x = 12$ are simply omitted in the Mathematics 10 and 13 courses. This must be ascribed to the advent of curriculum development. A coherent textbook would not have missed this. Geometry (except for analytic geometry) has been omitted. In *The College Mathematical Journal* (March 1991), Reuben Hersch says, "Acquaintance with Euclid is indispensable in the understanding of mathematical philosophy of centuries earlier than our own." This omission might be less crucial to the Mathematics 13 student than it is to

the more academically inclined student because the latter needs a broader understanding of our cultural heritage. At the level of international competition, our students are going to be no match for students of other nationalities.

The new program has been introduced with the motto *problem solving*. The problem-solving approach to the curriculum can only work if more Alberta teachers contribute to problem sections of mathematical journals. This is the only place where one can learn the art of problem solving in the mathematical sense because it is impossible to teach a technique with which one is unfamiliar. Any form of innovation in teaching mathematics must come from the practicing teachers themselves. Only teachers can make a curriculum work!

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