Actions and Fraction Patterns

Thomas E. Kieren

If whole numbers allow us to look at the world through the lens of *how many*, fractional numbers allow us to look at it through the lens of *how much*. The materials such as those shown here allow children to explore an image of fractions that says *fractions are numbers which show amounts*. The fraction activity called Covering Fractions, described and discussed here, is for young children and was used with Grade 3 students.

If one looks at the rational numbers mathematically, they form a field and have a formal additive and multiplicative nature. The same is true for fractional thinking in children. Before using the materials here which consider fractions as amounts (additive), it is useful to have children explore fractions as actions which generate them (multiplicative). For example, before using the Covering Fractions set, children should fold a sheet identified as one unit into 2, 4, 8 and 16 equal pieces. In particular, they should explore and discuss such concepts as *one eighth is one half of one fourth*, and *I can get sixteenths in four folds*. Such actions allow children to experience the multiplicative, exponential growth of the number of parts, compensated for by the multiplicative, exponential decline of the part size. That is, children should join in discussing the fact that folding into sixteenths gives more (4 times more) parts than a fold into fourths, but that these parts are smaller (one fourth is made up of 4 sixteenths).

In terms of the pieces that result as a product of the fold, or the fractional amounts, children should consider two kinds of questions:

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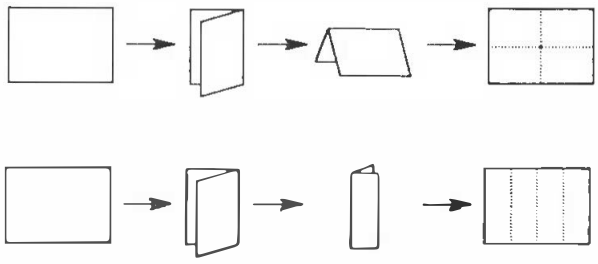
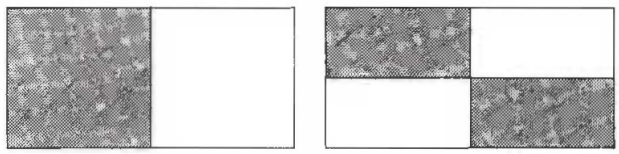
Can different fourth folds give the same sized *pieces?*



For children who do not conserve area, this is not a trivial sorting out; they should experiment and see that, at least for fourths, differently folded shapes can have the same size. It is size compared to the size of the unit that is important.

Can different combinations make the same amount?

A discussion of this idea could be started using the simple activity below:



Gina: "Those can't be the same: one is one half, the other is two fourths."

Theo: "Well, they make the same amount. You could just slide the pieces together."

Who do you think is right, Gina or Theo? Why?

Using Covering Fractions

Questions One and Two

The first two exercises in the set included at the end of this article introduce the idea of *covering* as *showing the same amount*. They also allow students to see and discuss the relationship between a unit and a part and the part and the unit. Children can be asked to complete these exercises orally or in writing or both. If work is done in writing, children should be encouraged to use either fraction words, for example, four eighths is one half, or symbols, 4/8 = ½. Different children will choose different options and some children will mix the use of words and symbols. What is important is what they say and how they reason about it.

Questions Three and Four

Items 3 and 4 allow students to act and talk about using different fractions to show the same amount. These exercises take advantage of the fact that children are aware of a general *equivalence* concept based on *is as much as*. Some coverings will use just one kind of piece. This action leads to the standard notion of equivalence. But the more general idea of *as much as* is a powerful notion that children should use and write and talk about.

Questions Five and Six

Items 5 and 6 generated much student discussion in the Grade 3 class. Children were able to evaluate coverings that they hadn't made. That is, students were able to imagine how another student had made a covering and comment on the work in physical and mathematical terms without actually doing the particular covering themselves.

Tom reported, "¾ = ¼ + 2/8 + 4/16."

Jodie said, "Oh he's right—each [fraction] makes one fourth."

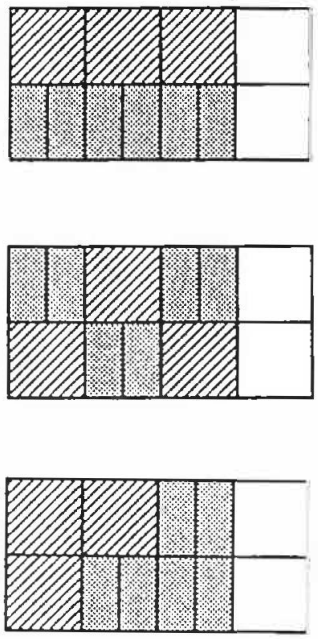
Here is another example of this kind of analysis.

George said, "Three sixteenths and one eighth and five sixteenths makes three fourths."

Kay responded, "No, that's one eighth too little. 8/16 is ½, you need two eighths more."

This activity allowed for the development of a healthy mathematical atmosphere where children generated, defended and discussed real mathematics.

Item 6 reflects student interest in the aesthetics of fraction patterns.



Students thought the middle pattern was "neat," while the covering at the top prompted two different discussions.

"¾ is 3 repeats of ⅛ and 2/16."

The above comment represents an intuitive sense of distributivity.

"¾ is made up now of half eighths and half sixteenths in this pattern."

Thus, in building fractional patterns, beauty and mathematical thinking go hand in hand.

Question Seven

Item 7 proved to be one of the most interesting and exciting. All except two of the children were able to generate, describe in fraction words or symbols, and defend their solutions to this pattern puzzle. These solutions allowed a discussion of order compared to one unit and also allowed students to compare coverings with one another.

"½ + 2/4 + 1/16 is closer to 1 than ¾ + ⅜. Yeah, it's 1/16 closer."

One numerically-facile boy wrote the following on the board:

½ + ¼ + ⅛ + 1/16 < 1

At this, the whole class blurted out:

"Oh yeah, only 1/16 more gets you to one," or something like it.

The teacher then asked the boy if he could make a covering which was even closer to one. He said,

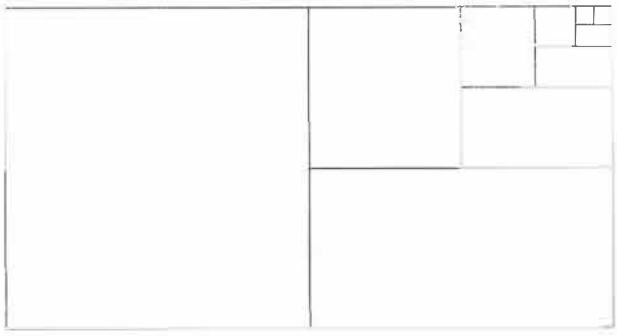
"Yup. Add 1/32."

To which Stacey added: "Then you could just cut a 1/32 piece in half and add it, but I don't know how to say it." A chorus of 10 children saying "one sixty-fourth" was heard in response. About six children pursued this further and reported, "We've got it out to 1/1024."

One then wrote on the board:

½ + ¼ + ⅛ + 1/16 + 1/32 + 1/64 + 1/128 + 1/256 + 1/512 + 1/1024

One of the group drew a picture of the covering as follows:



Aaron responded, "There'd just be a dust mote left."

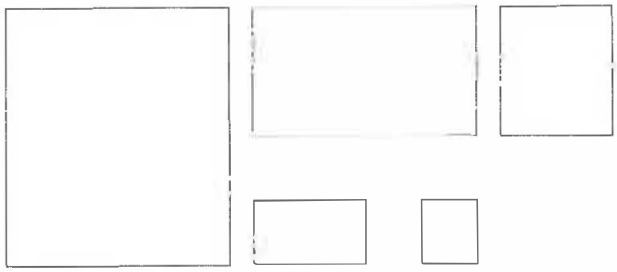
Summary

The materials discussed here allow children to actively build and discuss ideas of fractions. In this work they come to see an important property of fractions: different combinations of fractional numbers make the same amount. Using their own local individual properties, they looked at ideas such as equivalence, order, addition, fractions more than one and even the beginnings of patterns of the infinite. They used language in a powerful way, articulating actions and visual patterns. And they saw in their own work mathematical patterns—geometric and numerical—and their beauty.

Rather than a narrow, trivial, static view of fractions, these children saw fractions in terms of their own actions and of an imaginative world the materials allowed them to create.

Covering Fractions

Materials: A kit containing
- two unit, or one, pieces
- a number of ½ pieces
- a number of ¼ pieces
- a number of ⅛ pieces
- a number of 1/16 pieces



Instructions:

1. Open your kit. The largest piece is one unit or one.
   Find all of the other pieces.
   On each piece, write the fraction name which describes it.

   Example: one half or ½

2. Some questions:
   a) Making one:
   - How many one-half pieces do you need to make one?

   You can write:
   Two half pieces make one
   or
   2/2 = 1

   - How many one-eighth pieces make one?

   Write about it. _____

   - How many one-sixteenth pieces make one?

   Write about it. _____

b) Use just one other kind of piece to make or cover one half piece. Draw pictures of what you find in the space below:

Write about what you find.
Example: Eight sixteenths makes one half
8/16 = ½

3. Here is a puzzle:

It is one whole covered by 2 halves.

Show how you could replace one half and still cover one whole. Use the space below.

Can you do this using just one kind of piece? Show how.

Can you do this using three kinds of pieces? Show how.

4. Faye said, "1 = ½ + ¼ + 2/8."
Is she right? _____

Why? _____

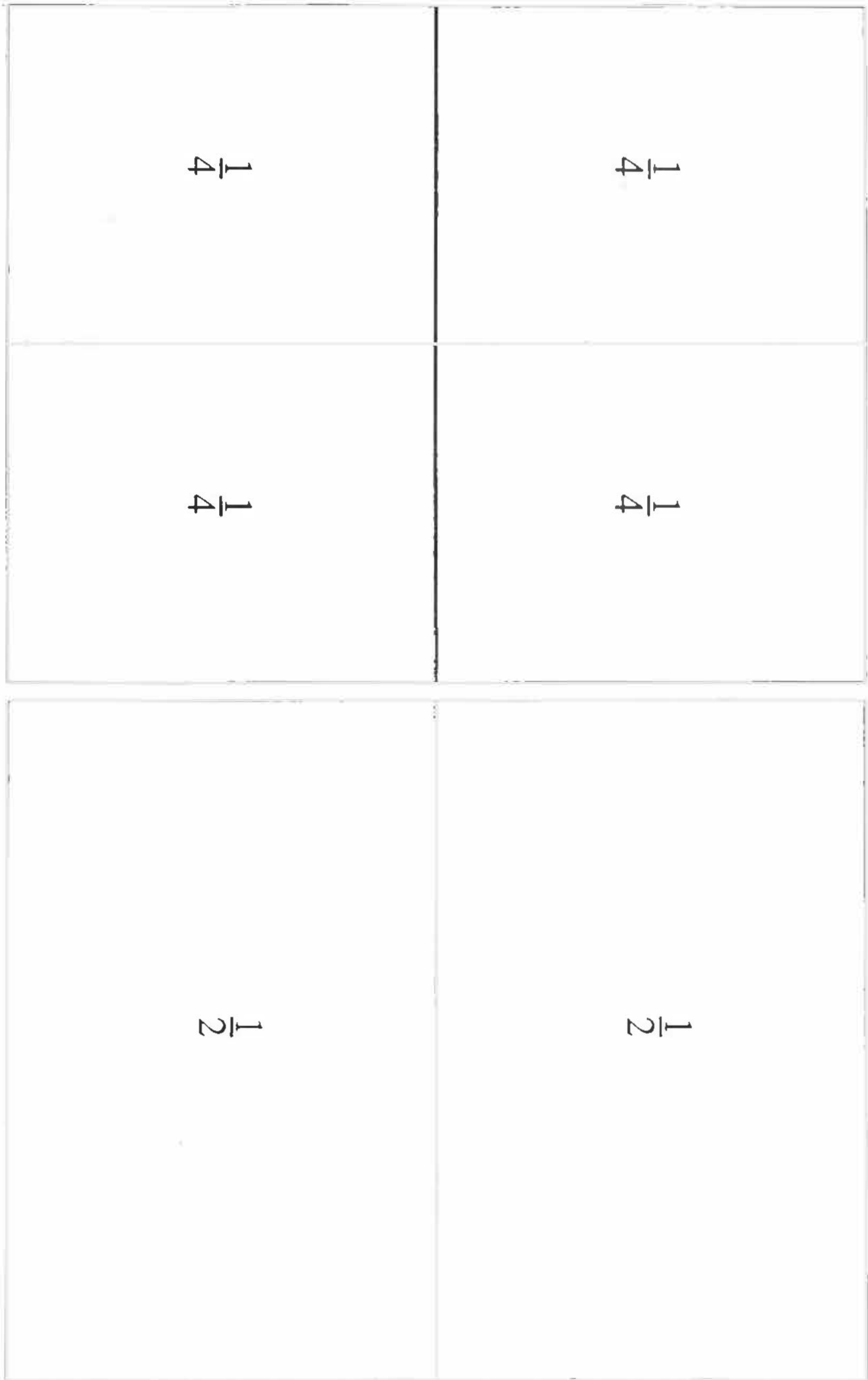
5. Take three quarters or three fourths out of your kit.
Cover three fourths (or three one-fourth pieces or ¾) in as many ways as you can.
Draw each covering. Write a sentence about each.
You can use words or symbols.

6. Draw your favorite covering of three fourths.

Why is it your favorite? _____

7. Make a covering which is almost equal to one whole but is a little more or a little less than one whole.
Draw it and write about it.

8. Lou said, "½ + ¼ + 2/16 is less than one."
Is he right? _____
Why? _____



Note: The teacher may wish to enlarge these fraction coverings to fill an entire page.

$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$
$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$
$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$
$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$

$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

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