A Bunch of Squares + Some Children = Lots of Mathematics

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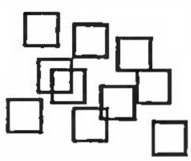
At times we become envious of all the interesting manipulative aids we see in catalogs and conference displays and described in articles. "I could sure do a better job of teaching mathematics if I had these!" we think. Indeed, in general, these aids are useful and can contribute to a better mathematics program. But all is not lost. If you do not have the aids you envy and there is no money left in the budget to purchase them, you can still be creative. For example, you can use the paper cutter to produce a bunch of squares (about 2 cm x 2 cm) from different colors of mayfair (or heavier) paper.

What can I do with a bunch of squares? Lots!!



Prenumber and Number

Sorting

1. In groups, children can sort the squares according to color.
2. For each group, prepare a set of squares of predominantly one color, but add a few of a different color. Ask, "Which does *not* belong?"

Patterning

Have children work in pairs. One student creates a pattern with the colored squares and the other demonstrates recognition of the pattern by extending it. The more capable students might be challenged to generate two-dimensional patterns (Figure 1).

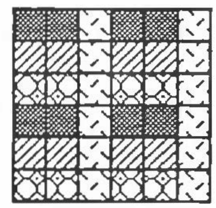


Figure 1. Two-Dimensional Pattern

Counting

The squares could, of course, be used for conventional counting experiences.

1. Arrange a set of squares in a linear order and ask certain children to count them beginning
   - at the left end,
   - at the right end, and
   - with a specific square, not at either end.
2. Arrange some squares in a random arrangement and ask certain children to count them.

Numeration

The squares can be used for grouping activities. Groups of five can be put together with paper clips and five groups of fives can be bundled with an elastic band. The same thing can be done with 10 and 10 tens. A group of 10 tens might be more easily handled by placing it in a small jar or box.

If the squares have been cut from stock of different colors, they can be traded. For example, in a small-group setting, children could play a trading game using a board as shown in Figure 2. The object is to get at least one blue square. Players take turns rolling a die. The die tells the player how many red squares to pick and place on the board under

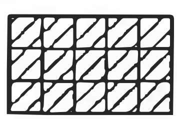
''Red.'' When five red squares are collected, they must be traded for one yellow square. Continue collecting red squares. When five yellow squares have been accumulated, they are traded for one blue. The first player to get a blue could be declared the winner or the game can continue until all players have a blue.

Alternatively, players could start with one blue square. The roll of the die tells how many red squares to remove. In order to remove red squares, the blue must be traded for five yellow, and at least one yellow traded for five reds. The object is to remove all squares from the board.

Blue	Yellow	Red

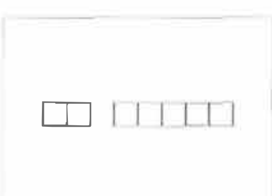
Figure 2. Trading Board

Whole Number Operations

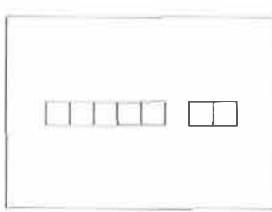


The squares can be used as counters to demonstrate the joining and separating action involved in the operations. The array interpretation of multiplication can also be shown easily with squares as illustrated here.

Properties such as the commutative property and the distributive property of multiplication over addition can be shown with the squares. Placing a set of two and a set of five (2 + 5) squares on a sheet of paper and rotating the sheet 180° should help children visualize the commutative property (Figure 3). Figure 4 suggests how the distributive property of multiplication over addition could be demonstrated with squares.

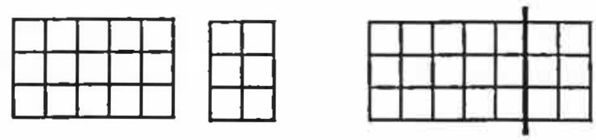


(a)



(b)

Figure 3. (a) 2 + 5 (b) Rotated To Show 5 + 2



$(3 \times 5) + (3 \times 2) = 3 \times (5 + 2)$

Figure 4. Squares Showing Distributive Property

Fractions and Ratio

One interpretation of a fraction is a subset of a set. This notion is normally introduced after the region model. Squares can be used in this context. Give children three red squares and one blue square. Ask, ''What fraction of the squares is blue?'' ''What fraction is red?''

Using the same setting, ask, ''What is the ratio of blue squares to red squares? What is the ratio of red squares to blue squares?''

Measurement

Length

A square could be used as a nonstandard unit of measure. ''How many squares long is your desk?'' The length of many familiar objects in the classroom (a book, pencil, chalkboard, computer screen and so on) could be determined.

The perimeter of certain objects could also be measured with squares. Students could align squares around their mathematics text, desk, table and so on, then count the squares to determine the perimeter in squares.

Area

A surface, such as the cover of a mathematics text, desk top or other convenient surface, could be covered with squares, which are then counted to find the area of the surface in squares.

Geometry

Polyominoes is but one example of many geometric concepts that children could explore with a set of squares. Children have to understand that, when placing squares together, the sides must be coincident as shown here.

correct incorrect

Have the children take one square and then ask, "How many different ways can it be arranged?" Take two squares. How many different ways can two squares be arranged? Here children will need to think about rotations. Is the vertical arrangement different from the horizontal arrangement? No, one is a 90° rotation of the other.

Use three squares. How many ways can they be arranged? Now children will also have to think about flips. In Figure 5, is arrangement (a) different from arrangement (b)? No, one is a flip of the other.

(a) (b)

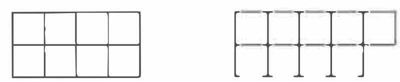
Figure 5

How many different arrangements are there for four squares? Five squares?

Number Theory

Odd/Even

The concepts of odd and even are frequently introduced in Grade 2. Young children can use the squares to help them understand these ideas. A set of squares that can be placed in a 2 x n array represent an even number. An odd number of squares results in a "missing piece" when arranged into a rectangle of width two squares.

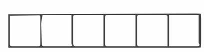
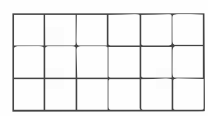


Even Odd

Prime and Composite Numbers

To help children understand the concepts of prime and composite and the difference between them, an activity like the following could be prepared.

Activity: Prime and Composite Numbers

A sidewalk:  A patio: 

Take the number of squares indicated in the left-hand column. Construct as many different sidewalks and patios as you can with the squares. Record the sizes in the middle column and shade the correct circle in the last column.

Squares	Sizes	Designs Made
2	Sidewalk: 1 x Patio sizes:	O Sidewalk only O Both sidewalk and patio
3	Sidewalk: 1 x ______ Patio sizes:	O Sidewalk only O Both sidewalk and patio
4	Sidewalk: 1 x Patio sizes:	O Sidewalk only O Both sidewalk and patio
5	Sidewalk: 1 x ______ Patio sizes:	O Sidewalk only O Both sidewalk and patio

. . . and so on

List the numbers that form sidewalks only:
(These numbers are called PRIME numbers.)

List the numbers that form both sidewalks and patios:
(These numbers are called COMPOSITE numbers.)

Figurate Numbers

Some patterns involving triangular and square numbers can be explored with a simple set of squares. Squares may not be suitable for exploring more complex figurate number patterns. The following three activities may be appropriate for middle or junior high school grades. The first activity, *stair steps*, develops the pattern for generating triangular numbers. The pattern is started at the right.

$$1 = 1$$
$$1 + 2 = 3$$
$$1 + 2 + 3 = 6$$
$$\ldots$$

Activity: Stair Steps

Use the squares to construct a series of stair steps. In the second column, record the number of additional squares needed to build the final step after the previous steps have been completed.

Number of Steps	Number of Additional Squares Needed for Final Step	Total Number of Squares Used
1	1	1
2	2	3
3		
. . . and so on		

Use the numbers in column 2 to get the total number in column 3.

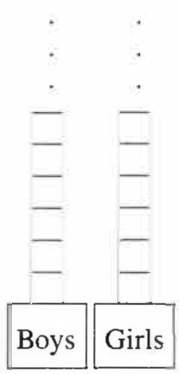
Describe the relationship that you found.

What geometic figure do your stair steps resemble?

The second activity, *squares*, explores the pattern involved in generating successive square numbers, and the third activity, *stairs and squares*, is an attempt to help children discover the relationship between triangular and square numbers.

Graphing

Graphing experiences for young children need to be concrete. Paper squares can be used in this setting as well. To compare the number of boys and girls in the class (or small group), each boy and girl could place a square, in a line, opposite the appropriate label as shown. Alternatively, the squares could be pinned on the bulletin board.



The squares could be used to represent almost any event and pinned on the bulletin board, laid end-to-end, or pasted on paper to form a bar graph. A design or symbol could be drawn on a set of the squares which could then be used as a pictograph.

Activity: Squares

Use the paper squares to construct a series of squares. In the second column, record the number of additional squares needed to construct the square using the previous one as a start.

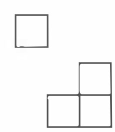
Size of Square	Number of Additional Squares Needed Over Previous Size	Total Number of Squares Used
1 x 1	1	1
2 x 2	3	4
3 x 3		
. . . and so on		

Use the numbers in column 2 to get the total number in column 3.

Describe the relationship that you found.

Activity: Stairs and Squares

The stairs you built can be placed together. Here the stairs with one square and three squares are being put together to form a 2 x 2 square.



Place the stairs with three and six squares together to form a square. What size square did you get?

Do the same with the six- and ten-square stairs.

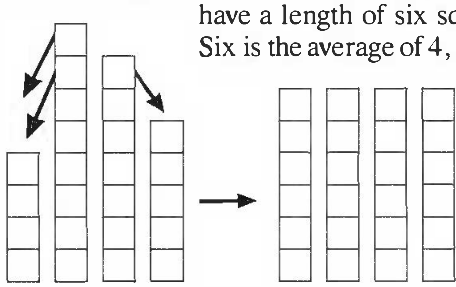
Try placing any two successive stairs together. Do you get a square? What size?

Write a sentence about the relationship between stairs (triangular numbers) and square numbers.

Statistics

Mean (Average)

The mean or arithmetic average is often taught in the late elementary years. A set of squares can be used to help children understand the concept of the mean. For example, the numbers 4, 8, 7 and 5 could be represented as bars of squares. To get the average, all bars are made the same length. Children could take two squares from the eight-bar and place them on the four-bar giving both bars a length of six squares. Then one square could be taken from the seven-bar and added to the five-bar. Now all four bars have a length of six squares. Six is the average of 4, 8, 7, 5.



Conclusion

Manipulative aids such as those displayed at conferences or described in publishers' catalogs are highly recommended. In the absence of a specific aid, materials that are easily collected or prepared can often serve as a valuable substitute to help children build an understanding of mathematical concepts. We have seen that a variety of topics can be explored with small squares cut from reasonably sturdy paper. They are not elaborate, but they will do the job! No doubt readers can suggest other examples of topics that can be demonstrated with a set of squares!