Birthdays: A Rich Source of Problems

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One of the happy things that most of us remember from our elementary school days was the attention that was given to birthdays. Apart from students with birthdays celebrated during school holidays, everyone had the pleasant experience of being "birthday girl" or "birthday boy" once a year. Birthdays can provide the basic numerical data which give rise to mathematical problems and activities related to graphing, probability and computation. The data can be presented in one or more of the concrete, pictorial or symbolic modes.

Graphing (Concrete)

The Alberta Program of Studies includes the objective at the Grade 1 level: "collects data from the immediate environment to construct graphs using pictures or objects, and discusses the results." The following illustrations indicate some ways in which readily available manipulative materials can be used to construct a three-dimensional graph. In Figure 1, interlocking beads are used on the numbers 1–31 on a Hundreds Board frame to indicate the day of the month on which the birthdays of a particular class occur. Figure 2 uses colored wooden cubes to display the various birthday months of the same group of children. A pair of curved wire abacuses are used in Figure 3 to illustrate the data for the day of the week on which each child was born. Grade 1 children will probably have to ask their parents for these data. Each child can participate in the building of these graphs by placing the bead, block and abacus counter in the appropriate place.

Graphing (Pictorial)

The Alberta *Program of Studies* (Alberta Education 1991) includes the objectives at the Grade 3 level: "collects, constructs and interprets pictographs and simple bar graphs" and "locates position of an object on a grid." The following illustrations present the birthday data from the previous activity in a two-dimensional format. All of the birthdays for a particular group of children can be charted on a calendar such as the one included as Figure 4. The source of this particular calendar was the March 1984 issue of *Student Math Notes*. Each child draws a small circle around the date representing his or her birthday. If two children share a particular date (not as unusual as you might think), a second circle is made for that date (for example, May 5). Figure 5 shows a grid that is another way of presenting the birthdays. Again it is necessary to have some way of indicating two birthdays that fall on a particular day. Figure 6 is a bar graph for each of the days 1–31. Figure 7 presents a bar graph for each of the 12 months and Figure 8 that of the number of children who were born on a particular day of the week. The data for the last bar graph could be replaced with data on the day of the week on which each child's birthday fell in 1991.

Probability (Symbolic)

Although the present Alberta *Program of Studies* does not formally include probability objectives prior to the junior high school grades, many teachers provide children with intuitive experiences in this area. It is quite likely that the next revision of the Program will include such concepts as the chance component of probability, using terms like *always*, *never*, *sometimes* and *maybe*.

A common question often asked about birthdays is: What is the likelihood of two people sharing a birthday (same month and day)? One way of explaining this at the elementary level is to consider the idea

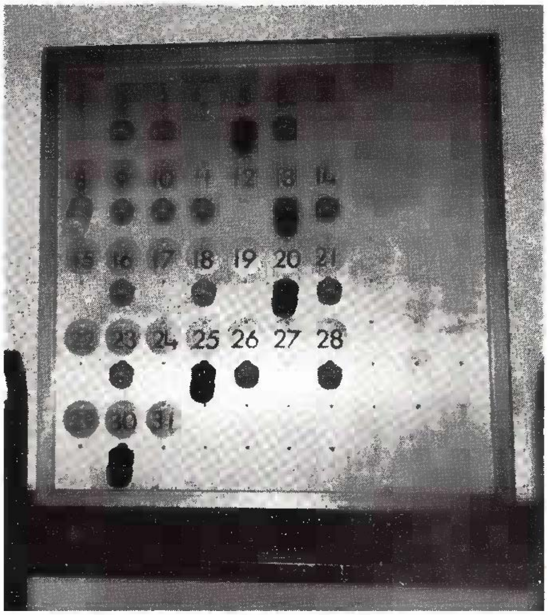


Figure 1

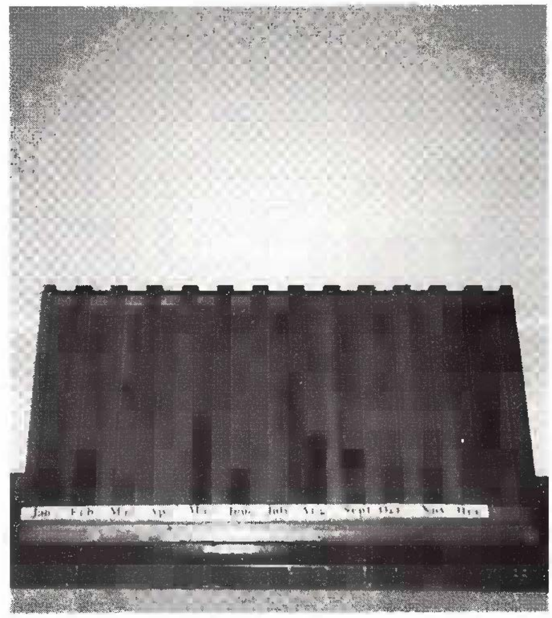


Figure 2

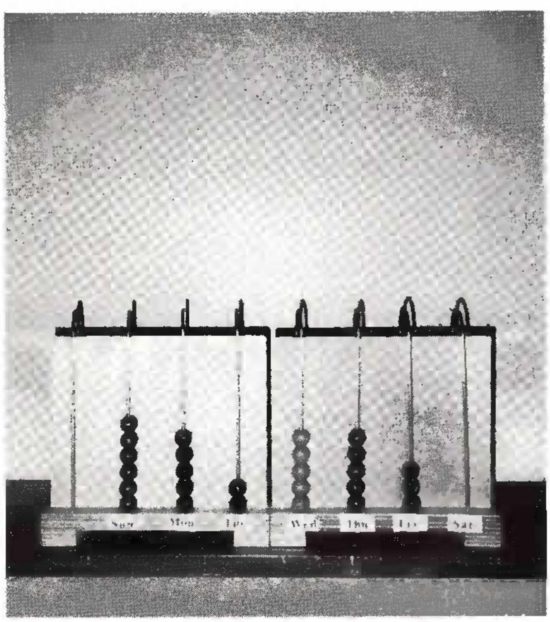
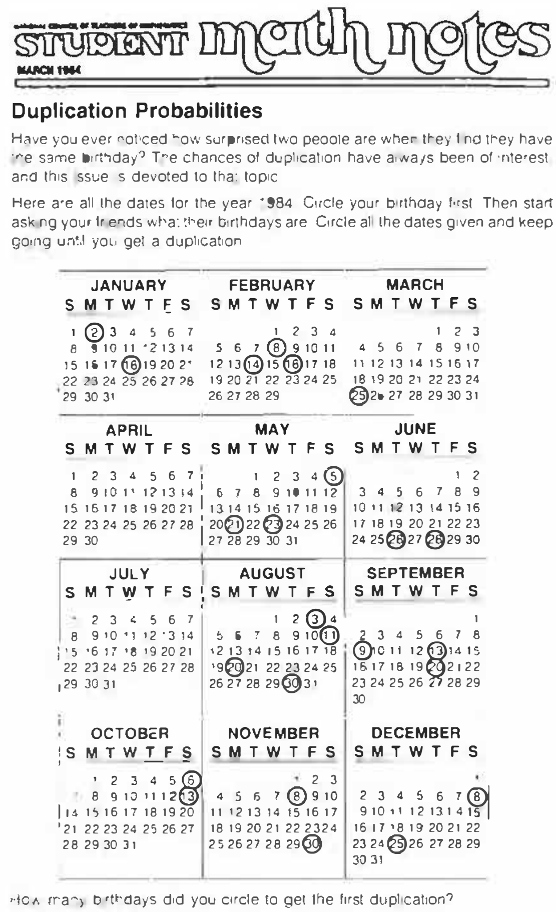


Figure 3



STUDENT math notes

MARCH 1984

Duplication Probabilities

Have you ever noticed how surprised two people are when they find they have the same birthday? The chances of duplication have always been of interest and this issue is devoted to that topic

Here are all the dates for the year 1984. Circle your birthday first. Then start asking your friends what their birthdays are. Circle all the dates given and keep going until you get a duplication

JANUARY

S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

FEBRUARY

S	M	T	W	T	F	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29			

MARCH

S	M	T	W	T	F	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

APRIL

S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

MAY

S	M	T	W	T	F	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

JUNE

S	M	T	W	T	F	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

JULY

S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

AUGUST

S	M	T	W	T	F	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

SEPTEMBER

S	M	T	W	T	F	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30						

OCTOBER

S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

NOVEMBER

S	M	T	W	T	F	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

DECEMBER

S	M	T	W	T	F	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

How many birthdays did you circle to get the first duplication?

Figure 4

Birthday Grid

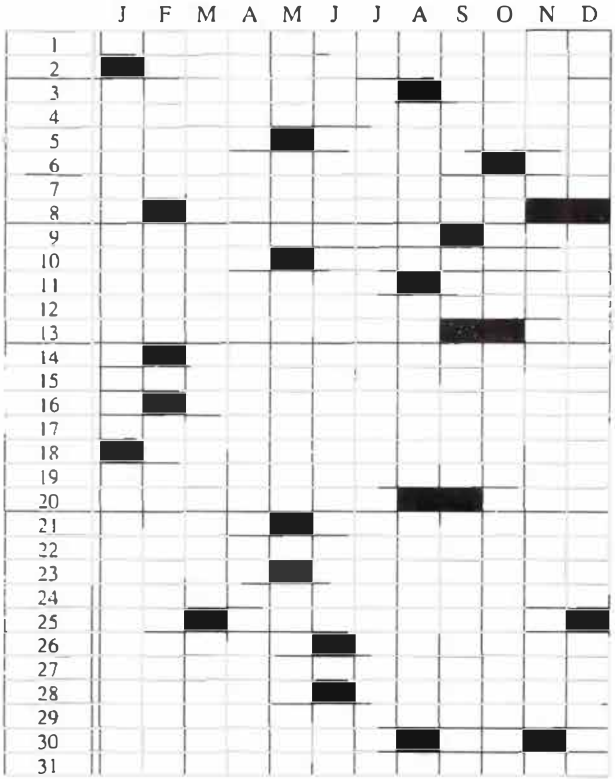


Figure 5

Same Day Number

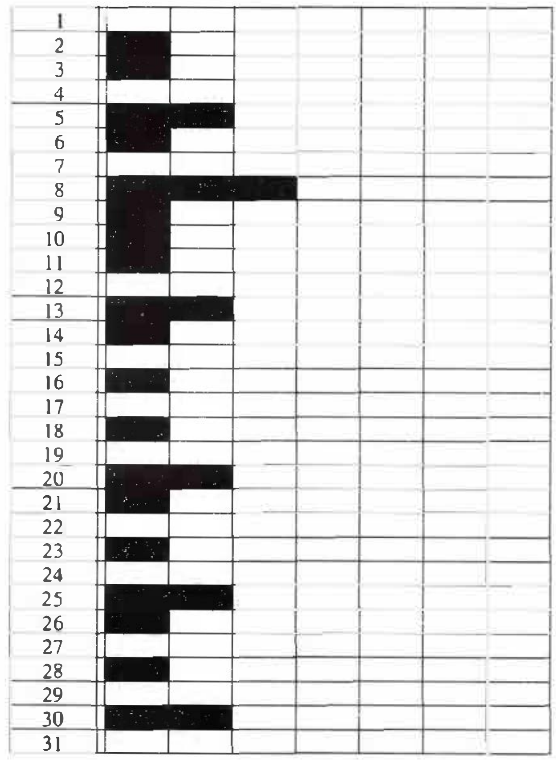


Figure 6

Same Month

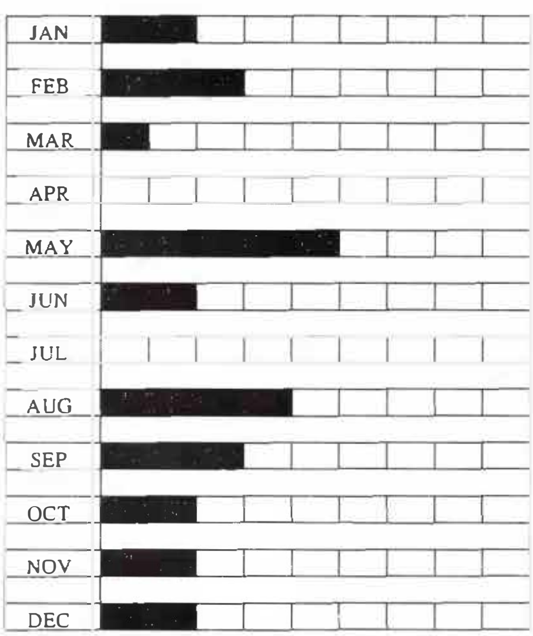


Figure 7

Same Day of Week

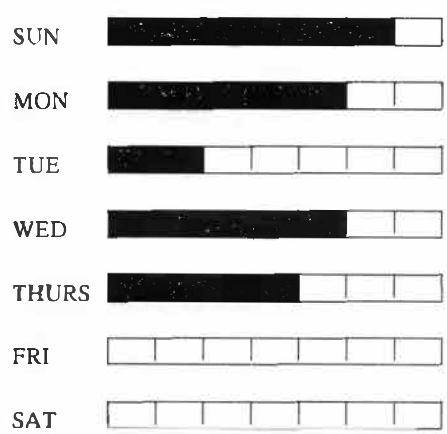


Figure 8

of pairs. How many pairs are there in a classroom of 25 children? An initial strategy would be to pose a less complex problem such as: How many pairs are there in a group of three children (for example, Mary, Bob and Ann)? We could have three pairs, Mary and Bob, Mary and Ann, or Bob and Ann. After considering other small groups one could note that the pattern is the same as in the familiar "handshake problem." The equation $[(n)(n-1)] \div 2 = p$ applies where n is the size of group and p is the number of pairs. Thus, 25 children would result in [25 x 24] 2 = 300 pairs. With 366 different possible birthdays (in a leap year), it becomes evident that the 300 different pairings in a group of 25 children is not very far from that number. In fact, a group of 27 or 28 would result in 351 or 378 pairs, respectively.

The data shown in Figure 5 were based on a particular class of 26 children. The number of pairs would be [26 x 25] 2 = 325. One could calculate the quotient $325 \div 366 = 0.89$ as the expected number of pairs. It is not surprising, then, that one of the pairs in this particular group of 26 did, in fact, share a birthday.

The data in Figure 7 can be used to answer the question: What is the likelihood of two people in a group of 26 sharing a particular month of birth? As there are 325 pairs and 12 months, the expected number of people sharing a particular month could be calculated as $325 \div 12 = 27$, rounded to the nearest whole number. In January two people have a birthday, giving us a pair. The three people who share a February birthday give us three more pairs. Because only one person has a birthday in March, there are no additional pairs. For the other months, the number of pairs are as follows: May (five people)–10 pairs, June (two people)–1 pair, August (four people)–6 pairs, September (three people)–3 pairs, and two people in each of October, November and December, resulting in 3 more pairs. The total number of actual pairs who share a particular month of birth is 27, exactly the same as the expected number.

The number of pairs for groups of sizes increasing by one each time also exhibits a pattern which children have probably encountered in other contexts. The differences increase by one each time to form the sequence: 1, 3, 6, 10, 15, 21, 28, 36, . . .

The data shown in Figure 6 can be used to answer the question: What is the likelihood of two people in a group of 26 sharing a particular day number (for example, both having a birthday on the fifth day of the month but not necessarily in the same month)? Because there are 325 pairs and 31 days, the expected number of people sharing the same day number could be calculated as $325 \div 31 = 10$, rounded to the nearest whole number. In the figure, each of the numbers 5, 13, 20, 25 and 30 is shared by a pair. The number eight is shared by three people, resulting in three more pairs for a total of eight pairs who share a day number. In this example, the actual number, eight, is less than the expected number, 10.

The data shown in Figure 8 can be used to answer the question: What is the likelihood of two people in a group of 26 being born on the same day of the week (for example, both being born on a Sunday)? As there are only seven possible days, it is obvious that a large number of pairs would share a birthday on a particular day of the week, many more than the number of pairs who share a particular day number or month name. The expected number of people sharing the same day of the week could be calculated as $325 \div 7 = 46$, rounded to the nearest whole number. The six people who share a Sunday birthday result in 15 pairs. For the other days of the week the numbers of pairs are as follows: Monday (five people)–10 pairs, Tuesday (two people)–1 pair, Wednesday (five people)–10 pairs, Thursday (five people)–10 pairs, Friday (three people)–3 pairs. The total number of actual pairs who share a particular day of the week is 49, slightly more than the expected number of 46.

Computation (Symbolic)

For children in the upper elementary grades who do not know the day of the week on which they were born, a few relatively simple computations will quickly reveal the desired information.

To find the day of the week on which you were born, do the following calculations, using as a sample birthdate July 1, 1867.

Last 2 digits of year ÷ 4 (ignore remainder)	$67 \div 4 = 16$
Month code (see below)	0
Day	1
Last 2 digits of year	67
Century code (see below)	2
SUM	86

Sum ÷ 7 $\quad 86 \div 7 = 12$ with a remainder of 2

Remainder code (see below) Monday

Thus, our country, Canada, was born on a Monday.

Codes

Month		Century	Remainder
Jan – 1 (0)**	July – 0		1 – Sunday
Feb – 4 (3)	Aug – 3	1700s – 4	2 – Monday
Mar – 4	Sept – 6	1800s – 2	3 – Tuesday
Apr – 0	Oct – 1	1900s – 0	4 – Wednesday
May – 2	Nov – 4	2000s – 6	5 – Thursday
June – 5	Dec – 6	2100s – 4	6 – Friday
			0 – Saturday

**(use bracketed number if you were born in a leap year)

Do the calculation above to find out on what day you were born, and then read the rhyme below to find out what kind of a child you were!

Monday's child is fair of face,
Tuesday's child is full of grace,
Wednesday's child is full of woe,
Thursday's child has far to go.
Friday's child is kind and giving,
Saturday's child works hard for a living.
But the child who is born on the Sabbath day
Is bonny and blithe, and good and gay.

Reference

Alberta Education. *Mathematics Component of the Program of Studies for Elementary Schools.* Edmonton: Author. 1991.