delta-k

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TECHNOLOGY

IN THE CLASSROOM

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EDITORIAL

In the late 1970s, the National Council of Teachers of Mathematics (NCTM) specified that the teaching and learning of problem solving skills and abilities should constitute a major focus and thrust of mathematics education in the coming decade. The major focus for the nineties is the instigation of new curriculum standards for mathematics education. These standards are based on five fundamental precepts (*Curriculum and Evaluation Standards for School Mathematics* 1989):

- Students should learn to value mathematics.
- Students must learn to become confident in their own abilities.
- Students should become mathematical problem-solvers.
- Students must learn to communicate mathematically.
- Students must learn to reason mathematically.

The reader will notice that these broad aims or goals for the mathematics curriculum deviate significantly from our traditional view of instruction: that students learn to perform mathematical computations accurately, with the ability to selectively recall and employ algorithms for the purpose of finding correct, efficient and elegant solutions to given problems or tasks. The NCTM standards represent a much greater interest in intellectual interaction with a dynamic and challenging curriculum—including reflection on past learning, synthesis and generalization of new concepts, and the development of powerful communication structures.

How do we as mathematics teachers then respond to the challenges of delivering such a curriculum? This issue focuses on providing instructional alternatives to activate and manipulate mathematical concepts.

Marie Hauk and Bryan Quinn provide an example of their attempts to move toward a more active environment including both manipulative activities and cooperative learning structures. They discuss their reactions to their experiment through three themes: seeking immediate rewards, coping with time constraints and looking for support.

Thomas Kieren describes some interesting examples of manipulatives useful for teaching fractions. These examples are drawn from his work with Grade 3 children.

George Cathcart gives several examples of activities using a very simple manipulative: cut paper squares. The topics that can be addressed with this manipulative include sorting and patterning activities, measurement activities, odd and even numbers, and prime and composite numbers.

Allen Neufeld uses the familiar and enjoyable context of children's birthdays as a way of introducing several different mathematics problems.

David Duncan and Bonnie Litwiller introduce *taxicab* geometry in which distances are measured only in vertical or horizontal moves between points. This activity provides an interesting extension to our traditional topics of distance and area found in the geometry curriculum.

In his article, Dale Burnett gives us another chance to consider or reconsider the role of the computer in mathematics classrooms.

In the Teaching Ideas section, Craig Loewen uses the framework of the NCTM Standards document to give two examples of some probability activities for use with middle school children.

Derek Gray and Craig Loewen describe an application game that can be used to teach estimating, measuring and constructing angles.

Enjoy!

A. Craig Loewen
John B. Percevault

Reference

Curriculum and Evaluation Standards for School Mathematics. Prepared by the Working Groups of the Commission on Standards for School Mathematics of the National Council of Teachers of Mathematics. Reston, Va., 1989.

Moving Out of the Comfort Zone

Marie Hauk and Bryan Quinn

The project had just begun, and already Bryan was experiencing self-doubt. Why was he rocking his boat and entering uncharted waters? He is a good math teacher; he works conscientiously toward having his students enjoy math *and* experience success. So, why was he choosing to depart from the safety of his familiar, traditional classroom structure? Why was he now standing in his classroom directing traffic as his Grade 7 students attempted to rearrange their desks into groups of four?

To begin with, the students had come to class unprepared. None had submitted lists for proposed group members, so class time was required to do this. Few had brought new folders or duotangs as requested the previous day. To top it off, class time had been shortened due to a school activity. Bryan had planned to have his students spend the whole period collaborating on an introductory poster activity in which they would identify, discuss and display the purposes of group work. Not only was there less time for this activity but also it had become evident that to keep the students on task, they needed more specific direction than anticipated.

Bryan capitulated. After all, it was Friday afternoon! Perhaps if he supplied the folders and duotangs for the students, they could get organized properly on Monday. Besides, he knew Marie would be there to help. Bryan and Marie were collaborating on all aspects of the planning and teaching of this fractions unit in which students would make extensive use of concrete materials within a small-group learning setup.

What motivates successful teachers to make significant changes in their teaching style? In other words, *if it ain't broke, why fix it*? Good teachers continue to be learners throughout their careers. Not only do they reflect on what they are doing or have done but also they seek alternatives. Current research and literature offer new directions as theories regarding teaching and learning change. For Bryan and Marie, it was a matter of making classroom practices consistent with their philosophical stance. They believe that students must have opportunities to be involved responsibly and actively in their own learning. They also believe that for students to truly understand, and appropriately apply mathematical concepts and skills, they must have concrete experiences in personally meaningful problem solving contexts.

The topic of fractions is notorious for being abstract and difficult. Many students who appear to have achieved success through traditional chalk-and-talk methods have often developed only a superficial understanding. For this reason, it seemed to be an appropriate area in which to introduce an alternative approach. While Bryan's previous use of concrete materials in teaching math was primarily for teacher demonstrations, he perceived that the manipulatives had facilitated his students' understanding. Ongoing use of concrete materials within a small-group setting for a unit in mathematics was a new experience for Bryan and the students.

One month was allocated for this unit. During this time, a major theme, *Out of the Comfort Zone*, emerged. While Bryan used this phrase explicitly on a number of occasions, it was descriptive of both the students' and teachers' experiences. The comfort zone was defined by a number of interwoven beliefs and practices regarding teaching and learning. The path for moving out of the comfort zone was neither smooth nor one-way; the urge to return remained strong.

The students' experience of moving away from what was familiar and comfortable centred on day-to-day personal gain and may be described as *Seeking Immediate Rewards*. From the teachers' perspective, however, concern with short-term personal effects was only one factor. As the innovators, they were accountable for moving themselves and the students out of the comfort zone. This demanded daily self-evaluation of both the practical and the philosophical aspects of the experience. Their movement was influenced by time and confidence, and may be categorized as *Coping with Time Constraints* and *Looking for Support*.

The Comfort Zone

Most teachers and students operate within a multifaceted comfort zone without being explicitly aware of its existence. Consciousness of this phenomenon occurs when any boundaries are crossed. This project revealed the following comfort zone fronts: curriculum and pedagogy, classroom organization, control of learning, learning resources and evaluation of learning. Movement requires a departure from traditional practices.

A particular curriculum focus tends to give rise to a compatible pedagogical style. When curriculum focuses on product-oriented objectives, pedagogy that produces easily measured short-term results may appear to be successful and is often viewed as desirable. A move toward emphasizing process over product, where the results are less tangible and often long-term, is viewed with suspicion by traditionalists.

Classroom organization sets the tone for learning. The traditional arrangement of desks in rows fosters a nonsocial approach to learning. A small-group arrangement creates a different atmosphere: no longer is talking among students taboo, it is required! Traditionalists view control of learning to be the prerogative of the teacher, who presents and explains the concepts and skills to the students. However, having students work in groups allows them to assume ownership of the learning process. They are held more accountable both for their own learning and for assisting others within their group.

The textbook is often taken for granted as a learning resource. Because of the security it can offer to teachers, students and parents alike, it may be the only resource used. In fact, too often, the quality of teaching is judged on the basis of what page the class is on. The textbook *becomes* the course even though its content may not be congruent with the prescribed curriculum. While a good textbook can be of value, ideally, it should be viewed as just one of many resources used in the classroom. Manipulatives, calculators and other print materials can enrich learning.

Evaluating mathematics learning traditionally has been done through measuring concepts and skills in an objective manner, usually by means of paper and pencil tests. However, learning is also subjective. Assessment strategies that focus on process and include the use of manipulatives and oral reasoning as well as written work result in more valid assessments.

Seeking Immediate Rewards

The second day of the unit began with a pleasant surprise for Bryan. Waiting to greet his students, he braced himself for impending confusion. However, the students entered the classroom, and, without waiting for instructions, quickly maneuvered their desks into their respective groups of four. The novelty of the situation, having two teachers in the classroom and the opportunity for social interaction, seemed to be motivational; certainly the students were eager to try the new setup for learning.

Another novel aspect for the students was keeping two-part logbooks. The first section was used for in-class activities and homework assignments, the second as a journal for personal writing (about group work, the use of concrete materials and sundry comments). Many journal entries indicated students' discomfort with the writing process, both with expression of ideas and spelling and grammar. Despite their difficulties with writing, many students made honest observations and offered insightful comments. To present the experience realistically from the students' point of view, examples of journal entries have not been edited. The students' real names have been changed to maintain their anonymity.

To capitalize on the students' desire to seek immediate rewards, a prize draw was instituted very early in the program. Students earned stamps for their group during the course of a week. A group received an entry for each five stamps earned. On the basis of the accumulated entries from the previous week, a draw was conducted each Monday. Stamps were given rather liberally for a wide variety of student behaviors—literally anything that contributed in a positive way to the functioning of the class (for example, arranging desks quickly and quietly, taking group attendance, working as a group rather than individually, being polite, assisting others, writing in logbooks). Despite the fact that the group prizes were mere token gifts, earning stamps proved to be an overwhelmingly powerful motivator for individual students while strengthening group dynamics.

The students' generally positive attitude toward the group arrangement remained throughout the unit. Their opinions fluctuated, however, depending on perceived personal benefits from day to day. Early on, Bruce commented in his journal, "I think every thing going prity good. Exept some times people act up. I like this more than the ragular class." At the end of the unit, Blair expressed a common opinion, "Today we are finished off your fractions and group work. I thought the group work was good and I hope that we have it next year.''

Most students were pleased with their selection of group members. In early journal entries, Mike commented, ''Today Rob was the leader. He was very nice,'' and Flora wrote, ''I do like Wally in my group.'' Bonding among group members became evident when a change was made early in the unit. When one girl in Group 4 left the class, Bryan replaced her with a girl who had been unhappy in Group 1, and he transferred one boy from Group 2, the only group of five, to Group 1. Bryan gave the five boys in the group the responsibility for deciding which one of them should move. They found the decision difficult and ended up putting their names in for a draw. Mike's concern was evident when, later that day, he wrote, ''Darcy had to move to group #1. He is so far coping with the group fine.'' Another day, he commented further, ''Darcy is doing fine with group #1.''

Using manipulatives for mathematics was a new experience for most of the students. As a result, these students used play time to become familiar with each of the materials. A certain amount of free play had to be tolerated before the students would use the manipulatives for the intended purposes. The students were inclined to handle and explore the colorful materials rather naturally. They often described the work with manipulatives as being fun. Lois wrote, ''Well, now I learned about cuisenaire rods. Their kinda fun!'' Helen similarly reflected, ''I learned that Cuisunaire block are used for fractions. They are used as telling sizes. It's easy and fun too.''

An unspoken question seemed to develop: ''If we're having fun, can we be learning?'' (Is it OK to have fun while learning?) Some dealt with this uncertainty about the legitimacy of *playing* in math class by differentiating between meaningful and non-productive types of play. This was expressed by Helen who wrote, ''It is fun to play with shapes and I just learned never to give Amanda any kind of blocks to play with because that is one of her favorite kinds of toys, building blocks.'' While having fun was important, Gloria's comment, ''I enjoy having to do this kind of work,'' suggests that the playing was goal-directed.

Having fun was important, but students gained more appreciation for the manipulatives as they became aware that the latter helped them develop greater understanding of the mathematics being explored. A number of journal entries reflected this attitude. Howard wrote, ''When we used the manipulatives it made the problems easier because it helped us understand them. The fraction strips was probably the most helpful manipulative.'' That some students were initially dubious was reflected by Wayne: ''Today we learned about what maded ⅓, whole, ½, & how to compare other rods. The thing that was fasinating was using the Cuisenaire rods to make it easier to understand. The thing I was surprised about was to talk with my group members and understand fractions much better.'' The visual aspect was appreciated by Lois, who wrote, ''Dividing fractions is pretty easy with cuisenaire rods. I guess it is better to see it than to think it in your head.'' The need for tactile experience was expressed by Shawn: ''Also we got to use little blocks cuisenaire rods bar graphs and stuff like that. Using thing you can Feel is better than just thinking about it because you can see them and Feel them and ther wright there in front of you bye your self you can only think about it.''

Students who previously have had only superficial exposure to procedures and algorithms may become frustrated when they are confronted with the more lofty goal of attaining a deeper understanding of concepts. Evidence of this was indicated by Melanie: ''Right know we are working on fraction bars. I don't understand so I'm behind the group. I don't find the directions straight forward and I just don't get it.'' The need for patience in this respect was recognized by Gloria, who wrote, ''Understanding fractions is a little bit hard. I used to know a lot about fractions but now it is getting harder. One thing I like about groups is that theres lots of things to do. We also worked with colored cubes. Its kind of neat because you get to do all sorts of things with shapes. I learned that its easy once you know what your doing. Not everyone thinks it easy but I think they should give it a try. At first this was hard for me but once you give it a try it can be easy. I just hope everyone gives everything a try.''

Probably the most motivating aspect for the students was the opportunity for social interaction. This enthusiasm was highlighted by Lois, who wrote, ''Boy this was fun! Today I got a double period! Woa dude! This is heavy! I like working in a group. Its totally dense!'' As for the concrete materials, fun was a common descriptor of group work, but there was also a sincere desire to learn, as expressed by Judy: ''I think group work is really fun and eciting. I honestly belive that this group work thing will really work and I'm sure that we will get a lot of work done. . . . This will probably be the best math classes that we ever had.'' She went on to address explicitly the importance of interpersonal relations by writing, "I love this working in group. I mean this is really fun. I find that working in groups you get to know people better. I think that math class is better now that we are working in groups too bad we cant stay like this forever." Douglas expressed similar thoughts: "Today I am going to write how I like groups better than working by myself. Its better I find it because you can shar ideas and meet new people and become better frends or learn more about them." The benefits of collaborating and sharing the responsibility of learning were often mentioned by the students. For example, Shawn wrote, "When we were working in groups I found it Fun. It was better than working buy yourself because when you do a question in a group you get to hear what everybody has to say and you can discuss it." Similarly, Howard said, "I think it was a good experience, working in groups. It made the work easier and I understood more that we worked together. It helped some people with their listening." Gloria agreed, "Groups are fun to work in because you can help people or they can help you."

The most common complaints about group work were noise and lack of cooperation. This was indicated by Gloria, who noted, "Some people hate groups because its destracting when some group members keep talking." However, this was not necessarily perceived as a reason to quit group work. Lois acknowledged the distractions, but was not discouraged: "I guess it was pretty noisy. The work was a lot easier and we learned to cooperate with each other. I think in the future, we should get into more groups!" Some of the students were unsure about the value of group work in terms of their own learning. This feeling was expressed by Billy, who wrote, "In this section I feel that I did not lern as much as I would if we were not in groups In other words I feel I would have lerned more by myself." Even Judy, who enjoyed the experience, said, "I think learning how to use fractions in a group was fun and I really dont know if I could have done better or worse if I would have been by myself."

At the end of the unit, the students generally spoke favorably about group work and indicated that they would prefer to continue learning that way. Cheryl wrote, "The fractions unit is over and I really enjoyed working in groups. We are now doing ratios and rates and now we work individually. I enjoyed sharing the load of work with others." Endorsement was offered by Mike, in his comment, "Today we are back to normal in math. BORING! I like group work way better. I learn better that way. The group is more better way to learn math."

Before the project, in terms of performance, most students in this class were either high achievers or low achievers. Although this pattern was not altered significantly by the project, it seemed as though the high achievers felt more challenged while the low achievers, despite low marks, felt as though they benefited in other ways. Not surprisingly, the bottom line for most students is marks. Those who did not achieve well on the tests expressed disappointment. Melanie wrote, "I used to think math was easy but ever since the first term my marks are slipping majorly! I don't know why I just don't understand THIS." Shawn related, "We had a test on fractions I thought that I would do O.K. but I didint I only got 48% Not to good at all. I try to study more and do better." Even some students who did well, relatively speaking, indicated that they had greater self-expectations. Rachel noted, "On February 18 we got our tests back. I got 72%. That's okay but is not so great. I don't understand how to multiply and divide fractions yet." Similarly, Lois commented, "Well, I didn't do very well on my fractions test. I got 72% Pretty bad, huh? I hate it. I'm a 70%-80% person. I'm so bad. Why can't I do any better. I guess I have to 'Apply' myself to it."

The disappointment, however, was not directed at the group setup. Gloria lamented, "On the test I didn't do very well. I use to be good at fractions but is differnt now. Some how its alot harder than the years before. I think I need some help with fractions." Shawn, in fact, supported group work in spite of his own performance: "We had a test on Fractions I thought that I would do pretty good on the test but I didint I only got 33% thats not to good but at least I tried. It was a Fun experienc being in groups I hope are class will get to do it again."

On the other hand, some students who were pleased with their marks did attribute their success to working in groups. Blair wrote, ". . . I liked the group work and we should have it again. I liked the fraction unit because I did good on the tests and I found it easy." Helen was most pleased with her results on both tests. After the first test, she enthused, ". . . all I can tell you is I GOT 79% ON MY MATH TEST!!! . . . I think I work much better in groups. I usually fail. I know I'm one of few people who are better in groups and I know we won't stay like this but I wish we could." Following the final test, she reported, "We don't have to go in groups anymore. I really enjoyed it though. I obviously did better in groups though I actually passed my 2 tests!!! I am happy, my mom is happy and if I get 80% on my whole report card I get a new bike!!!!!"

Coping with Time Constraints

Throughout the project, time was a significant factor. Both unit and daily planning required many hours. The planning began with consideration of the prescribed curriculum objectives, and then a tentative timeline was established. Because many of the concepts and skills were at the introductory level, at least four weeks seemed necessary.

The Ginn *Journeys in Mathematics 7* textbook was used as a resource, but the focus was on personally-developed cooperative group activities that required concrete materials. Appropriate materials were selected to facilitate learning each objective. An inventory of on-hand manipulatives was taken, then a decision was made as to what other materials were to be borrowed or purchased. The daily activities were developed as the unit progressed.

Time had to be allowed for orienting the students to group work. As cooperative learning in mathematics was a new experience for most students, they needed to appreciate that the occasion was to be used for more than socializing. Routines had to be established. Rearranging the desks at the beginning and end of the class became progressively more efficient. Math classes were held four days per week. Each of the four students in a group was assigned responsibility for obtaining the folders, activity sheets and trays of materials on a particular day.

The cooperative learning process required considerable time and patience for both the students and ourselves. To encourage the students to work together, only one activity sheet was given to each group initially. However, many of the students were not sure how to share the sheet; some groups circulated it for each one to read separately and did the activities as individuals. Other groups attempted to read in unison but found it awkward to reposition themselves. After discussing the problem, Bryan and Marie compromised by giving each group two activity sheets. Even then, continual prompting was necessary to discourage them from working as separate pairs within a group. Frequent reminders were given to the students so that group members would monitor and help each other to understand and finish the activities.

Some groups worked more efficiently than others. For accountability, all students were expected to record their activities (for example, draw the result). Yet, some used too much time needlessly documenting (using pictures and writings) each step in detail. Pacing the activities was difficult for some students, as they were not used to making such decisions. As time went on, Bryan took a greater role in monitoring the amount of time they spent on each activity. The small-group approach with teacher as facilitator was a new teaching style for him, and the urge to go back to a more teacher-controlled situation was strong.

According to the curriculum, much of the fraction work in Grade 7 should be at the concrete level. Not only did the exploratory hands-on nature of the activities place demands on time but, because few students had experience with concrete materials, more time had to be allowed for them to become familiar with the manipulatives. Journal writing was often short-changed. As this was deemed an important aspect of the experience, special effort was required to ensure that enough time was allowed. One solution was to start the class with journal writing on occasion.

Coping with time constraints was an up-hill battle. Class time passed quickly; 40-minute classes were too short for an activity-based program. The teachers' and students' inexperience with cooperative learning and extensive use of concrete materials put extra demands on the time required. Further flexibility was required to accommodate school timetable changes and classes missed due to holidays and the teachers' convention.

Looking for Support

People feel stress when *someone else* (someone in authority) changes the boundaries, but the degree of stress is greater when the change is self-initiated, because the decision itself is questioned both internally and externally and there is self-doubt in addition to criticism by others. For this reason, ongoing suport is essential. Bryan and Marie continually looked for support from within, from each other and from the students. Unsolicited support from colleagues was relished.

Support from school administration is essential when a teacher departs from traditional methodology. Bryan's principal took an active interest, endorsing the project and visiting the classroom to observe the students as they worked in groups. Affirmative collegial support is also significant. Such encouragement came from a language arts teacher who, on several occasions, looked in on the class and each time expressed her pleasure to Bryan at seeing group work and journal writing being done in math.

The value of motivation cannot be overestimated. Support from the students was manifested daily through their eagerness to enter class, set things up and use the materials enthusiastically. Even though they tended to react to the immediacy of a particular situation, their generally positive attitude was encouraging.

The availability and the practicality of using and storing the concrete materials was not a major problem: most of the students handled the materials reasonably and were accountable for their use. But, in view of the investment of money and teacher effort, Bryan found occasional lack of appreciation for the materials and subsequent misuse (at times from students in his other classes) frustrating.

High marks are often perceived as *concrete proof* of the success of a program. Because of this natural tendency to value test marks, Bryan and Marie were initially disappointed with the results of the first exam. Although the students' results were lower than hoped, they correlated with their marks from earlier in the year. Perhaps with a more traditional teaching and testing approach, the students may have been able to get more correct *answers* to routine questions, but Bryan and Marie doubt that the students would have had greater *understanding*.

Bryan admitted that even though long-term goals were critical and essential to him, a devil's advocate in the back of his mind kept telling him that he could get certain results faster with traditional methods. In fact, he pointed out that "good" students sometimes want the traditional approach because they have found it easy to be successful with it.

Bryan and Marie both considered the project a success. What they learned will guide their review and reteaching of the unit. Some of the benefits had immediate impact on Bryan's teaching style. Following this unit, he implemented cooperative learning in a Grade 9 unit on surface areas of prisms and cylinders, where he had previously used concrete materials. Not only has he shown more willingness to try innovative approaches with other classes but also he feels an increased responsibility to continue and broaden his teaching techniques. All teachers who experience similar anxiety should have confidence knowing that moving out of comfort zones is an indication of professional growth.

Actions and Fraction Patterns

Thomas E. Kieren

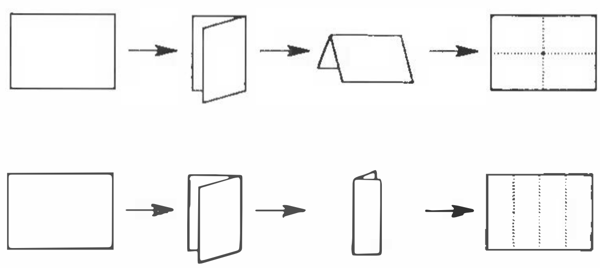
If whole numbers allow us to look at the world through the lens of *how many*, fractional numbers allow us to look at it through the lens of *how much*. The materials such as those shown here allow children to explore an image of fractions that says *fractions are numbers which show amounts*. The fraction activity called Covering Fractions, described and discussed here, is for young children and was used with Grade 3 students.

If one looks at the rational numbers mathematically, they form a field and have a formal additive and multiplicative nature. The same is true for fractional thinking in children. Before using the materials here which consider fractions as amounts (additive), it is useful to have children explore fractions as actions which generate them (multiplicative). For example, before using the Covering Fractions set, children should fold a sheet identified as one unit into 2, 4, 8 and 16 equal pieces. In particular, they should explore and discuss such concepts as *one eighth is one half of one fourth*, and *I can get sixteenths in four folds*. Such actions allow children to experience the multiplicative, exponential growth of the number of parts, compensated for by the multiplicative, exponential decline of the part size. That is, children should join in discussing the fact that folding into sixteenths gives more (4 times more) parts than a fold into fourths, but that these parts are smaller (one fourth is made up of 4 sixteenths).

In terms of the pieces that result as a product of the fold, or the fractional amounts, children should consider two kinds of questions:

Author's Note: This research was supported by a SERAC grant from the Faculty of Education at the University of Alberta.

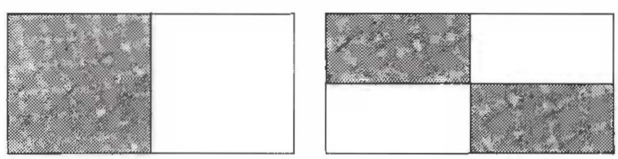
Can different fourth folds give the same sized *pieces?*



For children who do not conserve area, this is not a trivial sorting out; they should experiment and see that, at least for fourths, differently folded shapes can have the same size. It is size compared to the size of the unit that is important.

Can different combinations make the same amount?

A discussion of this idea could be started using the simple activity below:



Gina: "Those can't be the same; one is one half, the other is two fourths."

Theo: "Well, they make the same amount. You could just slide the pieces together."

Who do you think is right, Gina or Theo? Why?

Using Covering Fractions

Questions One and Two

The first two exercises in the set included at the end of this article introduce the idea of *covering* as *showing the same amount*. They also allow students to see and discuss the relationship between a unit and a part and the part and the unit. Children can be asked to complete these exercises orally or in writing or both. If work is done in writing, children should be encouraged to use either fraction words, for example, four eighths is one half, or symbols, 4/8 = ½. Different children will choose different options and some children will mix the use of words and symbols. What is important is what they say and how they reason about it.

Questions Three and Four

Items 3 and 4 allow students to act and talk about using different fractions to show the same amount. These exercises take advantage of the fact that children are aware of a general *equivalence* concept based on *is as much as*. Some coverings will use just one kind of piece. This action leads to the standard notion of equivalence. But the more general idea of *as much as* is a powerful notion that children should use and write and talk about.

Questions Five and Six

Items 5 and 6 generated much student discussion in the Grade 3 class. Children were able to evaluate coverings that they hadn't made. That is, students were able to imagine how another student had made a covering and comment on the work in physical and mathematical terms without actually doing the particular covering themselves.

Tom reported, "¾ = ¼ + 2/8 + 4/16."

Jodie said, "Oh he's right—each [fraction] makes one fourth."

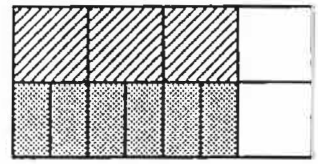
Here is another example of this kind of analysis.

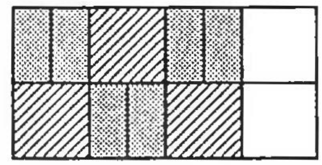
George said, "Three sixteenths and one eighth and five sixteenths makes three fourths."

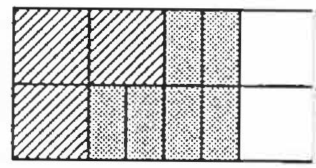
Kay responded, "No, that's one eighth too little. 8/16 is ½, you need two eighths more."

This activity allowed for the development of a healthy mathematical atmosphere where children generated, defended and discussed real mathematics.

Item 6 reflects student interest in the aesthetics of fraction patterns.







Students thought the middle pattern was "neat," while the covering at the top prompted two different discussions.

"¾ is 3 repeats of ⅛ and 2/16."

The above comment represents an intuitive sense of distributivity.

"¾ is made up now of half eighths and half sixteenths in this pattern."

Thus, in building fractional patterns, beauty and mathematical thinking go hand in hand.

Question Seven

Item 7 proved to be one of the most interesting and exciting. All except two of the children were able to generate, describe in fraction words or symbols, and defend their solutions to this pattern puzzle. These solutions allowed a discussion of order compared to one unit and also allowed students to compare coverings with one another.

"½ + 2/4 + 1/16 is closer to 1 than ¾ + ⅜. Yeah, it's 1/16 closer."

One numerically-facile boy wrote the following on the board:

½ + ¼ + ⅛ + 1/16 < 1

At this, the whole class blurted out:

"Oh yeah, only 1/16 more gets you to one," or something like it.

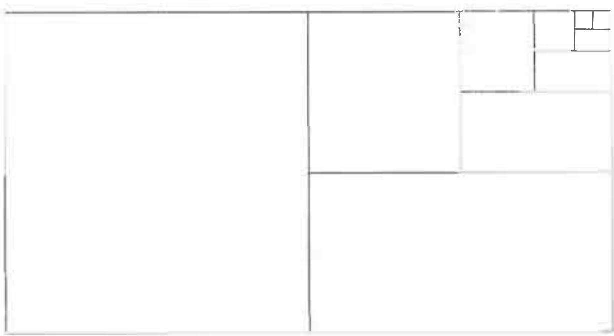
The teacher then asked the boy if he could make a covering which was even closer to one. He said, "Yup. Add 1/32."

To which Stacey added: "Then you could just cut a 1/32 piece in half and add it, but I don't know how to say it." A chorus of 10 children saying "one sixty-fourth" was heard in response. About six children pursued this further and reported, "We've got it out to 1/1024."

One then wrote on the board:

½ + ¼ + ⅛ + 1/16 + 1/32 + 1/64 + 1/128 + 1/256 + 1/512 + 1/1024

One of the group drew a picture of the covering as follows:



Aaron responded, "There'd just be a dust mote left."

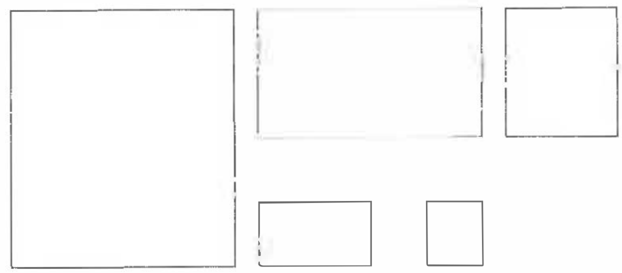
Summary

The materials discussed here allow children to actively build and discuss ideas of fractions. In this work they come to see an important property of fractions: different combinations of fractional numbers make the same amount. Using their own local individual properties, they looked at ideas such as equivalence, order, addition, fractions more than one and even the beginnings of patterns of the infinite. They used language in a powerful way, articulating actions and visual patterns. And they saw in their own work mathematical patterns—geometric and numerical—and their beauty.

Rather than a narrow, trivial, static view of fractions, these children saw fractions in terms of their own actions and of an imaginative world the materials allowed them to create.

Covering Fractions

Materials: A kit containing
- two unit, or one, pieces
- a number of ½ pieces
- a number of ¼ pieces
- a number of ⅛ pieces
- a number of 1/16 pieces



Instructions:

1. Open your kit. The largest piece is one unit or one.
   Find all of the other pieces.
   On each piece, write the fraction name which describes it.

   Example: one half or ½

2. Some questions:
   a) Making one:
   - How many one-half pieces do you need to make one?

   ____

   You can write:
   Two half pieces make one
   or
   2/2 = 1

   - How many one-eighth pieces make one?

   ____

   Write about it. ______________________

   ____

   - How many one-sixteenth pieces make one?

   ____

   Write about it. ______________________

   ____

b) Use just one other kind of piece to make or cover one half piece. Draw pictures of what you find in the space below:

Write about what you find.

Example: Eight sixteenths makes one half

$8/16 = \frac{1}{2}$

3. Here is a puzzle:

It is one whole covered by 2 halves.

Show how you could replace one half and still cover one whole. Use the space below.

Can you do this using just one kind of piece? Show how.

Can you do this using three kinds of pieces? Show how.

4. Faye said, "$1 = \frac{1}{2} + \frac{1}{4} + 2/8$."
Is she right? ______________________

Why? ______________________

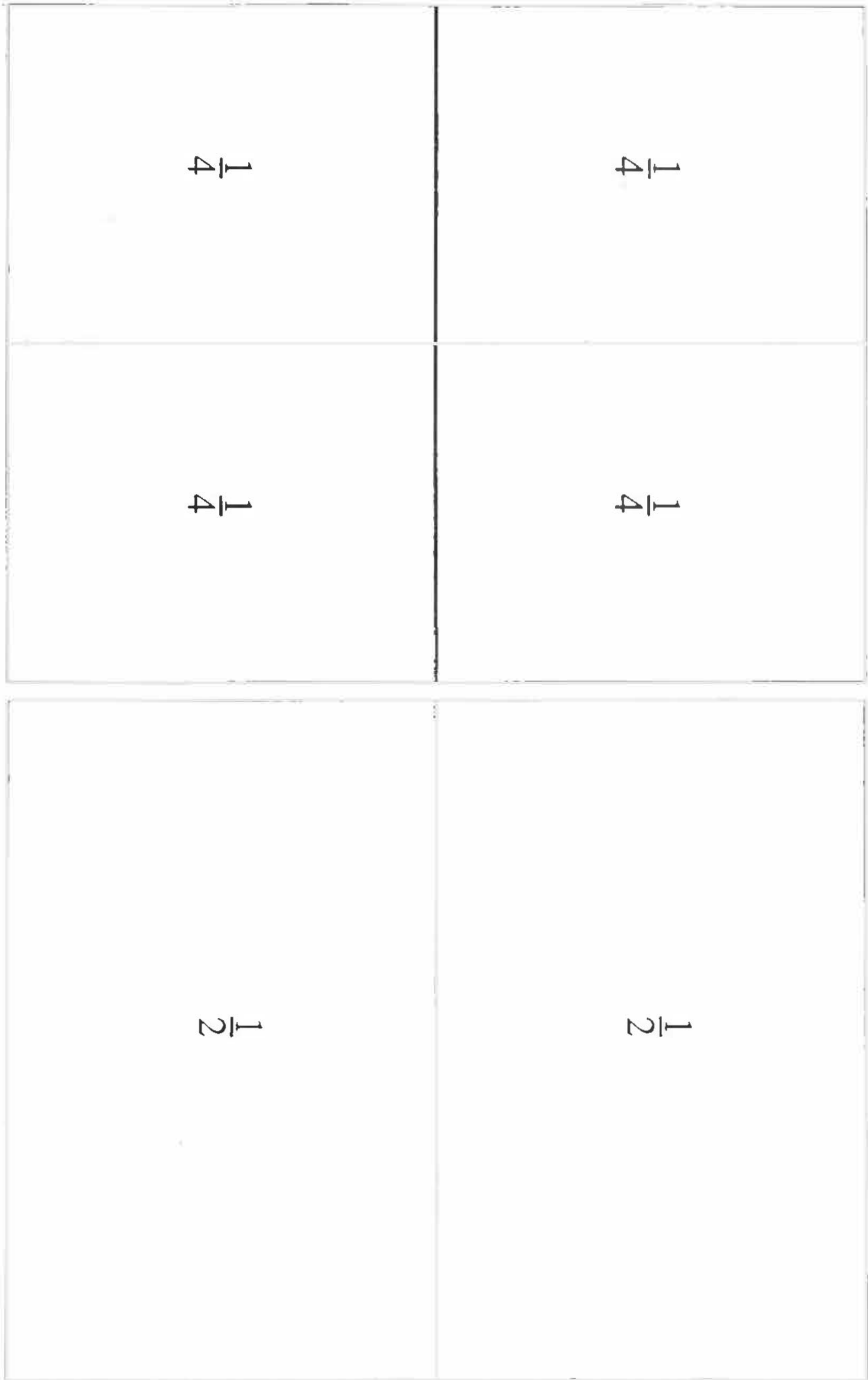
5. Take three quarters or three fourths out of your kit.
Cover three fourths (or three one-fourth pieces or $\frac{3}{4}$) in as many ways as you can.
Draw each covering. Write a sentence about each.
You can use words or symbols.

6. Draw your favorite covering of three fourths.

Why is it your favorite? ______________________

7. Make a covering which is almost equal to one whole but is a little more or a little less than one whole.
Draw it and write about it.

8. Lou said, "$\frac{1}{2} + \frac{1}{4} + 2/16$ is less than one."
Is he right? ______________________
Why? ______________________



Note: The teacher may wish to enlarge these fraction coverings to fill an entire page.

$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$
$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$
$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$
$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$

$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

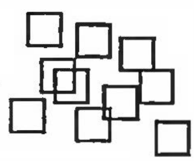
Note: The teacher may wish to enlarge these fraction coverings to fill an entire page.

A Bunch of Squares + Some Children = Lots of Mathematics

W. George Cathcart

At times we become envious of all the interesting manipulative aids we see in catalogs and conference displays and described in articles. "I could sure do a better job of teaching mathematics if I had these!" we think. Indeed, in general, these aids are useful and can contribute to a better mathematics program. But all is not lost. If you do not have the aids you envy and there is no money left in the budget to purchase them, you can still be creative. For example, you can use the paper cutter to produce a bunch of squares (about 2 cm x 2 cm) from different colors of mayfair (or heavier) paper.

What can I do with a bunch of squares? Lots!!



Prenumber and Number

Sorting

1. In groups, children can sort the squares according to color.
2. For each group, prepare a set of squares of predominantly one color, but add a few of a different color. Ask, "Which does *not* belong?"

Patterning

Have children work in pairs. One student creates a pattern with the colored squares and the other demonstrates recognition of the pattern by extending it. The more capable students might be challenged to generate two-dimensional patterns (Figure 1).

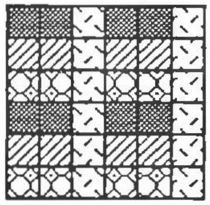


Figure 1. Two-Dimensional Pattern

Counting

The squares could, of course, be used for conventional counting experiences.

1. Arrange a set of squares in a linear order and ask certain children to count them beginning
   - at the left end,
   - at the right end, and
   - with a specific square, not at either end.
2. Arrange some squares in a random arrangement and ask certain children to count them.

Numeration

The squares can be used for grouping activities. Groups of five can be put together with paper clips and five groups of fives can be bundled with an elastic band. The same thing can be done with 10 and 10 tens. A group of 10 tens might be more easily handled by placing it in a small jar or box.

If the squares have been cut from stock of different colors, they can be traded. For example, in a small-group setting, children could play a trading game using a board as shown in Figure 2. The object is to get at least one blue square. Players take turns rolling a die. The die tells the player how many red squares to pick and place on the board under

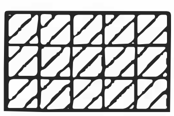
"Red." When five red squares are collected, they must be traded for one yellow square. Continue collecting red squares. When five yellow squares have been accumulated, they are traded for one blue. The first player to get a blue could be declared the winner or the game can continue until all players have a blue.

Alternatively, players could start with one blue square. The roll of the die tells how many red squares to remove. In order to remove red squares, the blue must be traded for five yellow, and at least one yellow traded for five reds. The object is to remove all squares from the board.

Blue	Yellow	Red

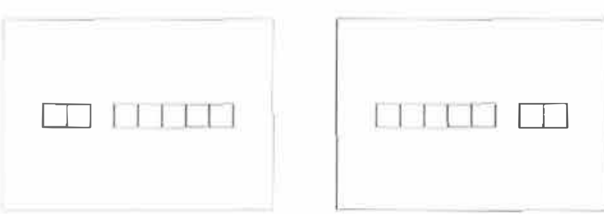
Figure 2. Trading Board

Whole Number Operations



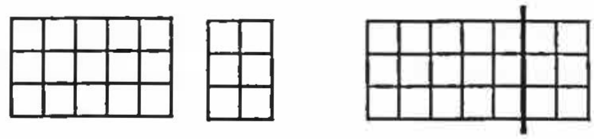
The squares can be used as counters to demonstrate the joining and separating action involved in the operations. The array interpretation of multiplication can also be shown easily with squares as illustrated here.

Properties such as the commutative property and the distributive property of multiplication over addition can be shown with the squares. Placing a set of two and a set of five (2 + 5) squares on a sheet of paper and rotating the sheet 180° should help children visualize the commutative property (Figure 3). Figure 4 suggests how the distributive property of multiplication over addition could be demonstrated with squares.



(a) (b)

Figure 3. (a) 2 + 5 (b) Rotated To Show 5 + 2



$(3 \times 5) + (3 \times 2) = 3 \times (5 + 2)$

Figure 4. Squares Showing Distributive Property

Fractions and Ratio

One interpretation of a fraction is a subset of a set. This notion is normally introduced after the region model. Squares can be used in this context. Give children three red squares and one blue square. Ask, "What fraction of the squares is blue?" "What fraction is red?"

Using the same setting, ask, "What is the ratio of blue squares to red squares? What is the ratio of red squares to blue squares?"

Measurement

Length

A square could be used as a nonstandard unit of measure. "How many squares long is your desk?" The length of many familiar objects in the classroom (a book, pencil, chalkboard, computer screen and so on) could be determined.

The perimeter of certain objects could also be measured with squares. Students could align squares around their mathematics text, desk, table and so on, then count the squares to determine the perimeter in squares.

Area

A surface, such as the cover of a mathematics text, desk top or other convenient surface, could be covered with squares, which are then counted to find the area of the surface in squares.

Geometry

Polyominoes is but one example of many geometric concepts that children could explore with a set of squares. Children have to understand that, when placing squares together, the sides must be coincident as shown here.

correct incorrect

Have the children take one square and then ask, "How many different ways can it be arranged?" Take two squares. How many different ways can two squares be arranged? Here children will need to think about rotations. Is the vertical arrangement different from the horizontal arrangement? No, one is a 90° rotation of the other.

Use three squares. How many ways can they be arranged? Now children will also have to think about flips. In Figure 5, is arrangement (a) different from arrangement (b)? No, one is a flip of the other.

(a) (b)

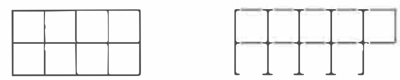
Figure 5

How many different arrangements are there for four squares? Five squares?

Number Theory

Odd/Even

The concepts of odd and even are frequently introduced in Grade 2. Young children can use the squares to help them understand these ideas. A set of squares that can be placed in a 2 x *n* array represent an even number. An odd number of squares results in a "missing piece" when arranged into a rectangle of width two squares.

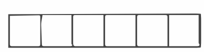


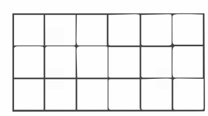
Even Odd

Prime and Composite Numbers

To help children understand the concepts of prime and composite and the difference between them, an activity like the following could be prepared.

Activity: Prime and Composite Numbers

A sidewalk: 

A patio: 

Take the number of squares indicated in the left-hand column. Construct as many different sidewalks and patios as you can with the squares. Record the sizes in the middle column and shade the correct circle in the last column.

Squares	Sizes	Designs Made
2	Sidewalk: 1 x Patio sizes:	O Sidewalk only O Both sidewalk and patio
3	Sidewalk: 1 x ______ Patio sizes:	O Sidewalk only O Both sidewalk and patio
4	Sidewalk: 1 x Patio sizes:	O Sidewalk only O Both sidewalk and patio
5	Sidewalk: 1 x ______ Patio sizes:	O Sidewalk only O Both sidewalk and patio

. . . and so on

List the numbers that form sidewalks only:
(These numbers are called PRIME numbers.)

List the numbers that form both sidewalks and patios:
(These numbers are called COMPOSITE numbers.)

Figurate Numbers

Some patterns involving triangular and square numbers can be explored with a simple set of squares. Squares may not be suitable for exploring more complex figurate number patterns. The following three activities may be appropriate for middle or junior high school grades. The first activity, *stair steps*, develops the pattern for generating triangular numbers. The pattern is started at the right.

$$1 = 1$$
$$1 + 2 = 3$$
$$1 + 2 + 3 = 6$$
$$\cdots$$

Activity: Stair Steps

Use the squares to construct a series of stair steps. In the second column, record the number of additional squares needed to build the final step after the previous steps have been completed.

Number of Steps	Number of Additional Squares Needed for Final Step	Total Number of Squares Used
1	1	1
2	2	3
3		
. . . and so on		

Use the numbers in column 2 to get the total number in column 3.

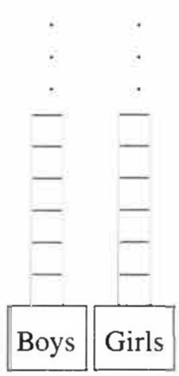
Describe the relationship that you found.

What geometic figure do your stair steps resemble?

The second activity, *squares*, explores the pattern involved in generating successive square numbers, and the third activity, *stairs and squares*, is an attempt to help children discover the relationship between triangular and square numbers.

Graphing

Graphing experiences for young children need to be concrete. Paper squares can be used in this setting as well. To compare the number of boys and girls in the class (or small group), each boy and girl could place a square, in a line, opposite the appropriate label as shown. Alternatively, the squares could be pinned on the bulletin board.



The squares could be used to represent almost any event and pinned on the bulletin board, laid end-to-end, or pasted on paper to form a bar graph. A design or symbol could be drawn on a set of the squares which could then be used as a pictograph.

Activity: Squares

Use the paper squares to construct a series of squares. In the second column, record the number of additional squares needed to construct the square using the previous one as a start.

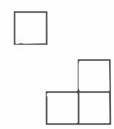
Size of Square	Number of Additional Squares Needed Over Previous Size	Total Number of Squares Used
1 x 1	1	1
2 x 2	3	4
3 x 3		
. . . and so on		

Use the numbers in column 2 to get the total number in column 3.

Describe the relationship that you found.

Activity: Stairs and Squares

The stairs you built can be placed together. Here the stairs with one square and three squares are being put together to form a 2 x 2 square.



Place the stairs with three and six squares together to form a square. What size square did you get?

Do the same with the six- and ten-square stairs.

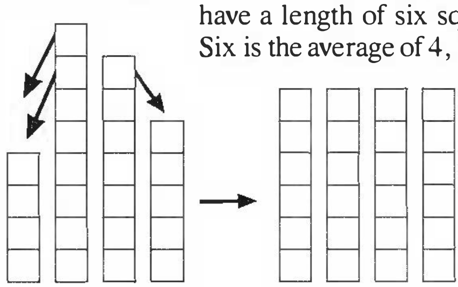
Try placing any two successive stairs together. Do you get a square? What size?

Write a sentence about the relationship between stairs (triangular numbers) and square numbers.

Statistics

Mean (Average)

The mean or arithmetic average is often taught in the late elementary years. A set of squares can be used to help children understand the concept of the mean. For example, the numbers 4, 8, 7 and 5 could be represented as bars of squares. To get the average, all bars are made the same length. Children could take two squares from the eight-bar and place them on the four-bar giving both bars a length of six squares. Then one square could be taken from the seven-bar and added to the five-bar. Now all four bars have a length of six squares. Six is the average of 4, 8, 7, 5.



Conclusion

Manipulative aids such as those displayed at conferences or described in publishers' catalogs are highly recommended. In the absence of a specific aid, materials that are easily collected or prepared can often serve as a valuable substitute to help children build an understanding of mathematical concepts. We have seen that a variety of topics can be explored with small squares cut from reasonably sturdy paper. They are not elaborate, but they will do the job! No doubt readers can suggest other examples of topics that can be demonstrated with a set of squares!

Birthdays: A Rich Source of Problems

K. Allen Neufeld

One of the happy things that most of us remember from our elementary school days was the attention that was given to birthdays. Apart from students with birthdays celebrated during school holidays, everyone had the pleasant experience of being "birthday girl" or "birthday boy" once a year. Birthdays can provide the basic numerical data which give rise to mathematical problems and activities related to graphing, probability and computation. The data can be presented in one or more of the concrete, pictorial or symbolic modes.

Graphing (Concrete)

The Alberta Program of Studies includes the objective at the Grade 1 level: "collects data from the immediate environment to construct graphs using pictures or objects, and discusses the results." The following illustrations indicate some ways in which readily available manipulative materials can be used to construct a three-dimensional graph. In Figure 1, interlocking beads are used on the numbers 1–31 on a Hundreds Board frame to indicate the day of the month on which the birthdays of a particular class occur. Figure 2 uses colored wooden cubes to display the various birthday months of the same group of children. A pair of curved wire abacuses are used in Figure 3 to illustrate the data for the day of the week on which each child was born. Grade 1 children will probably have to ask their parents for these data. Each child can participate in the building of these graphs by placing the bead, block and abacus counter in the appropriate place.

Graphing (Pictorial)

The Alberta *Program of Studies* (Alberta Education 1991) includes the objectives at the Grade 3 level: "collects, constructs and interprets pictographs and simple bar graphs" and "locates position of an object on a grid." The following illustrations present the birthday data from the previous activity in a two-dimensional format. All of the birthdays for a particular group of children can be charted on a calendar such as the one included as Figure 4. The source of this particular calendar was the March 1984 issue of *Student Math Notes*. Each child draws a small circle around the date representing his or her birthday. If two children share a particular date (not as unusual as you might think), a second circle is made for that date (for example, May 5). Figure 5 shows a grid that is another way of presenting the birthdays. Again it is necessary to have some way of indicating two birthdays that fall on a particular day. Figure 6 is a bar graph for each of the days 1–31. Figure 7 presents a bar graph for each of the 12 months and Figure 8 that of the number of children who were born on a particular day of the week. The data for the last bar graph could be replaced with data on the day of the week on which each child's birthday fell in 1991.

Probability (Symbolic)

Although the present Alberta *Program of Studies* does not formally include probability objectives prior to the junior high school grades, many teachers provide children with intuitive experiences in this area. It is quite likely that the next revision of the Program will include such concepts as the chance component of probability, using terms like *always*, *never*, *sometimes* and *maybe*.

A common question often asked about birthdays is: What is the likelihood of two people sharing a birthday (same month and day)? One way of explaining this at the elementary level is to consider the idea

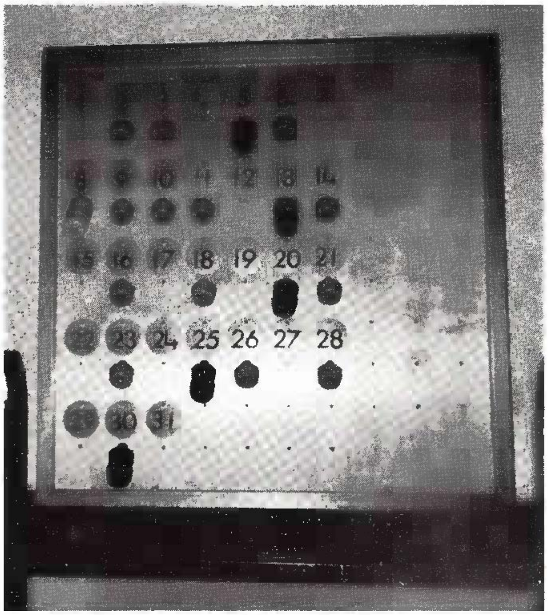


Figure 1

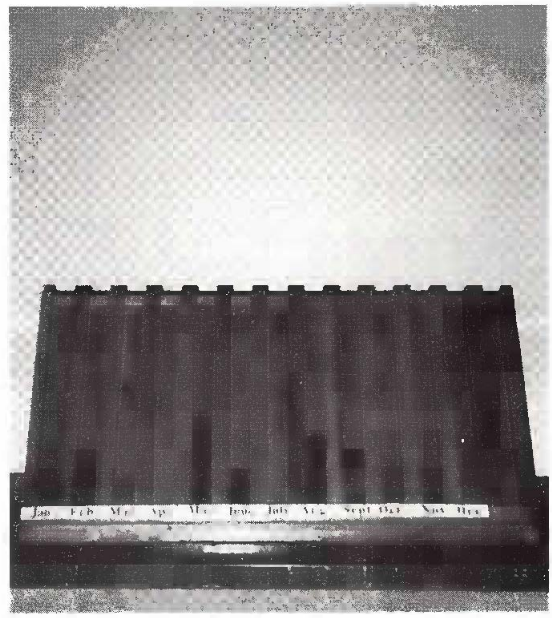


Figure 2

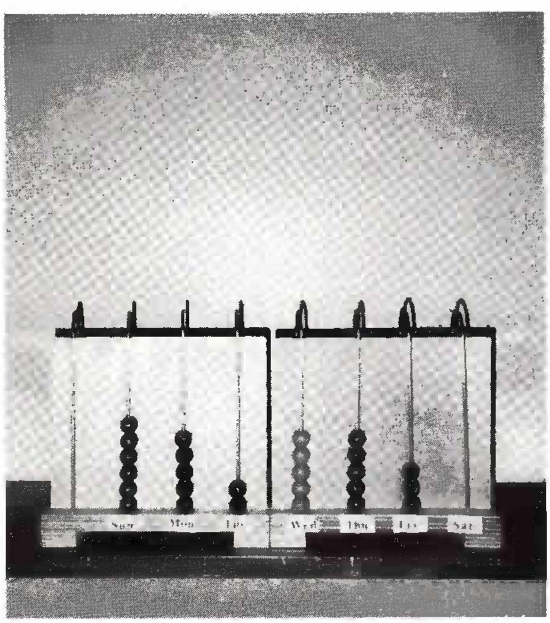
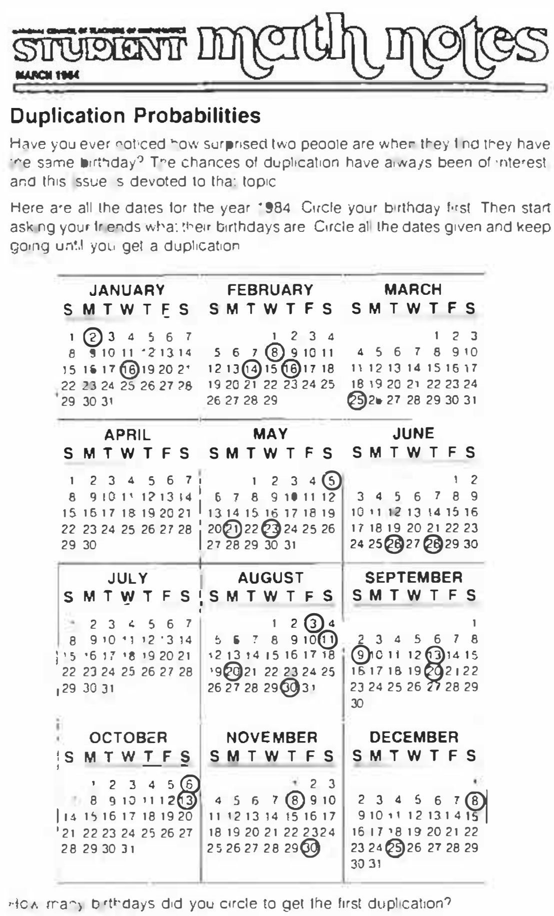


Figure 3



STUDENT math notes

MARCH 1984

Duplication Probabilities

Have you ever noticed how surprised two people are when they find they have the same birthday? The chances of duplication have always been of interest and this issue is devoted to that topic

Here are all the dates for the year 1984 Circle your birthday first Then start asking your friends what their birthdays are Circle all the dates given and keep going until you get a duplication

JANUARY

S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

FEBRUARY

S	M	T	W	T	F	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29			

MARCH

S	M	T	W	T	F	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

APRIL

S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

MAY

S	M	T	W	T	F	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

JUNE

S	M	T	W	T	F	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

JULY

S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

AUGUST

S	M	T	W	T	F	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

SEPTEMBER

S	M	T	W	T	F	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30						

OCTOBER

S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

NOVEMBER

S	M	T	W	T	F	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

DECEMBER

S	M	T	W	T	F	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

How many birthdays did you circle to get the first duplication?

Figure 4

Birthday Grid

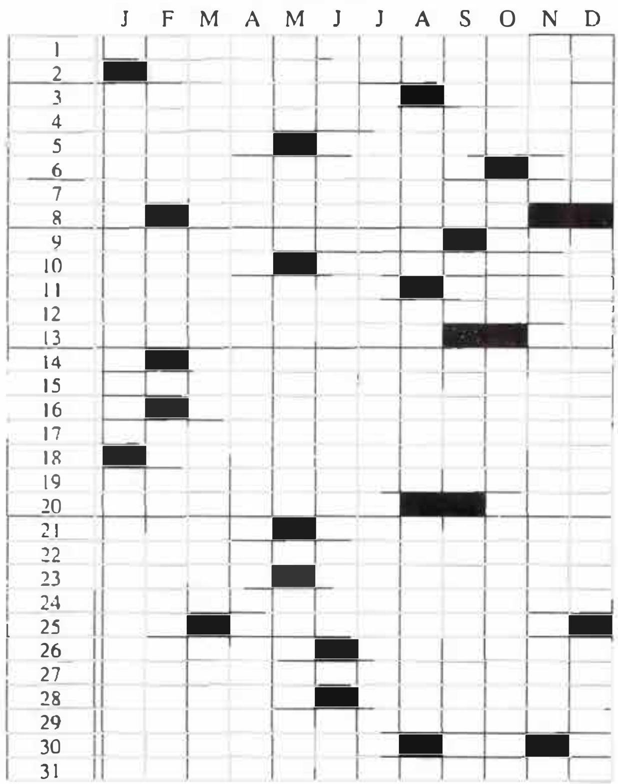


Figure 5

Same Day Number

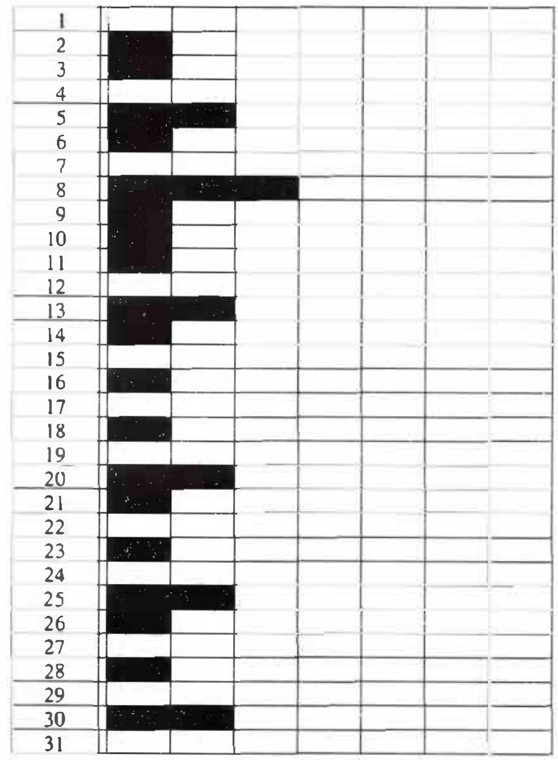


Figure 6

Same Month

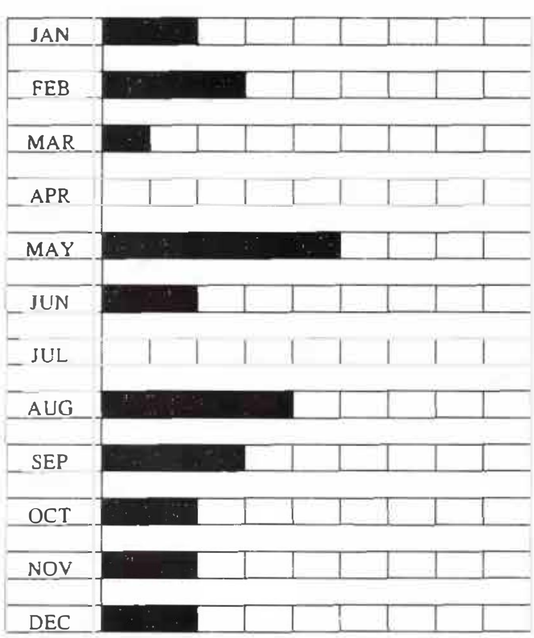


Figure 7

Same Day of Week

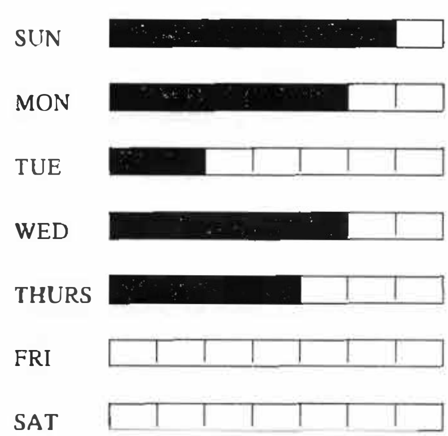


Figure 8

of pairs. How many pairs are there in a classroom of 25 children? An initial strategy would be to pose a less complex problem such as: How many pairs are there in a group of three children (for example, Mary, Bob and Ann)? We could have three pairs, Mary and Bob, Mary and Ann, or Bob and Ann. After considering other small groups one could note that the pattern is the same as in the familiar "handshake problem." The equation $[(n)(n-1)] \div 2 = p$ applies where n is the size of group and p is the number of pairs. Thus, 25 children would result in [25 x 24] 2 = 300 pairs. With 366 different possible birthdays (in a leap year), it becomes evident that the 300 different pairings in a group of 25 children is not very far from that number. In fact, a group of 27 or 28 would result in 351 or 378 pairs, respectively.

The data shown in Figure 5 were based on a particular class of 26 children. The number of pairs would be [26 x 25] 2 = 325. One could calculate the quotient 325 ÷ 366 = 0.89 as the expected number of pairs. It is not surprising, then, that one of the pairs in this particular group of 26 did, in fact, share a birthday.

The data in Figure 7 can be used to answer the question: What is the likelihood of two people in a group of 26 sharing a particular month of birth? As there are 325 pairs and 12 months, the expected number of people sharing a particular month could be calculated as 325 ÷ 12 =27, rounded to the nearest whole number. In January two people have a birthday, giving us a pair. The three people who share a February birthday give us three more pairs. Because only one person has a birthday in March, there are no additional pairs. For the other months, the number of pairs are as follows: May (five people)–10 pairs, June (two people)–1 pair, August (four people)–6 pairs, September (three people)–3 pairs, and two people in each of October, November and December, resulting in 3 more pairs. The total number of actual pairs who share a particular month of birth is 27, exactly the same as the expected number.

The number of pairs for groups of sizes increasing by one each time also exhibits a pattern which children have probably encountered in other contexts. The differences increase by one each time to form the sequence: 1, 3, 6, 10, 15, 21, 28, 36, . . .

The data shown in Figure 6 can be used to answer the question: What is the likelihood of two people in a group of 26 sharing a particular day number (for example, both having a birthday on the fifth day of the month but not necessarily in the same month)? Because there are 325 pairs and 31 days, the expected number of people sharing the same day number could be calculated as 325 ÷ 31 =10, rounded to the nearest whole number. In the figure, each of the numbers 5, 13, 20, 25 and 30 is shared by a pair. The number eight is shared by three people, resulting in three more pairs for a total of eight pairs who share a day number. In this example, the actual number, eight, is less than the expected number, 10.

The data shown in Figure 8 can be used to answer the question: What is the likelihood of two people in a group of 26 being born on the same day of the week (for example, both being born on a Sunday)? As there are only seven possible days, it is obvious that a large number of pairs would share a birthday on a particular day of the week, many more than the number of pairs who share a particular day number or month name. The expected number of people sharing the same day of the week could be calculated as 325 ÷ 7 = 46, rounded to the nearest whole number. The six people who share a Sunday birthday result in 15 pairs. For the other days of the week the numbers of pairs are as follows: Monday (five people)–10 pairs, Tuesday (two people)–1 pair, Wednesday (five people)–10 pairs, Thursday (five people)–10 pairs, Friday (three people)–3 pairs. The total number of actual pairs who share a particular day of the week is 49, slightly more than the expected number of 46.

Computation (Symbolic)

For children in the upper elementary grades who do not know the day of the week on which they were born, a few relatively simple computations will quickly reveal the desired information.

To find the day of the week on which you were born, do the following calculations, using as a sample birthdate July 1, 1867.

Last 2 digits of year ÷ 4 (ignore remainder)	67 ÷ 4 = 16
Month code (see below)	0
Day	1
Last 2 digits of year	67
Century code (see below)	2
SUM	86

Sum ÷ 7   86 ÷ 7 = 12 with a remainder of 2

Remainder code (see below)   Monday

Thus, our country, Canada, was born on a Monday.

Codes

Month		Century	Remainder
Jan – 1 (0)**	July – 0		1 – Sunday
Feb – 4 (3)	Aug – 3	1700s – 4	2 – Monday
Mar – 4	Sept – 6	1800s – 2	3 – Tuesday
Apr – 0	Oct – 1	1900s – 0	4 – Wednesday
May – 2	Nov – 4	2000s – 6	5 – Thursday
June – 5	Dec – 6	2100s – 4	6 – Friday
			0 – Saturday

**(use bracketed number if you were born in a leap year)

Do the calculation above to find out on what day you were born, and then read the rhyme below to find out what kind of a child you were!

Monday's child is fair of face,
Tuesday's child is full of grace,
Wednesday's child is full of woe,
Thursday's child has far to go.
Friday's child is kind and giving,
Saturday's child works hard for a living.
But the child who is born on the Sabbath day
Is bonny and blithe, and good and gay.

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Alberta Education. *Mathematics Component of the Program of Studies for Elementary Schools.* Edmonton: Author. 1991.

Area of *Taxicab* Geometry Circles

David R. Duncan and Bonnie H. Litwiller

Mathematics teachers are always interested in finding activities that lead their students to apply familiar concepts in novel settings. Geometry provides a rich source of such activities.

The concept of the area of a circle is a familiar geometric idea. Most students can recall the familiar formula, $A = \pi r^2$, where A is the area of the circle, r is its radius, and π is the constant = 3.1416.

What would happen if these same concepts were considered in a *taxicab* geometry setting, that is, a setting in which distances are only measured along preestablished horizontal and vertical routes? We shall consider two examples of taxicab geometry, using square dot paper and isometric dot paper.

Square Dot Paper

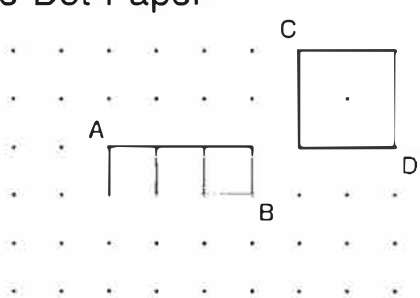

Figure 1

Figure 1 depicts square dot paper with all possible route lines of minimal length drawn from A to B. In this illustration the distance between A and B is 4, and there are four separate routes of that minimal length. The distance between C and D is also 4, but there are now six different routes of that length. Also note that there are many additional routes from A to B whose lengths are greater than 4 units; however, a shortest route will define the distance between A and B or between any two distinct points.

Recall that a circle is the set of points equidistant from a fixed point, called its centre. Using this definition, what would a circle look like in a square dot domain? Figure 2 shows Circle 1 of radius 3 with centre *0*. Each of the 12 points that lie on the circle has a *taxicab* distance of 3 units from *0*. We have connected the 12 points in Circle 2 for counting units of area.

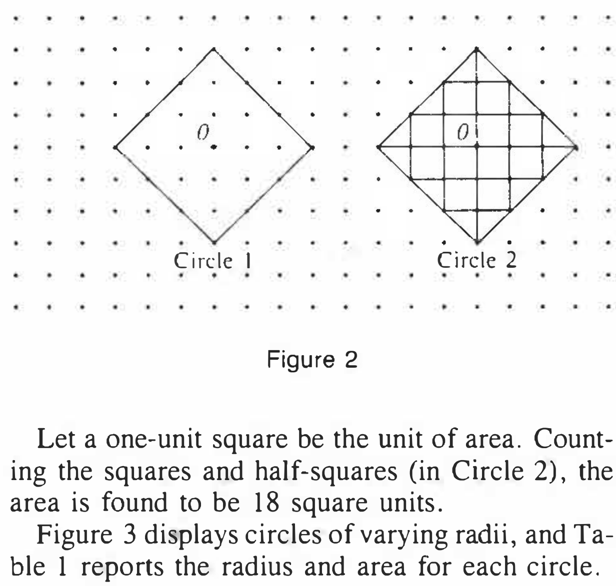

Figure 2

Let a one-unit square be the unit of area. Counting the squares and half-squares (in Circle 2), the area is found to be 18 square units.

Figure 3 displays circles of varying radii, and Table 1 reports the radius and area for each circle.

Table 1

Circle	Radius	Area in Squares
A	1	2
B	2	8
C	3	18
D	4	32
E	5	50
F	6	72

What general formula could be used to compute these areas? Can your students conjecture, from the data in Table 1, that $A = 2r^2$? This taxicab formula has the same form as the familiar $A = \pi r^2$ in Euclidean geometry, with π replaced by the constant 2.

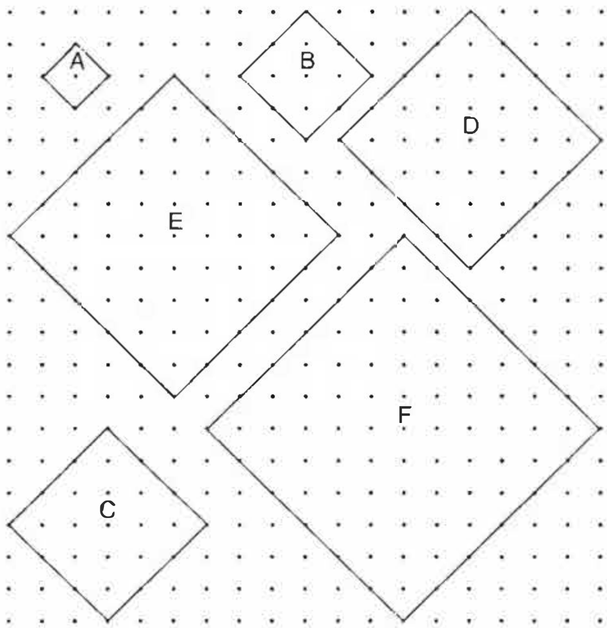

Figure 3

Isometric Dot Paper

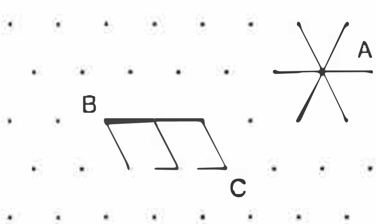

Figure 4

Figure 4 displays the basics of line segments on isometric graph paper. There are six possible one-unit line segments from point A, as shown. Figure 4 also depicts all possible routes drawn from B to C. The distance between B and C is three because the shortest distance from B to C uses three units. Note that there are three separate routes of that minimal length. We also note that there are many additional routes whose lengths are greater than three units. Again, the shortest route will define the distance between A and B or between any two points.

In this setting, what is the analogue to the Euclidean circle? Figure 5 shows Circle 1 of radius 2 with centre *0*. Each of the 12 points that lie on the circle has a taxicab distance of two units from *0*.

Let an equilateral triangle of side one-unit be the unit of area. Counting the triangles in Circle 2 of Figure 5 yields an area of 24 triangular units.

Figure 6 displays circles of varying radii, and Table 2 reports the radius and area for each circle.

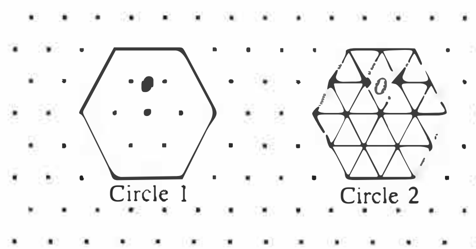

Figure 5

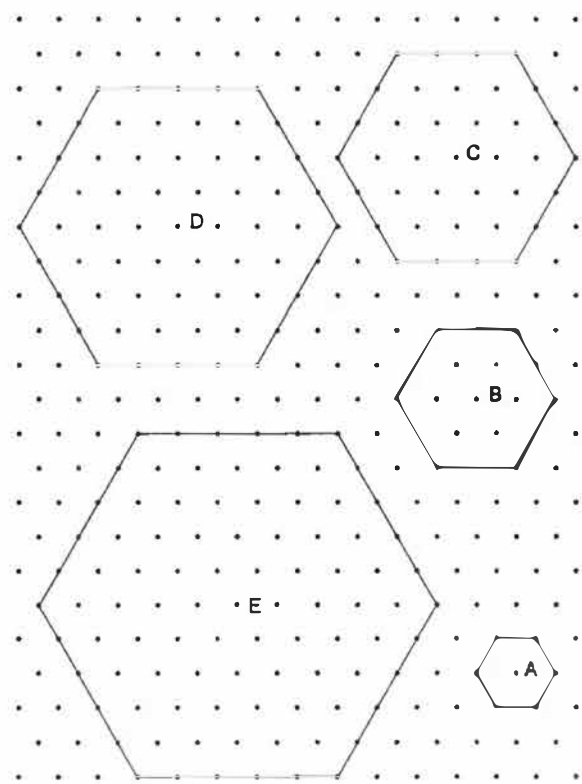

Figure 6

Table 2

Circle	Radius	Area in Triangular Squares
A	1	6
B	2	24
C	3	54
D	4	96
E	5	150

Using the data from Table 2, can your students conjecture that $A = 6r^2$? Again, this taxicab formula is similar to the familiar $A = \pi r^2$, except that π is replaced by the constant 6 in this new setting.

Challenges for the reader and his/her students:

1) Check the formulas that we conjectured using circles of other radii.
2) Conjecture formulas for the circumference of the taxicab circles of Figures 3 and 6.
3) Are there three-space, four-space or *n*-space analogues to the point lattices discussed in this article?

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Hi-Tech Schools: The Path of the Paddle

J. Dale Burnett

I often begin an article such as this with a quotation. I would like to begin this with a name—Mr. James Grieve. He was my high school physics teacher in Jasper.

My primary interest is education, which I care about more deeply than I could ever have imagined. I am watching a system evolve that is increasingly unresponsive to the needs and desires of individual teachers and learners, while, according to the rhetoric, "this is the best of times" for our system. Technology per se is not the problem, but in the hands of unphilosophic kings it is an awesome weapon. However, unlike atomic weapons, which seem to have few redeeming social values, computer technology may have two sides to it.

I will highlight a few issues related to the use of technology in education. This is an important article because I am trying to influence you. I may or may not succeed, but I can assure you that I want to succeed. I would like to raise the level of discussion and debate on these issues because I consider them important in the evolution of our society.

We are approaching an era in which computing hardware is close to meeting our every wish. The question then becomes, so what? (It is true that thoughtful educators have been asking this since the 1960s; Bushnell and Allen 1967.) We also have telephones, television, automobiles, aircraft and weapons technologies that have been developed to a high degree, yet have had minimal impact on classrooms. We usually do not criticize educators for failing to adopt these technologies. Should we criticize them for failing to adopt computer technology? Are all forms of technology inherently irrelevant to education? Or is computer technology somehow different?

An earlier form of this article was presented as a paper at "Curriculum at the Centre," a national conference on curriculum, instruction and leadership, in Montreal in 1989.

Upstream: The Last 10 Years

Microcomputers have been with us for about a decade, although today's micro looks very little like the machines of just five years ago. The response of the educational community to this development was generally positive and enthusiastic. Two basic themes prevailed. One, it was politically desirable to have at least one microcomputer in every school. Two, a new set of courses was created, with names like Computer Literacy, Computer Studies and Informatiques. The basic content of these courses focused on the history of computing and on programming, usually in BASIC. Software developers produced a plethora of products, most of the drill and practice variety. In just a few years we have realized that having a computer in a school can be more of an embarrassment than a source of pride. Computer literacy courses are also on the ropes. It is becoming clear that computer programming is not a necessary skill for the profitable use of computers —in many ways it is actually an impediment because it ties up student time that might be better spent learning to use application packages such as word processors, data base systems and spreadsheets. Most of the claims from the software developers have a hollow ring today. There were widespread calls for increased teacher training, but little was actually done, except by those self-motivated teachers who became involved on their own time with their own money.

I view the last decade with a sense of pride, even though most of the "solutions" have not turned out too well. The pride comes from noting the sense of willingness to try new ventures. In the long haul, the results from the first few outings may not be as important as knowing you are playing the game. However, it would be naive to think that we have the techniques under control.

Around the Bend: The Impossible

Davis and Park (1987) have edited a delightful book of essays on what is impossible in various disciplines. The topics include mountaineering, biology, medicine, chemistry, computer science, technology, physics, mathematics, law, politics, economics, psychology, education, poetry, music and philosophy. It is worth listing these topics, if only to remind ourselves of the wealth of diverse activities that constitute human knowledge and are thus part of the educational enterprise. The chapters on technology and education are particularly relevant to considerations of future educational milieus. The technology chapter (Sturge 1987, 120) considers limits to computers and concludes:

> The principles of a reversible quantum-mechanical computer have been sketched out by Feynman. Its performance is not limited by quantum mechanics or by thermodynamics. Unless the particular technology by which the computer will be realized is specified, no level of performance can be ruled out as "impossible."

It is important to recognize that this conclusion refers to the machine, not the software or the human use of the machine, aspects of the total picture to which I will return for a second glimpse.

Iano (1987, 256) begins his chapter on the impossible in education:

> Everyday living is filled with the uncertain, the unpredictable and the ambiguous. The same is true of the everyday world of educational practice. Yet many educators have been trying to turn this uncertain world of educational practice into a world of definitive knowledge.

Iano proceeds to show why such definitive knowledge is impossible. A similar conclusion is arrived at by Dreyfus and Dreyfus (1986) in their review of artificial intelligence and expert system approaches. Today, one of the difficulties facing a person who is interested in the role of technology in education, is that of obtaining a balanced perspective between the various claims and counterclaims of different authors, who often have a vested interest in the outcome.

The Map: Where Are We Going?

Most of us realize that to arrive at a destination it is not only important to move but also to move in the right direction. I have extracted some illustrative goal statements from Alberta Education's policy documents.

> The aim of education is to develop the knowledge, the skills and the positive attitudes of individuals, so that they will be self-confident, capable and committed to setting goals, making informed choices and acting in ways that will improve their own lives and the life of their community. (p. 7)
>
> . . . develop the ability to think conceptually, critically and creatively, to acquire and apply problem-solving skills, to apply principles of logic, and to use different modes of inquiry. . . .
>
> . . . master effective language and communication skills, including the ability to use communications technology. . . (p. 13).
>
> (Alberta Education 1985)

I would like to contrast these statements with the following table:

Mathematics 163
MWF 10 Sec. 3B 301 Manse
Prof. R. B. Smith TA: F. Jones
Final Exam

Student ID	Grade
072-36-7345	78
140-47-7262	75
149-87-4850	88
241-01-5033	62
362-22-8625	91
384-98-9098	75
509-15-5143	94
522-17-1276	88
791-35-0107	79
798-45-6063	55
807-89-0229	72
936-01-3145	85
987-03-2678	82

Av = 78.769231
Sigma = 10.821303
Median = 79.000000

(Davis and Hersh 1986, 55)

The starkness of the chart is frightening. It would make a fitting epitaph for Education 1992. We must continue to reflect on our educational goals. A job not worth doing is not worth doing well.

A series of three books have appeared on successful management practices in the United States. I think of them as The Excellence Trilogy. *In Search of Excellence* (Peters and Waterman 1982), *A Passion for Excellence* (Peters and Austin 1985) and *Thriving on Chaos* (Peters 1987) have all become best-sellers. Not without their critics, these books nonetheless appear to have an important message for educators as well: listen to your customer. A quote from the middle book gives one the flavor of the authors' message. Commenting on a series of reports on mismanagement, they say on page xviii, "All [reviews] put the knock on mindless systems analysis and [the reviews] began the examination of a misplaced emphasis on paper rather than on people."

Maps are important. However, the map is not the river. Curriculum guidelines are important. However, guidelines are not the curriculum.

The Journey

On a river there are long stretches of calm water and deep pools. Many educators, including planners and administrators, prefer this type of situation. Control seems possible and even desirable. The curriculum remains relatively static and last year's plans still work. However, this view is not shared by all. A number of recent reviews of education have come to similar conclusions, such as "School is boring" (for example, Sizer 1984). Computer technology may become part of the solution, but not because of the glitz surrounding it—we have been misled by the arcade phenomenon. Rather, the appeal will lie in the intensity of the student's involvement with the ideas in the curriculum. Word processors permit the student to develop his/her own voice. We will have word processing across the curriculum, but it will not simply be the transferring of the term paper from one medium to another. It will be apparent (when each student has ready access to the technology) that it represents a totally new way of composing—not something to be handed in and graded, but a dynamic, flexible medium for clarifying one's thoughts. Even this paper, which may appear relatively lifeless to the reader, has been through a continuous state of change as ideas and points swirl about. For example, the canoeing metaphor emerged rather late in the process. Yet word processing is relatively benign. The content of the curriculum can remain much the same as before—it is just that students can play with this content more easily. I suspect that this is where the major impact of the technology will be felt. It will be subtle but pervasive, as we are all carried along in the current. A number of examples show that elementary school students have developed keyboarding skills that permit them to type faster than they can write, often in excess of 20 words per minute. Thus they are now free to concentrate on the content.

We must be alert to hidden hazards lurking just below the surface and capable of tipping the canoe at a moment's notice. I call these hazards preformed curriculum packages. Not all rocks are hazardous, but those that provide mindless, repetitive exercises (even though they are coated with technological moss such as flashing, colored graphics and loud sounds) should be approached with caution. Whirlpools may also capture the unwary. Software packages abound that presume that learners all learn in essentially the same way and will thus respond in a desired manner to carefully constructed and sequenced screen displays. Do not be misled by the claims that the programs branch to different displays depending upon the response. The criteria for such branches are usually superficial, and the student feels that he is trapped in a vicious circle of preplanned activities, from which there is no escape.

Then come stretches of white water, where the current is faster. Here the curriculum is changing more rapidly. This is best represented by the integration of spreadsheets, data bases and computer-based simulations into the curriculum. It is much more exciting out here. Burnett has outlined potential spreadsheet uses for mathematics (1987) as well as across the curriculum (1988a, 1988c). Schwarz, Lademann and Christmas (1989) have pulled together a set of materials for teachers on the use of data bases in the curriculum. Computer-based simulations have received a lot of publicity and promise to be an area in which many will practise their white-water skills. As intimated earlier, new topics are also emerging: Logo (Burnett 1988d), Lego-logo, fractals, chaos, turtle geometry and perhaps even low temperature fusion. The technology also provides new ways of looking at familiar topics (Burnett 1988b; Clayson 1988; Goldenberg and Feurzeig 1987).

However, even the experienced canoeist can run into difficulties. Hoyles (1988) has outlined ways in which gender role stereotyping can creep into computer-based activities. Quiet students may remain quiet, and thus escape the attention of the teacher (Pye 1988). Anxiety surrounding the technology, although more apparent with older people, must also be addressed. Because access to computer technology is related to money, the children of more affluent parents are likely to benefit more by having computers in the home.

The Canoe: Invisible and Empty

The dominant use of computer technology in the next decade should be with systems that are invisible and empty. Invisibility is important. We want learners to be able to focus on the topic of interest and not be diverted by features of the environment. The medium should not be the only message. Invisibility is closely tied to the idea of "ease of use." We are slowly beginning to realize that most people do not want to think like a computer. They want a computer that responds to normal human signals, much as another human would. This is not a perverse desire: we simply have more experience communicating with one another and want to draw on that experience. Communicating with other life forms and with machines is not our strength. Pointing at an image on a screen is a genuine improvement over typing and having to be concerned with strange syntactic conventions. We want to focus on ideas inherent in the topic, not to have to pose questions like "How do I get the computer to do this?" which detract from important trains of thought. An invisible computer is one that is so easy to use we are not aware of its existence.

Emptiness is an even stranger concept for educators. Curriculum developers abhor a vacuum. Yet, for many reasons, an empty computer may be preferable to a full one. Rather than providing a student with a computer packed with curriculum materials, the idea is to provide the student with a sophisticated tool for placing, organizing and manipulating his/her own understandings. Potential examples include word processors, spreadsheets, data base programs, graphics packages, statistical packages and programming languages. An invisible, empty computer is a device that facilitates playing with student-generated ideas.

The Paddler

Even more paradoxical, in one important sense an empty teacher may be preferable to a full one. Let me now recount a story from my high school days. In my last year of high school, a new teacher was assigned to teach us physics. He was an excellent, experienced teacher—but not in physics. Physics was as new to him as it was to us. Of all the courses I took in school, this one stands out as the best. Why?

Reason, not authority, carried the day. Everyone was encouraged to share their view of why a particular idea or approach was appropriate in a given context. The teacher had his say but was comfortable with having it overturned by a better explanation from someone in the class. Knowing there was no authority, we had to work harder at making sure that we really understood because if we had a misunderstanding we would likely carry it with us into the departmental exam. Looking back on it, I think Socrates would have approved of this class. Another memory of this teacher is also vivid. We lived in a small town, and when we kids used to walk home after a movie and a coke, we passed the high school. Every night, and I do mean every night, we saw this teacher working in his classroom, preparing for the next day's class. What did I learn in his physics class? Some physics, definitely. But much more. By total accident I met this teacher in 1988 after 25 years. It was just like yesterday for both of us.

The teacher is the key.

Postscript: Mr. James Grieve died in 1990.

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Two Simple (and Not So Simple) Probability Activities

A. Craig Loewen

The NCTM's *Curriculum and Evaluation Standards for School Mathematics* (1989) has recommended that statistics and probability be included in the Grades 5 to 8 curriculum. Specifically, they recommend that students should

- model situations by devising and carrying out experiments or simulations to determine probabilities;
- model situations by constructing a sample space to determine probabilities;
- appreciate the power of using a probability model by comparing experimental results with mathematical expectations;
- make predictions that are based on experimental or theoretical probabilities;
- develop an appreciation for the pervasive use of probability in the real world (p. 109).

The introduction of probability concepts in the elementary grades (Grades 5 and 6) represents a significant addition to our present curriculum. Such a change necessitates further teacher education and the development of new teaching resources.

In this article we present two simple teaching activities that address two specific objectives taken from Alberta Education's new *Program of Studies* (1991). Both activities have adopted the familiar context of rolling dice and have been designed to stress simple probability principles, including the analysis of sample space as a means to predict and understand the outcomes of an experiment. In the first activity, The Biased Dice, four six-sided dice are used to introduce some important terms associated with probability experiments. In this activity each die has two unequal possible outcomes that control the movement of a colored marker making it easy to compare the various effects of the biased dice. In the second activity, The Mystery Dice, a problem-solving situation is presented through a computer simulation to stress the importance of sample space and the critical relationship between the conditions of the experiment and the range of possible outcomes. Each activity is described along with a discussion of the expected results, teaching hints, and a list of possible challenges or extensions. Answers to the challenges are not provided here but are left for the reader to enjoy figuring out.

Activity One: The Biased Dice

Objective

Develops probability concepts through experiments—identifies impossible events, certain events, uncertain events, equally likely events and unequally likely events (two or more events) (Alberta Education 1991).

Materials

Four blank six-sided dice, colored labels (dots) for file folders (blue, green, red and yellow), game board (see Figure 1) and four beans or other markers.

Activity

In this activity, the student begins by making four biased dice. Leave the first blank die blank. On two sides of the second die place a red dot. On four sides of the third die place a yellow dot, and on all six sides of the fourth die place a green dot.

To begin the activity, place the beans (or other markers) on the start squares of the game board. Roll all four dice simultaneously. Every time a dot of a given color is rolled, move the marker for that color one square further along its track. Continue rolling all four dice until one marker reaches the end of its track.

Questions

- *Predict which marker will make it to the end of its track first. Explain your prediction. Predict which color will finish second, third and fourth. Experiment to test your predictions.*
- *Using your experiment, give an example of each of the following: an impossible event, a likely event, a certain event, an uncertain event and two unequally likely events.*
- *Is it possible for the red marker to make it to the end of its track before the yellow marker does? Why or why not?*
- *Is it possible for the yellow marker to make it to the end of its track before the green marker? Why or why not?*
- *If you conduct the experiment again, will you get the same results? Why or why not?*

Discussion

In this activity, students are gaining experience with fundamental principles of probability. After a very few rolls, it will become obvious that a green dot will appear on every roll, while a blue dot will never appear. The yellow dot and red dot do not always appear, but the yellow dot appears more frequently than the red.

Understanding the results of this experiment requires considering the probability of rolling any one color in a given turn. This discussion may best proceed through an analysis of the *sample space* of each die. The sample space is defined as all the possible outcomes of an experiment. For each die, there are six possible sides that may surface when the die is rolled; therefore, there are six possible outcomes for each color and thus a sample space of six elements for each die.

Consider first the die with the green dots. We placed six dots on it and thus have a probability of 6 out of 6 for rolling a green dot. This is considered a *certain event*; that is, we know that whenever we roll this die a green dot will surface. We say that this die expresses a probability of 1 (or 6/6) of showing a green dot.

The die without any dots controls the movement of the blue marker. In our experiment the blue marker did not leave the start space. Because this die does not have any blue dots on it, it expresses a probability of 0 out of 6 (or simply 0) for rolling a blue dot (or any other color for that matter). This is considered an *impossible event*. That is, we know that no matter how many times we roll this die, it is impossible that a blue dot will ever surface.

The remaining two dice (the die with the red dots and the die with the yellow dots) provide us with examples of uncertain events. An *uncertain event* is one which cannot be guaranteed or predicted with any assurance. In the examples above, we knew that we would roll a green dot on each roll and that we would never roll a blue dot, no matter how many times we tried. We were certain about these events. With the remaining two dice, it was possible to roll a colored dot, but it was also possible to roll nothing, and thus we could not be certain about the actual outcomes. We call these uncertain events.

The die with the red dots provides us with an example of an *unlikely event*. On this die, we covered two sides with red dots, and thus we knew that the chance of rolling a red dot is 2 out of 6. It is more likely when rolling this die that we will have a blank side than a side with a red dot. This means that the rolling of a red dot is an unlikely event.

Now consider the remaining die, of which we covered four of the six sides with yellow dots. This die expresses the probability of 4 out of 6 for rolling a yellow dot. The chances of rolling a yellow dot are greater than the chances of having a blank side up with this die, and thus rolling a yellow dot is a *likely event*.

We have now described two likely events. The event of rolling a yellow dot is likely (4 chances out of 6), while that of rolling a green dot is very likely (in fact, certain) at 6 out of 6. These events are considered *unequally likely*. That is, they are both likely, but one is more likely than the other.

Some interesting questions arise in our analysis of the experiment. For example, is it possible for the red marker to make it to the end of its track before the yellow marker does? The occurrence of both the red dot and the yellow are uncertain events, and we cannot therefore answer this question with any guarantee; however, we also know that the event of rolling a yellow dot is more likely than the event of rolling a red dot (in fact, two times more likely). Therefore, it is *possible* for the red marker to reach the end point first, but also quite *unlikely*.

Is it possible for the yellow marker to reach the end of its track before the green marker does? We know that we will roll a green dot on each turn (a certain event), but the event of rolling a yellow dot is uncertain (although likely). If we were really lucky and rolled a yellow dot on every turn, then the green and yellow markers would reach the end point at the same time. This means that the yellow marker would never beat the green marker—the best it can do is finish at the same time (a tie). Because rolling a yellow dot is an uncertain event, while rolling a green dot is certain, it is probable that the green marker will almost always reach the finish line before the yellow marker (that is, a tie would be rare).

Would we get the same results if we ran the experiment again? This question is a fun question to ask in almost any probability experiment. In this case, we know that the green marker will always make it to the finish line, and we know that the blue marker will never make it to the finish line (in fact, it will never leave the start space!). It is more difficult to predict the behaviors of the red and yellow markers, however. The events of rolling either a red or yellow dot are both uncertain; therefore, if we were to run the experiment again, we would predict that both markers would make it somewhere along the track by the time the green marker finished, but not necessarily to the same space they reached in the previous experiment. Because rolling a red is less likely than rolling a yellow, it is *probable* (no guarantees however!) that the yellow marker will move further along its track than the red marker every time we conduct the experiment.

Hints for the Classroom

Hint One: Have students create the dice themselves; this is simple to do and helpful in classroom discussions. In creating the dice, the students will pay attention to what is actually found on the faces of the dice, an analysis that is critical to understanding the activity.

Hint Two: Ask many "why" questions while the students are conducting the experiment. You may wish to ask questions such as these:

- *Why does the green marker seem to move faster than the others?*
- *Why does the red marker seem to move faster than the yellow?*
- *Why does the blue marker never seem to move at all?*

"Why" questions help the students synthesize the information demonstrated through the experiment and force them to draw generalizations from the activity. Remember that it is not of particular interest whether

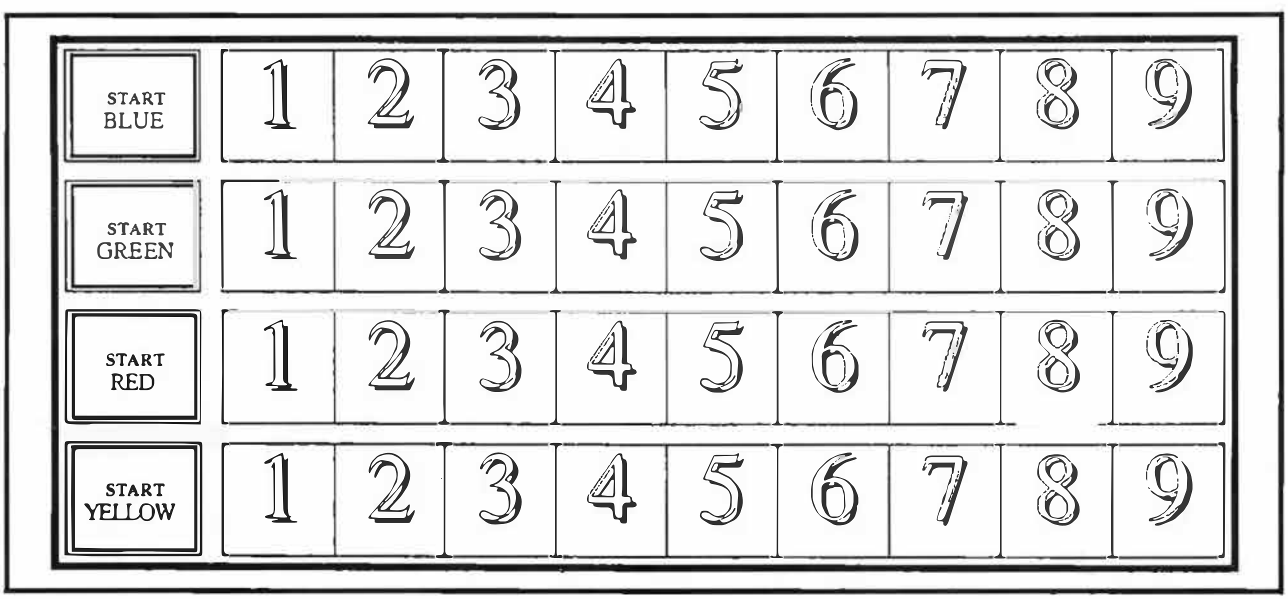

Figure 1, Game Board for the Biased Dice

or not students can correctly predict or describe the outcomes of the experiment; we are more interested in whether students can explain the outcomes. Ability to explain the outcomes of the experiment attests well to the students' mastery over and understanding of the experience. "Why" questions are helpful because they demand explanations.

Hint Three: Try to build the activity up from a simple to a more complex level. In rolling all four dice at the same time, students are forced to draw comparisons between four simultaneous events, and this large number of comparisons may be overwhelming for some of them. As an alternative, start with only two dice (say the die with the green dots and the die with the red dots) and conduct the experiment. When students become comfortable with their ability to explain the results, then you may introduce other dice (either adding a die or using different dice).

You may find it interesting to have the students complete the activity using only the dice with the red and yellow dots. It may happen that one of the groups has the red marker finish first, which could lead to an interesting discussion of the notion of chance.

Hint Four: A good test of how well students understand this activity may be found in asking them to construct some of their own dice. For example, ask the students to construct a die that (a) makes rolling a yellow dot an uncertain and unlikely event, or (b) makes rolling a green dot an uncertain but likely event. These tasks may be good tests of learning in that they require students to apply their new ideas in familiar contexts.

Challenges

Here are some ways to extend the activity as it has been described above. Find the answers for your own amusement.

- *Can you create a single six-sided die that makes rolling a yellow dot an uncertain but unlikely event and rolling a green dot an uncertain but likely event? How many such dice can you make?*
- *Can you create a single six-sided die that makes rolling a yellow dot a certain event and rolling a green dot an uncertain but likely event?*
- *Can a single six-sided die be created to show both a certain and an impossible event?*
- *Can a single six-sided die be created to show two equally likely events?*
- *Create new dice for the original experiment that would theoretically enable yellow to finish first, green second, blue third and red not at all. How many such combinations of dice can you make? Find several different possibilities.*

Activity Two: The Mystery Dice

Objective

Develops probability concepts through experiments—identifies all the possible outcomes of a probability experiment (Alberta Education 1991).

Materials

Hidden Dice program (see Figure 2), Apple II*e* computer, pencil, paper, one four-sided die, one six-sided die and one eight-sided die.

Activity

This activity is a computer simulation of rolling an unknown number of hidden dice. In this activity we are rolling four-, six- and eight-sided dice only. The computer secretly selects either one or two dice from a collection of two four-sided, two six-sided and two eight-sided dice. If the computer selects two dice, it never selects two of the same kind of die (that is, it will never pick two four-sided dice or two six-sided dice or two eight-sided dice). The computer rolls the dice for you and reports the value of the roll. If it is rolling only one die, then it simply tells you the value rolled on that die. If it is rolling two dice, then it adds the values of the two faces and reports this sum. By rolling the dice many times and observing the various outcomes of the rolls, you can figure out how many dice the computer is rolling, and whether the dice have four, six or eight sides.

Every time you ask the computer to roll the dice for you, it costs you one point. You may roll as often as you like, and the computer will keep track of your score. When you think you know how many dice the computer selected, and the number of faces on the die or the dice, you may opt to input your guess. If you guess incorrectly, 50 points will be added to your score. The computer will also draw a frequency histogram showing how often each value has been rolled, but this clue costs you 10 points. Of course, your objective is to discover the identity of the hidden dice with as low a score as possible.

Questions

- *What clue or clues tell you that the computer has selected only a single die?*

- *What clue or clues let you know that the computer has selected two dice?*
- *What clue or clues indicate that the computer has selected a six- and an eight-sided die?*
- *When you run the program the first five values the computer reports are 3, 7, 4, 1, 6. Do you know how many dice have been selected? Do you know which one or more dice have been selected? Explain.*
- *When you run the program the first five values the computer reports are 2, 2, 3, 4, 3. Do you have enough information to solve the problem (identify the number and nature of the one or more dice)? Explain.*
- *Could the computer generate these numbers in its report: 2, 1, 9, 5, 2? Explain.*
- *How does the sample space for the simultaneous rolling of a six- and a four-sided die differ from the sample space of an eight-sided die?*

Getting Started

You will need to begin this activity by entering the program into your Apple IIe computer. This can be accomplished by first booting the machine with a Disk Operating System (DOS) disk, and then typing NEW and pressing return.

You are now ready to type the program as listed in Figure 2. After typing it in, try running the program to ensure that you have not made any typing errors. Any line that has an error has to be retyped. Once the program runs well from beginning to end, save the program on a formatted disk so that you do not have to retype and debug it each time. When you are ready to begin your experiment, you merely have to type the word RUN and press return.

Operation of the program is very simple. You will see a statement of the first value rolled at the top of the screen along with your score so far. You may now opt to roll the die or dice again (simply press the R key, but make sure the caps lock key is down), guess at the number and type of dice (press the G key), or have the computer draw a histogram of the values rolled to date (press the H key). If you opt to guess the number and type of dice, you will be asked the following questions:

NUMBER OF DICE: (to which you respond either 1 or 2)

If you indicate that there is only one die, the computer will ask:

NUMBER OF SIDES ON DIE: (you respond with 4 or 6 or 8)

If you indicate that there are two dice, the computer will ask how many sides for each die:

NUMBER OF SIDES ON FIRST DIE: (either 4 or 6 or 8)

NUMBER OF SIDES ON OTHER DIE: (either 4 or 6 or 8)

If you are correct you may opt to play again or quit. If you are incorrect you will be assessed 50 points and asked to choose between rolling, guessing again and drawing the histogram.

Figure 2: BASIC Program for the Mystery Dice Activity

```
10 REM WRITTEN BY A. CRAIG LOEWEN
20 REM MYSTERY DICE PROGRAM
30 REM JANUARY 10, 1992
40 DICE = INT(RND(1) * 2 + 1)
50 D1 = INT(RND(1) * 3 + 2) * 2
60 D2 = INT(RND(1) * 3 + 2) * 2
65 IF D1 = D2 THEN 60
70 IF DICE = 1 THEN D2 = 0
80 HOME: SC = 0: DIM R(16)
90 PRINT "DIE OR DICE SELECTED!":PRINT
100 X = INT(RND(1) * D1 + 1)
110 Y = INT(RND(1) * D2 + 1)
120 IF DICE = 1 THEN Y = 0
130 ROLL = X + Y: R(ROLL) = R(ROLL) + 1
140 SC = SC + 1
150 PRINT "VALUE ROLLED: ";: INVERSE: PRINT "*  ";ROLL;"  *": NORMAL
160 PRINT "SCORE:  ";SC
```

Figure 2 continued overleaf

```
170 PRINT "----------------------------------------"
180 PRINT "ROLL (R), GUESS (G), OR HISTOGRAM (H)";
190 GET A$
200 IF A$ = "R" THEN PRINT: GOTO 100
210 IF A$ < > "G" THEN 560
220 HOME
230 PRINT "NUMBER OF DICE:  ";
240 GET A$
250 IF A$ < "1" OR A$ > "2" THEN PRINT CHR$(7);: GOTO 240
260 PRINT A$: A = VAL (A$)
270 IF A = 2 THEN 340
280 PRINT "NUMBER OF SIDES ON DIE:  ";
290 GET A$
300 IF A$ < > "4" AND A$ < > "6" AND A$ < > "8" THEN PRINT CHR$(7);: GOTO 290
310 PRINT A$: B = VAL (A$)
320 IF A = DICE AND B = D1 AND DICE = 1 THEN 480
330 GOTO 440
340 PRINT "NUMBER OF SIDES ON FIRST DIE:  ";
350 GET A$
360 IF A$ < > "4" AND A$ < > "6" AND A$ < > "8" THEN PRINT CHR$(7);: GOTO 350
370 PRINT A$: B = VAL (A$)
380 PRINT "NUMBER OF SIDES ON OTHER DIE:  ";
390 GET A$
400 IF A$ < > "4" AND A$ < > "6" AND A$ < > "8" THEN PRINT CHR$(7);: GOTO 390
410 PRINT A$: C = VAL (A$)
420 IF A = DICE AND B = D1 AND C = D2 AND DICE = 2 THEN 480
430 IF A = DICE AND B = D2 AND C = D1 AND DICE = 2 THEN 480
440 PRINT
450 PRINT "OOOOOPS!!": SC = SC + 50
460 PRINT "SCORE:  ";SC
470 GOTO 170
480 PRINT "CORRECT!!"
490 PRINT "SCORE:  ";SC
500 PRINT "----------------------------------------"
510 PRINT "PLAY AGAIN? (Y/N):  ";
520 GET A$
530 IF A$ = "Y" THEN RUN
540 IF A$ = "N" THEN HOME: END
550 PRINT CHR$(7);: GOTO 520
560 IF A$ < > "H" THEN PRINT CHR$(7);: GOTO 190
570 HOME
580 PRINT "HISTOGRAM:"
590 FOR X = 1 TO 16
600 IF X < 10 THEN PRINT "  ";
610 PRINT X;"|";
620 N = R(X): IF N > 35 THEN N = 35
625 IF N = 0 THEN PRINT: GOTO 660
630 FOR Y = 1 TO N
640 PRINT "*";
650 NEXT Y: PRINT
660 NEXT X
670 PRINT
680 SC = SC + 10: PRINT "SCORE:  ";SC
690 GOTO 170
```

Example

Assume that the program has been correctly placed in the computer's memory and has been started. The screen shows:

```
VALUE ROLLED: *4*
SCORE: 1
-------------
ROLL(R), GUESS(G), OR HISTOGRAM(H)
VALUE ROLLED: *1*
SCORE: 2
-------------
ROLL(R), GUESS(G), OR HISTOGRAM(H)
VALUE ROLLED: *7*
SCORE: 3
-------------
ROLL(R), GUESS(G), OR HISTOGRAM(H)
```

The first roll told you very little, but the second roll was important. It told you that the computer opted to roll only one die. (Why does a 1 tell you that?) The third roll was also important as it told you that the computer was rolling an eight-sided die. (Why does a 7 give this away?) You now know all the information you need to make your guess so you may select "G" and input your guess.

Discussion

As in the previous experiment, understanding this activity requires an analysis of sample space or the list of all the possible outcomes of an experiment. To determine the sample space in each case, you need merely ask what the lowest and highest values are that can be rolled. For example, with a single six-sided die, the lowest value is a 1, and the highest value is a 6. The sample space for a six-sided die includes six possible numbers: the values from 1 to 6. The sample space for a four-sided die will include all the values from 1 to 4. The sample space for an eight-sided die will include all the values from 1 to 8.

The sample space produced from rolling two dice is slightly more complicated, but the task can be simplified by asking what the smallest and largest possible sums are. The smallest sum can be determined by adding the two smallest values on the two dice. Likewise, the largest sum can be determined by adding the two largest values on the two dice. If you are rolling a four-sided and a six-sided die, then the sample space includes values ranging from 2 to 10. A six-sided and an eight-sided die together have a sample space with values ranging from 2 to 14. Notice that the value 1 can be rolled only with a single die. Therefore, if the computer reports that a 1 has been rolled, you know that the computer has selected only one die. We also know that the only combination of dice that can give us values of 13 and 14 are a six- and an eight-sided die together.

The problem that the computer presents can lead to some difficult questions. For example, the computer has reported several values ranging from 2 to 8; do we know what die or dice are being rolled? In this case the reported range alone does not identify the die or dice. It is possible that the computer is rolling an eight-sided die but, just by coincidence, the number 1 has yet to appear. However, it is also possible that the computer is rolling a four-sided and a six-sided die, but, just by coincidence, the numbers 9 and 10 have yet to appear. Because we do not yet know what dice are being rolled, we will have to roll again.

Consider another hypothetical situation. The computer has reported six values all ranging from 1 to 4; do we know what die or dice are being rolled? We know that the computer is rolling a single die given that the value 1 has appeared. We do not know whether the computer is rolling a four-, six- or eight-sided die. We must ask ourselves this question: Is it possible that the computer could roll a six-sided die six times, and not roll a 5 or 6? The answer is yes. Given that the roll of a die is a chance event, it is quite possible that after so few rolls we may not have seen either a 5 or 6. We must therefore ask ourselves another question: How many more rolls would convince us that the computer is rolling a four-sided die? Obviously, the larger the number of rolls made, the more likely it is that we would get a 5 or a 6 if the computer is rolling a six-sided die. The exact number of rolls necessary to convince each individual will vary: there is no single "correct" number of rolls. (Incidentally, statisticians would argue that approximately eight more rolls without obtaining a 5 or 6 would constitute a very good argument for rejecting the possibility of a six-sided die, but an explanation of their argument is beyond the scope of this paper.)

Hints for the Classroom

Hint One: When introducing this activity to students, start with a single six-sided die and ask a few important questions to establish the sample space (or range of possible outcomes) of rolling the die. For example: What is the largest possible number you could roll? What is the smallest possible number you could roll? How many different possible numbers could we roll? You may wish to familiarize students with the four- and eight-sided dice as well. Stress the fact that different outcomes result from rolling different dice.

Hint Two: After students are familiar with the range of possible outcomes from rolling a single die, introduce two dice. Again, we will need to determine our range of possible outcomes for rolling two dice. Be sure students understand how we determine the smallest possible outcome and the largest possible outcome.

Hint Three: Try playing the game a few times with the students as a whole while you (or a student) hide(s) behind a partition while rolling the dice. This activity familiarizes the students with the activity as well as allowing them to form initial hypotheses.

Hint Four: Create a low score list on the blackboard to emphasize that the task is to find out about the dice in as few rolls as possible. You will want to treat each combination of dice separately, as it is easier to identify a single die than to identify any combination of two dice.

Hint Five: You may wish to create a chart of the sample space for rolling two dice, such as for a four- and an eight-sided die (see Figure 3). This chart helps to clarify what the largest and smallest possible values are within the range of possible outcomes, and will help explain why some values are generated more often than others. For example, we notice in Figure 3 that there is only one way to roll a 2 (by rolling a 1 on each die), while there are four ways to roll a 7:

- roll a 1 on the four-sided die and a 6 on the eight-sided die
- roll a 2 on the four-sided die and a 5 on the eight-sided die
- roll a 3 on the four-sided die and a 4 on the eight-sided die
- roll a 4 on the four-sided die and a 3 on the eight-sided die

Therefore, if we conduct this experiment many times, we would expect that the value 7 would appear about four times more often than the value 2. The ability to create and interpret this sample space chart is important for the challenge activity described below.

		Possible rolls of the eight-sided die							
		1	2	3	4	5	6	7	8
Possible rolls of the four-sided die	1	2	3	4	5	6	7	8	9
	2	3	4	5	6	7	8	9	10
	3	4	5	6	7	8	9	10	11
	4	5	6	7	8	9	10	11	12

Figure 3: Sample Space for the Simultaneous Rolling of a Four- and an Eight-Sided Die

Challenges

When selecting two dice, the computer never selects two dice with the same number of sides, always two different dice. This criterion has been included so that one can determine the dice rolled through simple inspection of the range of possible outcomes without considering the frequency of each element within the sample space. If we remove this criterion, it becomes possible for the computer to pick two six-sided dice. Notice that the range of possible outcomes (2 to 12) for two six-sided dice is exactly the same as the range of possible outcomes for a four- and an eight-sided die. How then could we tell the difference between rolling a four- and an eight-sided die and rolling two six-sided dice?

To free the computer to select two dice with the same number of sides, simply remove line 65 from the program by typing the number 65 after the "}" prompt and pressing return. *Hint:* You will probably want to draw out the sample space for rolling two six-sided dice, and you will probably need to exercise the option to have the computer draw a histogram for you. Good luck!

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Cut-Throat: A Game Using Junior High School Geometry

Derek Gray and A. Craig Loewen

Cut-Throat is a game based on the application of angles in billiards and provides students with an opportunity to use angles in a practical and enjoyable context. This activity addresses many instructional methods and objectives of the mathematics curriculum, including the estimation, measurement and construction of angles of a given measure.

Cut-Throat is played in groups of three or four players. Each group requires a game board, a die and a protractor. Each player requires a colored marker, a ruler and a pencil or crayon. The game board is a facsimile of a billiard table, with six pockets and straight rails (see Figure 1). The game board may be larger if desired for demonstration purposes or for use in a whole-class setting.

The game begins as each player places his/her marker at a random point on the billiard table. The objective of the game is to bank opponents' balls off one rail into a pocket. Once a player's ball is banked into a pocket, that player is out of the game. The winner is the player whose ball is the last to remain on the table. In Cut-Throat there are two important assumptions: (1) balls bounce off the rails at exactly the same angle with which they hit the rail, that is, *the angle of incidence always equals the angle of reflection*, and (2) balls cannot be bounced off a pocket.

Rules for Playing the Game

- Each player places his/her marker at a random point on the board.
- A die is rolled to determine who will go first. Play continues in a clockwise direction from the first player.
- The first player chooses any one of his/her opponents' balls and verbally specifies: (a) into which pocket he/she will attempt to sink the ball, and (b) which rail he/she will use. Remember: all balls must be banked once (and only once) off one rail.
- To shoot the ball, the player selects the point along the rail which the ball will hit and places a pencil mark at that point.
- A line is drawn connecting the centre of the opponent's ball to this point on the rail.
- The angle formed by this line and the rail is found using the protractor, and a line showing the rebound path of the ball is constructed. Remember: the angle of incidence must always equal the angle of reflection.
- The rebound line is extended until it either enters one of the six pockets or strikes a second rail.
- If the rebound line enters one of the six pockets, the ball is removed from the table, and the owner of this ball is now out of the game (see Figure 2 for an example). Play proceeds to the left.
- If the rebound line strikes a second rail rather than a pocket, the ball is placed at the halfway point along the rebound line (see Figure 2 for an example). Play proceeds to the left.

Teachers may find using an overhead projector is a good idea to list the rules and provide some examples when the game is first introduced. This discussion will help eliminate confusion and enable the students to enter into the game quickly. The teacher may find it necessary to review the steps for construction of an angle, and may want to give several examples of how the rule the angle of incidence equals the angle of reflection applies in this context.

This game has several variations and uses. For example, the board may simply be used as an individual activity, giving students practice at measuring and constructing angles. As students become increasingly adept at playing Cut-Throat, the teacher may wish to change some of the rules to increase the challenge. For example, balls must bank off two (or more) rails, or balls may bank off pockets.

If the teacher wishes to extend the activity to include a further discussion or summary session, he or she can explore some interesting questions that arise from playing this game. Is it always possible to bank a ball into *any* pocket from *any* position on the table? Why or why not? Is there a method of calculating the correct and exact angle at which to bank a ball into a pocket? Describe how this might be done.

Whenever possible, it is beneficial to provide examples and applications when teaching mathematical concepts. Application games such as Cut-Throat help promote interest in the topic and (perhaps more importantly) provide a sense of relevance to student learning. Cut-Throat represents an attempt to increase students' motivation and heighten their sense of the usefulness of angles in the "real world."

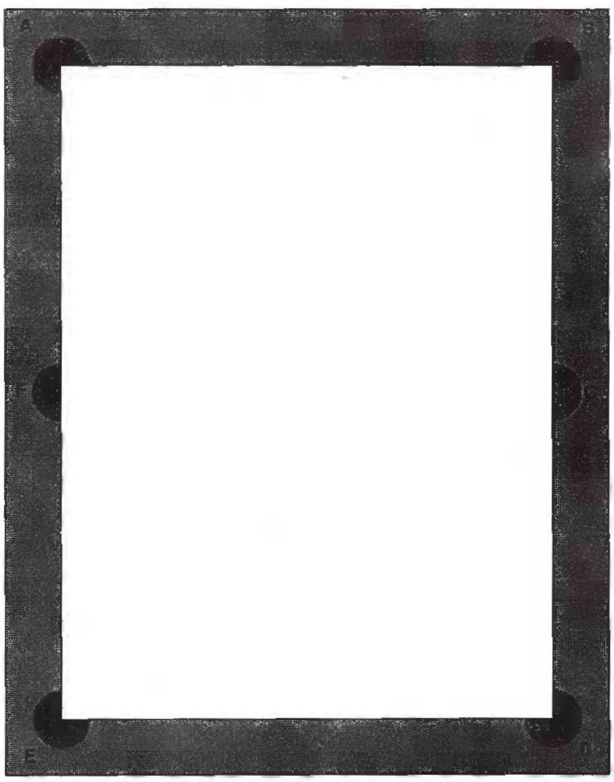

Figure 1. The Billiard Table (Game Board)

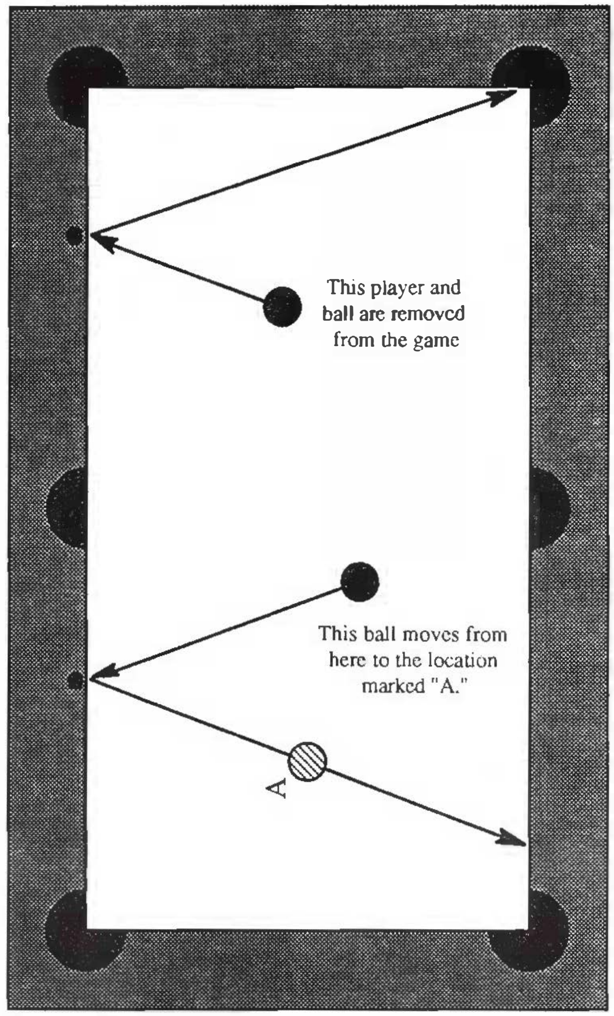

Figure 2. Examples

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