# Unit III: Fractions

## Teachers' Notes on Meaning Activities

### Activity 1

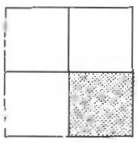
Rectangular dimensions are excellent models for thinking of factors. Of course, a prime number has only one possible and somewhat uninteresting rectangle associated with it.

### Activity 2

Using correct terminology for "cars" and "trains" is essential. This activity requires a high level of teacher participation. Even after students understand what is happening, they should practise to reinforce the connection.

### Activity 3

Students must clearly understand that the shaded part is the representation of the fraction. Other shadings such as ¼ are permitted as shown:



### Activity 4

The fraction bar sheets can be cut up, and they become quite useful. Students can use them to do the fraction bar questions and can draw sketches to find the answers. For example,

2 2/5 is 1 + 1 + 2/5 = 12/5

### Activity 5

Teacher help is essential in this activity. This question serves to emphasize what it means to ask how many sixths there are in 2 ⅓. In this case, 2 ⅓ means twice around and one third more.

### Activities 6 and 7

These activities stress the meaning of multiplying fractions as well as a means of finding answers. What is ½ of ⅔? The product of the numerators over the product of the denominators is the answer. These activities should help in explaining why that is so.

### Activity 8

This teacher sheet would make a good overhead transparency or blackboard demonstration.

### Activity 9

Understanding division is easier when we have unit fractions such as ⅓.

### Activity 10

Meaning is emphasized by switching from one mode of naming to another. We have many ways of referring to the same numbers.

Activities 11–18 require each student (or student group) to have fraction bar strips. These can be cut from duplicates of the fraction bar sheet. Note that each strip represents one unit. Strips are more durable if duplicated on heavier paper. This fraction bar sheet can be used to make a transparency.

### Activity 11

For two fractions such as ½ and 3/5, the division lines match the division lines on the 1/10 bar. This is the common denominator. The easy visual comparison of ½ and 3/5 with the bars should be integrated with the mathematical idea of common denominators. So, ½ is 5/10 and 3/5 is 6/10.

### Activity 12

Note the same bar shows the ¼ and the ¾. (Some fraction bar packages have strips which distinguish between ¼ and ¾.)

### Activity 13

Students will need two of the ¼ strips to add ¼ + ½ = ¾ because the lowest common denominator is back on the ¼ strip. Demonstrations with the overhead are easier to present if the strips are colored differently.

### Activity 14

To solve ⅔ − ⅓, you need two ⅓ strips. Suggested language use: "How much of the ⅔ is left after we take away the ⅓?" or "How much more is ⅔ than ⅓?"

### Activity 15

Solutions for these exercises should be demonstrated by the teacher. For example, ⅔ + ¼. Using the ⅓ and ¼ bars, find the bar (1/6) whose division lines match the division lines of both the ⅓ and the ¼ strips. Find the ⅔ point on the ⅓ bar and place the ¼ bar to add its length to that of the ⅔. The division line at the addition of ⅔ and ¼ matches which division line on the 1/6 bar? Answer: 5/6.

### Activity 16

Students should be encouraged to use the fraction bars for these activities even if they know shortcut methods. The use of the bars reinforces the concept of addition and subtraction and the use of the common denominator.

## Meaning Activity 1 Greatest Common Factor

1. On grid paper, draw all possible rectangles of area 12 units and area 18 units.
   (a) From these rectangles, write down the factors of 12 and of 18.
   (b) Why do factors always occur in pairs?
   (c) Which are the common factors of 12 and 18?
   (d) Which of these factors is the greatest?
2. Do question 1 again for rectangles of 9 units and 24 units.
3. Do question 1 again for rectangles of 12 units and 30 units.
4. Find out what a prime factorization is.
5. Write out prime factorizations for 12 and 18.
   (a) Match the prime factors of these two numbers. For example,
   12 = 1 x 2 x 2 x 3
   18 = 1 x 2 x 3 x 3
   (b) Find the product of all of the matched numbers. Why does this product give you the greatest common factor (GCF)?
6. Do question 5 using 9 and 24.
7. Do question 5 using 8 and 18.
8. Do question 5 using 15 and 25.
9. Do question 5 using 48 and 72.

## Meaning Activity 2 Least Common Multiple

1. Make cars of 3-cm and 5-cm lengths using centimetre cubes of two different colors.
2. By adding 3-cm and 5-cm cars together, build trains of equal length.
   (a) How many 3-cm cars did you use?
   (b) How many 5-cm cars did you use?
3. Do question 2 again using 4-cm and 6-cm cars. Now make the shortest train possible using the 4-cm and 6-cm cars.
4. Do question 2 again using 6-cm and 8-cm cars. Do not actually make trains, but sketch your answer in your notebook.
5. These trains are multiples of the car numbers. Trains of equal length are common multiples. We are usually interested in the least common multiple (LCM). Why?

## Meaning Activity 3 Adding and Subtracting Fractions

Do the following sums using diagrams only. Be sure to express your answers as basic fractions. Example: ¼ + 2/4 = ¾

1. (a) 1/3 + 1/3 =
   (b) 1/6 + 2/3 =
   (c) 1/4 + 1/3 =
   (d) 3/10 + 2/5 =
   (e) 3/4 + 1/6 =
   (f) 1/3 + 1/6 =

2. (a) 4/5 − 1/5 = ☐ − ☐ = ☐

(b) 5/6 − 5/12 = ☐ − ☐ = ☐

(c) 5/6 − 1/4 = ☐ − ☐ = ☐

(d) 7/8 − 3/4 = ☐ − ☐ = ☐

(e) 7/10 − 2/5 = ☐ − ☐ = ☐

(f) 4/5 − 1/3 = ☐ − ☐ = ☐

## Meaning Activity 4 Fractions and Mixed Numbers

1. Lay out one whole bar and two thirds.

 = 1 2/3

Using thirds, find out how many thirds are in a whole. ____

How many thirds are there in all?
3/3 + 2/3 = /3.

2. Lay out two whole bars and three tenths.

 = 2 3/10

How many tenths are in each whole bar? ___

In two whole bars? ___

How many tenths are there in two whole bars and three tenths?___

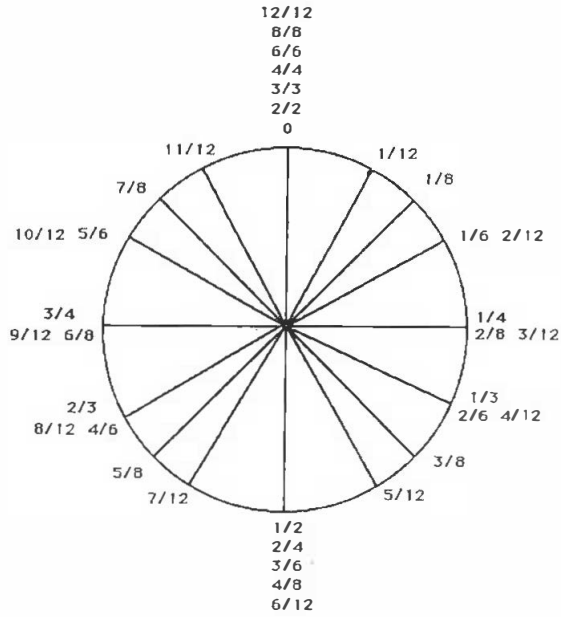
$$\frac{(2 \times 10) + 3}{10} = \frac{\quad}{10}$$

3. Following the pattern set in question 2, change
   (a) 3 2/7 to /7.
   (b) 2 2/5 to /5.
   (c) 1 8/9 to /9.

## Meaning Activity 5 Fractions and Mixed Numbers

Use the following circle to answer these questions.

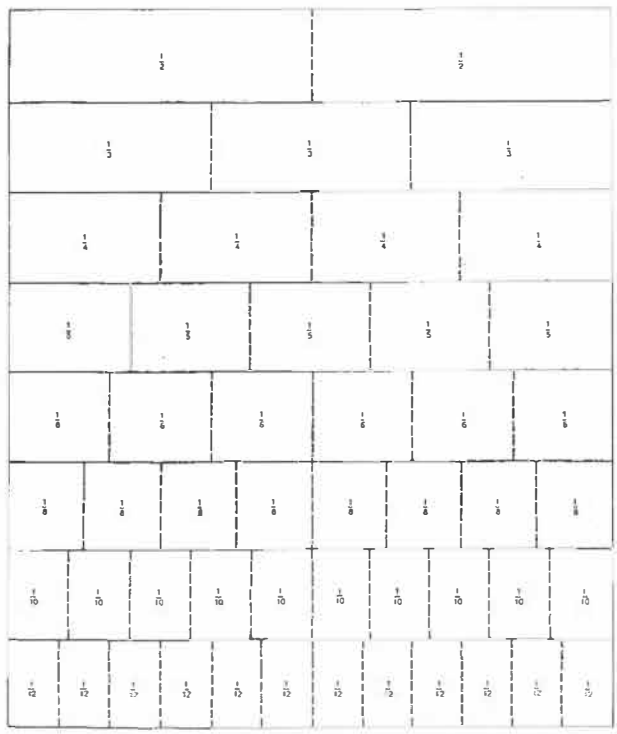
1. Using the large circle, how many halves are in a whole?
2. How many thirds are in a whole?
3. 5/2 is how many wholes?
4. 9/4 is how many wholes?
5. How many sixths are there in 2 1/3?
6. How many eighths are there in 2 1/4?




Use the circle to answer questions 7–9. Sketch your answers.

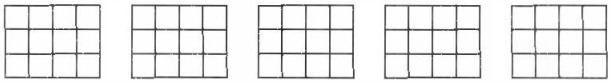
7. 2 1/2 + 1 1/3 =
8. 2 3/4 − 1 2/3 =
9. 2 1/2 − 1 2/3=

**Fraction Bar Sheet**



## Meaning Activity 6 Multiplying Fractions

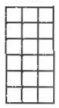
1. Shade in the answer:



½ of 12 ⅓ of 12 ¼ of 12 ⅔ of 12 ¾ of 12

2. (a) 
5
(i) Shade in ½ of 12.
(ii) How many did you shade in?

(b) 4 
3
(i) Shade in ¾ of 20.
(ii) How many did you shade in? (Compare)

(c) 6 
5
(i) Shade in ⅔ of 18.
(ii) How many did you shade in? (Compare)

(d) 4 
(i) Shade in 1/5 of 20.
(ii) How many did you shade in? (Compare)

3. (a) [IIIIIII]
(i) Shade in ½.
(ii) Shade in ⅓ of the area you just shaded in. What is ⅓ of ½?

(b) [IIIIIIIII]
(i) Shade in ¾.
(ii) Shade in ½ of the area you just shaded in. What is ½ of ¾?

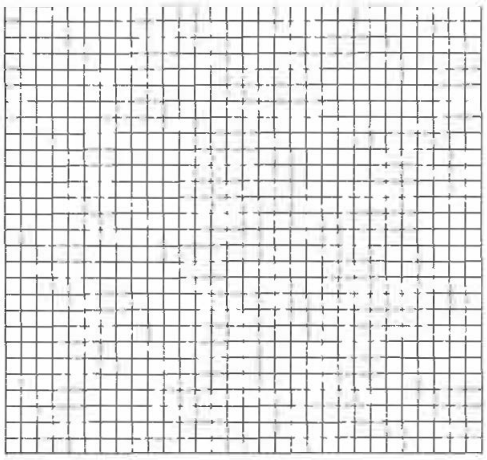
(c) [IIIIIIIII]
(i) Shade in ½.
(ii) Shade in ¾ of the area you just shaded in. What is ¾ of ½?

4. The following represents 10/3 or 3 ⅓. If we took ½ of 10/3, what do we get?

½ x 10/3 = _____ or ½ x 3 ⅓ = _____

## Meaning Activity 7 Multiplying Fractions

1. Using any size grid paper, show the following multiplication questions. Place the answer after each question.
   (a) ½ of ⅓ = ___
   (b) ½ of ¼ = ___
   (c) ¼ of 1/6 = ___
   (d) ¾ of ⅓ = ___
   (e) ¾ of 1 ½ = ___
   (f) ⅓ of 2 ⅔ = ___



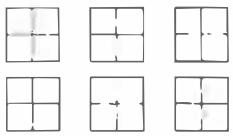
2. Looking at your answers, how could you obtain the same results by using the given fractions?

## Meaning Activity 8 Dividing by a Unit Fraction

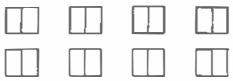
**Teacher Sheet**

1. 1 ÷ ¼ = ____. How many quarters?

2. 6 ÷ ¼ = ____. How many quarters?



3. 8 ÷ ½ = ____. How many halves?

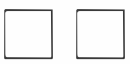


This shows that when you divide by a fraction, the result is the same as multiplying by the reciprocal of that fraction.

4. 4 ÷ ⅔ = ____. Draw four units divided into thirds. How many groups of two thirds can we get?

## Meaning Activity 9 Dividing Fractions

The teacher will assist you in doing these questions.

1.  2 ÷ ½ = ___ = 2 x ___. How many halves are there in 2?

2.   $2 \div \frac{1}{3} = \_\_ = 2 \times \_\_$. How many thirds are there in 2?

3.   $2 \div \frac{1}{4} = \_\_ = 2 \times \_\_$. How many quarters are there in 2?

4.  $1 \div \frac{1}{2} = \_\_ = 1 \times \_\_$. How many halves are there in 1?

5.  $\frac{1}{2} \div \frac{1}{2} = \_\_ = \frac{1}{2} \times \_\_$. How many halves are there in $\frac{1}{2}$?

6.  $1 \div \frac{1}{4} = \_\_ = 1 \times \_\_$. How many quarters are there in 1?

7.  $\frac{1}{2} \div \frac{1}{4} = \_\_ = \frac{1}{2} \times \_\_$. How many quarters are there in $\frac{1}{2}$?

8. In your *own* words, describe how to do the following:
   (a) $1 \div 1/5 =$
   (b) $\frac{1}{2} \div 1/5 =$

## Meaning Activity 10 Fractions and Decimals

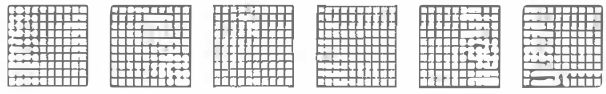
1. Write these decimals as words, fractions, reduced fractions, words and an equation. The first one is done as an example for you.

| Decimal | Words | Fraction | Reduced Equation | Words | Equation |
|---|---|---|---|---|---|
| 0.5 | five tenths | 5/10 | ½ | one half | 0.5 = ½ |
| 0.7 | | | | | |
| 0.25 | | | | | |
| 5.8 | | | | | |
| 0.12 | | | | | |
| 0.10 | | | | | |
| 0.04 | | | | | |
| 3.4 | | | | | |

2. On the hundredths square:
   (a) Shade in 1/5. Now write 1/5 as a decimal: ___
   (b) Shade in ½. Now write ½ as a decimal: ___

3. Shade in a hundredths square for each fraction below.

¼ 3/5 ¾ ⅛ 2/8 ⅝



## Meaning Activity 11 Comparing and Ordering Fraction Bars

1. Select the ⅓, ⅔, 2/6, 4/6, 4/12 and 8/12 fraction bars. Arrange them in ascending order.
2. Select the ⅔, ¾ and the 12ths bar.
   (a) How many 12ths are in ⅔?
   (b) How many 12ths are in ¾?
   (c) Which is greater numerically?
3. Select the ½ and the 3/5 bars.
   (a) Find another bar with division lines that coincide with the endpoints of each of these bars. Which bar is it?
   (b) Write the fractions ½ and 3/5 as fractions that have the same denominator.
   ½ = ___ 3/5 = ___
4. Select the ¾ and the 5/6 bars.
   (a) Repeat 3a and 3b above.
   (b) Find a method to help decide which bar to choose.
   (c) What is this bar number called?
5. Select the 1/5 and ¼ bars.
   (a) Repeat 4a and 4b above.
   (b) What method did you use for choosing the correct bar?
   (c) What bar number is this?

## Meaning Activity 12 Equivalent Fractions

1. (a) Select the fraction bar for ½.
   (b) Find by comparison all those fraction bars that have the same length as the one-half bar.
   (c) Write these as fractions.
   (d) What does the term *equivalent* mean?
   (e) Repeat parts a, b and c for the following bars:

⅓ _____ ⅔ _____
¼ _____ 2/4 _____
¾ _____ 2/5 _____
4/5 _____ 1/6 _____
5/6 _____

2. Write eight equivalent fraction pairs that you can find from your fraction bars. For example, 2/10 = 1/5.

## Meaning Activity 13 Adding Fractions

**Teacher Sheet**

A demonstration with fraction bars.

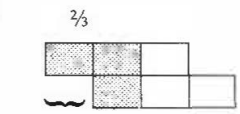
1. Start with the same denominators.
   (a) 2/4 + ¼ = ¾
   (b) 3/5 + 1/5 = 4/5
   (c) 7/10 + 3/10 = 10/10 = 1
   Observe: In a, b and c, the answer has the same denominator as the sum. When the numerator and the denominator are the same, the result is 1.
2. Next use different denominators (for example, 1/5 + ½ = 7/10). Match the fraction bars with another bar that has a division which lines up with that of the sum.
   (a) ¼ + ½ = ¾
   (b) 1/6 + ⅓ = ½
   (c) ¼ + ⅔ = 11/12
   (d) ⅓ + ½ = 5/6
   Observe: The answer's denominator might (i) be the same as that of one part of the sum or (ii) not be the same as either part of the sum.
   Hence: The idea of finding the lowest common denominator (LCD), which is the same as finding the lowest common multiple (LCM).
3. Give students several examples of their own to work out on fraction strips.

## Meaning Activity 14 Subtraction of Fractions

**Teacher Sheet**

Use the procedure in Meaning Activity 13 but make the end lines of the two given fractions line up so that one fraction bar connects with the other bar to give the idea of taking away (subtracting) one section from the other section. Find a fraction bar with a division that matches the length of the remaining section.

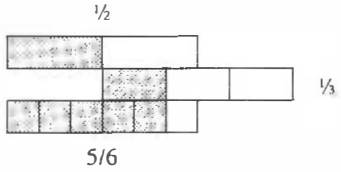
For example,




(a) ⅔ − ⅓ = ⅓
(b) 4/5 − 2/5 = 2/5
(c) 5/6 − ⅓ = 3/6
(d) ¾ − ½ = ¼
(e) ⅝ − ¼ = ⅜
(f) 5/6 − ¾ = 1/12
(g) ½ − ⅓ = 1/6

## Meaning Activity 15 Adding and Subtracting Fractions

1. Take a ½ bar and a ⅓ bar. Match along the ¼ and other bars until both match evenly on one of the bar sheets. For example, the ½ bar will match evenly along the ¼, 1/6, ⅛, 1/10 and 1/12 sheet. The ⅓ bar will match evenly on the 1/6 and 1/12 sheet. Thus, use the 1/6 bars. Place the ½ bar alongside the ⅓ bar overlapping on a 1/6 bar sheet. Count the sections of the 1/6 bar sheet covered.




Using the above procedure, try

(a) ½ + ¼ = ___ (d) ⅔ + ¼ = ___
(b) ½ + 1/5 = ___ (e) 1/5 + 1/10 = ___
(c) ½ + 1/6 = ___ (f) ½ + ¾ = ___

2. Follow the procedure in question 1 to locate the common bar sheet. Now place the ½ bar along the 1/6 bar sheet. Place the ⅓ bar below the ½ bar on the 1/6 bar sheet. The amount of the ½ bar showing beyond the ⅓ bar is the difference. Now try the following:

(a) ¾ − ½ = ___ (e) 7/10 − 1/5 = ___
(b) ½ − 1/6 = ___ (f) ⅔ − ½ = ___
(c) 5/6 − ½ = ___ (g) 5/12 − ⅓ = ___
(d) ⅜ − ¼ = ___ (h) 7/12 − ¼ = ___

3. Follow the procedure from question 1 again. This time write the equivalent fractions, for example, ½ as 3/6, ⅓ as 2/6, and then add or subtract the numerators as required.

| | Equivalent Fractions | Solution |
|---|---|---|
| (a) ⅜ + ½ = | _____ | = _____ |
| (b) ¼ + ⅔ = | _____ | = _____ |

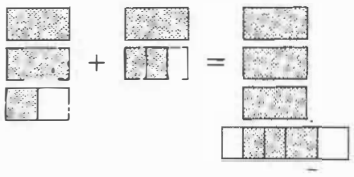
(c) 3/5 − 1/10 = _____ = _____

(d) ¾ − ⅔ = _____ = _____

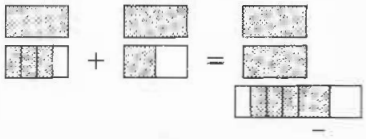
## Meaning Activity 16 Adding and Subtracting Mixed Numbers

Lay your fraction bars in colors on your desk. Do the following with your fraction bars:

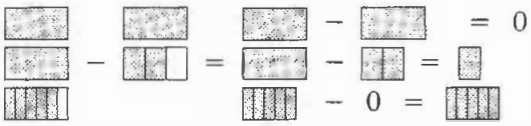
1. Add: 2 ½ + 1 ⅔ = 4 plus a bit. Find the bar that equals the bit left over.



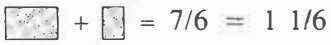
2. Add: 1 ¾ + 1 ½ = 3 ¼ or 3 2/8



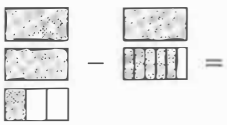
3. Subtract: 2 5/6 − 1 ⅔ =



adding the leftover bits



4. Subtract: 2 ⅓ − 1 5/6 =



5. Using the above examples, show the solutions to the following by drawing fraction bars.

(a) 1 ½ + 2 ⅓ =

(b) 2 ¼ + 2 ½ =

(c) 1 ¾ + 1 ½ =

(d) 2 ⅔ + 1 ½ =

(e) 3 ½ + 1 3/5 =

(f) 4 ¾ − 1 ½ =

(g) 4 ⅔ − 2 ½ =

(h) 4 ⅝ − 3 ¼ =

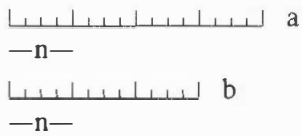
(i) 2 ¼ − 1 ⅞ =

(j) 1 1/5 − ½ =

## Problem Solving Activity 1 GCF and LCM

1. What is meant by the greatest common factor (GCF)?
2. What is meant by the lowest common multiple (LCM)?
3. What is the GCF of two numbers if one of them is triple the other? What is their LCM?
4. Find two numbers that fit the description in question 3.
5. What is pictured below? What is the GCF of a and b? The LCM of a and b?

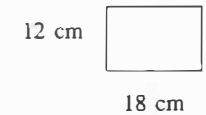

6. What is pictured below?




n is the LCM of a and b?
n is the GCF of a and b?

## Problem Solving Activity 2 GCF and LCM

1. During the past two years, Laura wore out a pair of runners every 12 months, a pair of socks every 6 months and a t-shirt every 8 months. At these rates, how often will she have to replace all of these items at the same time?
2. Bus #32 takes 45 minutes to complete one trip around its route. Bus #36 takes 60 minutes to complete one trip around its route. If they both leave the Southgate terminal at 9:00 a.m., when will they both be there together again?
3. Holly went to a card store and bought several packages of stickers. She got 75 stickers altogether. Darlene bought 125 stickers. Each package has the same number of stickers. How many stickers are there in each package?
4. Divide the shape into the minimum number of squares which are all the same size. How long is the side of each square?



## Problem Solving Activity 3 Equivalent Fractions

1. Estimate what fractions are represented by the shaded areas:

a) 

b) 

c) 

d) 

2. What are equivalent fractions?
3. Make up a problem which has an answer which is a fraction.
4. Make up an equality that fits $\frac{x}{y} = \frac{4x}{4y}$
5. Find values of x and y so that $\frac{x}{y} = \frac{5x}{5y}$
6. How many quarters are necessary to make 7/4 dollars?

## Problem Solving Activity 4 Comparing and Ordering

1. Of the 30 students in Mr. Lee's class, 3 were absent on Monday. On the same day, 2 students were absent out of 25 in Mr. Burton's class. Which class had the smaller fraction of students absent?
2. While playing basketball, Corey sank six out of thirteen shots into the basket. Paul sank five out of eight shots. Who got the greater fraction of shots into the hoop?
3. If $\frac{a}{b} > \frac{c}{b}$, what can you conclude about a and c?
4. Give an example that fits $\frac{a}{b} > \frac{c}{b}$.
5. If $\frac{a}{b}$ and $\frac{c}{d}$, how do b and c compare?
6. Give an example that fits $\frac{a}{b} < \frac{a}{c}$.
7. Two fractions, $\frac{a}{b}$ and $\frac{c}{d}$, are between zero and one. $\frac{a}{b}$ is closer to 1 than $\frac{c}{d}$. Which fraction is greater?
8. Make up a problem that requires comparing two fractions.

## Problem Solving Activity 5 Adding and Subtracting Fractions

1. In a hall, one third of the people present are men, one quarter are women and the rest are children. If 1,152 people are in the hall, how many children are present?
2. Michael has two containers, A and B. Container B holds twice as much as container A. A is ½-filled and B is ⅓-filled with syrup. Michael fills the rest of each container with water. He then pours the contents of containers A and B into a third container. What fraction of the total content is water?
3. Perform the following operations:

(a) $\frac{a}{b} + \frac{c}{b} =$

(b) $\frac{a}{b} - \frac{c}{b} =$

(c) $\frac{a}{a} + \frac{b}{a} =$

(d) $\frac{a}{b} + \frac{c}{d} =$

(e) $\frac{a}{b} - \frac{c}{d} =$

## Problem Solving Activity 6 Fractions and Mixed Numbers

1. After a farmer died, his will was read. He left instructions that his three sons were to divide up 17 cows as follows: "Tom gets ½, or 8 ½ cows; Dick gets ⅓, or 5 ⅔ cows; Harry gets 1/9, or 1 8/9 cows." The sons were each able to get their full shares, and all the cows lived. How did they do this? (Hint: Borrow 1 cow from a neighbor, then divide.)
2. Estimate first, then solve the following:

(a) 4 ⅔ + 1 ¾ = (5 + 2 = 7) =

(b) 2 ½ + 3 ¾ =

(c) 20 4/5 + 3 ⅓ =

(d) 6 ⅞ - 1 ⅓ =

3. Find four fractions (numbers less than one) whose sum is between two and three.

4. Use long division to create a mixed number; for example:

$$5\overline{)16} \quad 3 \text{ r } 1 \qquad 8\overline{)47} \qquad 6\overline{)52}$$

$$\underline{15}$$

$$1$$

5. Given n/d, what type of fraction do you have if $n < d$, $n > d$, $n = d$? Give an example of each.

## Problem Solving Activity 7 Multiplying Fractions

1. Karen had 15 posters. She sold 2/5 of her posters for $0.50 each and the remaining posters for $0.35 each. How much money did she get for the 15 posters?
2. Tyler receives $30 allowance monthly. He spends 2/5 of his money on snacks, 3/10 on entertainment, 1/10 on comics and the remainder is saved. How much money does Tyler save each month? In a year?
3. The Boston Bruins played 80 regular season hockey games. During the first half of the season, they won ¾ of their games. In the second half, they won ⅞ of their games. How many games did they win over the season?

## Problem Solving Activity 8 Dividing Fractions

1. On the weekend, Jackie worked 9 ¾ hours and earned $39. What was her hourly wage?
2. Theresa has a part-time job at McDonald's. It takes her ¾ of a minute to assemble a Big Mac. If she worked nonstop for 7 ½ hours, how many Big Macs would Theresa make?
3. Jim bought four rectangular ice cream cakes for his class party. Estimate how many pieces he must cut so that each student receives 1/12 of the cakes.

## Problem Solving Activity 9 Problems/ Operations

1. Goalie Grant Fuhr played 7 ⅓ games without allowing a goal. How many periods did he play?
2. The Edmonton Eskimos played 17 quarters without getting a field goal. How many games did they play?
3. Your teacher wants to give 1/6 of a chocolate bar to each member of your class. How many bars will be needed?
4. Your teacher wants to give one and a half chocolate bars to each member of your class. How many bars will be needed?
5. Did you solve questions 3 and 4 the same way? Explain.