Prologue

Alton T. Olson and Sol E. Sigurdson

The Meaning in Mathematics Teaching (MMT) project was completed just before the National Council of Teachers of Mathematics (NCTM) published the *Curriculum and Evaluation Standards for School Mathematics*. However, the spirit of this project fits very well with the NCTM goals. For example, the following statement is taken from the NCTM standards document and represents the goals of the students:

> Educational goals for students must reflect the importance of mathematical literacy. Toward this end, the K–12 standards articulate five general goals for all students: (1) that they learn to value mathematics, (2) that they become confident in their ability to do mathematics, (3) that they become mathematical problem solvers, (4) that they learn to communicate mathematically, and (5) that they learn to reason mathematically.

The MMT project goals, though phrased differently, could be reformed easily to encompass these student goals.

The Meaning in Mathematics Teaching project was funded by the Social Science and Humanities Research Council (SSHRC) and involved 55 Grade 8 mathematics teachers. The study assessed the importance of teaching mathematics "with meaning" within a direct instruction context. The data supports use of meaning in teaching mathematics in that students taught in this way achieved higher marks (than other students) on a test including knowledge, comprehension and problem solving items. This issue of *delta-K* provides a resource manual of meaning activities of the type followed by teachers participating in the teaching-with-meaning group.

Within the project, another group of teachers used daily problem solving activities in addition to meaning. The effect of these activities on student learning was not clear. At this point, it seems the activities do not add to student achievement, but they do not detract from student learning and can be a motivational element in the classroom. The problem solving activities here were used by project teachers and, for the most part, relate closely to the content of the unit in which they are given.

We did not, within the project, develop common mental computation exercises, but the ones given here in the "Extension" and "Applications" sections will serve as useful illustrations.

In looking at these meaning, problem solving and mental computation activities, the reader should remember that they were used in conjunction with the Missouri mathematics project lesson format outlined in the "Summary of Key Instructional Behaviors" section below.

Summary of Key Instructional Behaviors

1. Daily review (first 8 minutes, except Mondays)
   - A. Review the concepts and skills associated with the homework
   - B. Collect and deal with homework assignments
   - C. Do several mental computation exercises
2. Development (about 20 minutes)
   - A. Focus briefly on prerequisite skills and concepts
   - B. Focus on meaning and on promoting student understanding using lively explanations, demonstrations, process explanations, illustrations and so on

     Aids to understanding include
     (1) concrete materials,
     (2) concrete examples,
     (3) comparisons and
     (4) class discussion.

C. Student Comprehension Assessment
    1. Use process/product questions (active interaction)
    2. Use controlled practice, correcting misunderstandings
D. Repetition and elaboration of the meaning portion as necessary

3. Seatwork (about 15 minutes)
    A. Uninterrupted successful practice
    B. Momentum—keep the ball rolling, get everyone involved and sustain involvement
    C. Alerting—let students know their work will be checked at the end of the period
    D. Accountability—check the students' work
4. Homework
    A. Assign homework on a regular basis at the end of each math class (possibly excepting Fridays)
    B. Include one or two review problems
5. Special reviews
    A. Weekly review/maintenance
        1. Conduct during the first 20 minutes each Monday
        2. Focus on skills and concepts covered during the previous week
    B. Monthly review/maintenance
        1. Conduct every fourth Monday
        2. Focus on skills and concepts covered since the last monthly review

In addition to this format, the Missouri mathematics project recommends highly interactive teaching. Although problem solving is not mentioned in this format, teachers in the problem solving group were asked to do 10 minutes of problem solving activities daily, near the beginning of each lesson.

Meaning Activities

"Meaning in mathematics teaching" is a fancy label difficult to define. *Meaning* as used in this study includes relationships of formal mathematics to

1. other prerequisite mathematical knowledge;
2. concrete representations including physical objects and pictures;
3. practical uses of mathematics, within the students' world and in wider uses;
4. broader mathematical structures and the generality of concepts.

In this way, meaning might include a logical understanding of mathematics but even more importantly connects mathematics to images, the physical world, practical uses and other student knowledge. This view of mathematical learning is supported by the new, cognitive psychological view of learning that the learner continually develops knowledge networks which must connect inevitably to existing knowledge.

A major problem with such a definition is that meaning is not bounded. How is the teacher to know when meaning has been achieved? Indeed, is "true meaning" ever achieved or do knowledge networks continue to grow? These are important theoretical questions. However, in a practical sense for Grade 8 mathematics, the activities here give an appropriate scope for meaning development in mathematics.

In the development part of the lessons, meaning activities were set in an interactive teaching context, followed by seatwork in which students "practised" through assignments. In our particular project, teachers used meaning activities to supplement a textbook which formed the instruction core. The teachers used these activities where they thought the development of meaning was important to students' knowledge. It is probably fair to say that meaning was considered an add-on, as perhaps it must always be.

To illustrate the add-on meaning, suppose you have learned that $6 + 7 = 13$. The fact is easy to learn. Then a teacher encourages you to write this as $(3 + 3) + 7 = 3 + (3 + 7) = 3 + 10$. Now average 10-year-olds might feel they "understand why" $6 + 7 = 13$. They have "added meaning" to their knowledge. It does not follow that all meaning is add-on, but a case can be made that a good part of meaning is aimed at after the fact.

When to Use Meaning Activities

Where do meaning activities come in the learning sequence? Use your professional judgment. One aspect of this decision became clear during the project. Students in Grade 8 know a lot about what we teach them. Even in the study of percent, not only do they know fractions, decimals and ratio but also they have studied percent before. No mathematics teaching at the junior high level is done on virgin territory. In effect, *all* the teaching we do (and not simply the meaning) is add-on.

Motivational factors may be equally important in considering when to use the meaning activities.

Meaning Practice

Students must learn to do mathematics by practising it. They also learn meaning the same way. To use the previous example, showing students that (3 + 3) + 7 = 3 + (3 + 7) = 3 + 10 is not enough. They must be encouraged to practise breaking 6 up into its various parts and to see that only one "breaking" works. Students must then practise this process on other numbers to see number breakup as a useful tool to be applied in different situations. Even work with physical models requires not so much *insight* on the students' part as *practice* in fully understanding the relationship. If "*practice* with meaning" is omitted, a much smaller percentage of students will establish the appropriate relationships.

Unless we are intentional about our meaning implementation, it will not happen. [Our study showed substantial differences in teacher ability to implement a meaning approach.] Teachers must expect students to reproduce meaning learning. It is not enough at the end of a unit to ask 6 + 7 = ____. We want to know if students think of 6 + 7 as 3 + 3 + 7. Do they have this particular meaning? If we are interested in meaning, students must be accountable for it. All tests should include meaning items. How else will students come to think of meaning as an essential ingredient in mathematics?

How to Use Meaning Activities

The meaning activities here can be used as is. Copies can be made and handed out to students. However, none are stand-alone activities; all are meant to be used interactively. Teacher-student interaction should begin every use of these activities. As students become aware of the purpose of the activity and how to do it, they can complete the activity on their own. Every meaning activity presents its own challenges, and each should end with an interactive summation of the activity's results.

The meaning activities included here are not a complete set but rather the ones project teachers felt were relevant. They have not been perfected, and some may be found wanting. Become familiar with them before you use them. Even then, students will surprise you. Like any useful tool, meaning activities perform best in the hands of a skilled user.

Problem Solving Questions

The problem solving exercises presented here were project developed. Teachers were urged to include problem solving in the early stages of a lesson, either to start or after the homework correction and mental computation. Although in our project student achievement was not noticeably enhanced, we think judicious use of these activities can only benefit students.

Theoretically, problem solving is just as difficult to define as meaning. The project took a conservative view, looking at problems that made use of the following ideas:

1. Drawing diagrams
2. Using complex and/or simple numbers
3. Incorporating an overall plan: understand, plan, execute, look back
4. Problems without numbers
5. Estimation of answers
6. Focus on reading skills
7. Two different problems—with the same structure
8. Students making up problems
9. Translating to open sentences

The other consideration we made was to use problems related to unit content. We did not encourage use of classic problems such as "locker" or "checkerboard" problems. A final consideration was to use an interactive approach. Students were not left for the 10-minute period to work on their own. Teachers dealt with problems as a class interaction. Our rationale was to make problem solving processes explicit and up-front in every mathematics period, with the idea that this would influence the remainder of the class. The downside of this approach is that the 10 minutes of problem solving might detract from otherwise valuable time. Although improvement in student learning did not show up on student test scores, many project teachers feel this was an important aspect of the mathematics class.

Mental Computation

We have included several examples of mental computation. The idea behind mental computation is not so much mental drill as computation without pencil and paper. Given a question like 1 ½ x 6 = __ , on paper one writes 3/2 x 6 = 3/1 x 3 = 9. Mentally, one is liable to proceed by thinking of one 6 plus half of 6, 6 + 3 = 9.

By accurate answers achieved mentally, mental computation encourages a number sense in students, not an estimation sense. Although we do not know precisely how this affects student learning, project teachers identify this as positive classroom activity. The project teachers experimented with how to conduct mental computation. Most teachers eventually had students write down answers for later correction. Otherwise, students who are rapid calculators dominate classroom proceedings. Some teachers wrote the questions on overhead transparencies. What is essential is that students understand the activity is to be done mentally.

Message to Project Teachers

These activities are laid out according to the several chapters of *Journeys in Math*, Grade 8. Most of them are applicable in any junior high grade. Your using them will help repay you and your students for your much-appreciated efforts during the MMT project.