Unit I: Perimeter and Area

Teachers' Notes on Meaning Activities

Activity 1

These three questions establish perimeter as independent of shape and size. Question 2 shows that removing some squares does not change perimeter. Question 3 relates to formula work.

Activity 2

This simple activity can be turned into a meaning activity through discussion. The symmetries of these shapes are worth noting. Perimeters of symmetric figures in the grid context are even numbers.

Activity 3

In the first section, gaining a sense of the length of a metre is essential for interpreting measurement situations. The second section reinforces the meaning of regular polygons. Sketching figures in notebooks formalizes and consolidates student learning.

Activity 4

In addition to conveying meaning about π, experiments and demonstrations have an "episodic" value. The episode can help students remember the lesson.

Activity 5

All parallelograms can be divided up and made to fit into the related rectangle. The concept of "rearranging areas" is important for junior high school mathematics. The related rectangle should have the same base and altitude as the parallelogram.

Activity 6

This activity is similar to Activity 5. This time related rectangles are found for given triangles.

Activity 7

Here the idea of area as the number of square units is stressed. If areas of parallelograms and triangles are known, students can check their answers by other means. One of the "definitions" of meaning is having two ways to do things, both correct but useful in different situations.

Activity 8

This is a reinforcement sheet relating square units to rectangles, triangles and parallelograms. Students can be asked to draw any parallelogram (triangle) on their grid paper and to find its area or to draw a parallelogram (triangle) with area 24 square units.

Activity 9

Any polygon can be thought of as a combination of simpler shapes. This is an especially useful concept in some of the complex area problems.

Activity 10

The most interesting polygons are regular. These are usually divided into isosceles triangles. Here students can make use of their isosceles triangle knowledge.

Activity 11

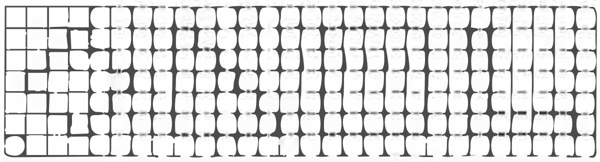
This is the ultimate activity in dividing regular polygons into triangles. Some caution is recommended in this activity as students do not easily see the connections.

Activity 12

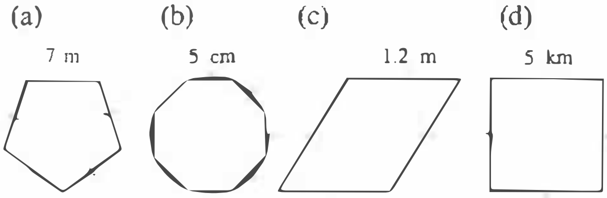
Three interesting and important ways of looking at the area of a circle are dealt with.

Meaning Activity 1 Perimeter

1. On grid paper, draw five rectangular shapes with perimeters of 24 grid units.
2. Find the shape on the following page that has the largest perimeter. What happens to the perimeter of a shape if a grid square is removed?



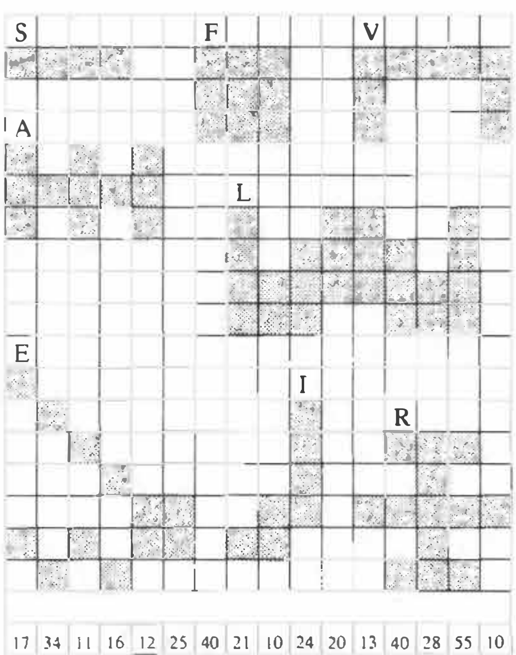
3. Show how you would find the perimeter of each figure below in two different ways.



Meaning Activity 2 Perimeter

Perimeter Code

What do you buy if you're dying and only have a dime? To answer this important question, find the perimeters of the shapes below. Find each answer in the code at the bottom of the page. Each time the answer appears in the code, write the letter of that shape above it.



Perimeter Code

What patterns do you notice in these perimeters?

Meaning Activity 3 Perimeter

Teacher Sheet

A. Perimeter (Adaptable to teacher demo)

Required materials: a 1-m length of string per student.

Teacher provides a list of objects available in the classroom. Use the string to determine if a given object has a perimeter of 1 m, less than 1 m or more than 1 m. Suggested objects:

1. desk top
2. overhead projector
3. door
4. math book
5. loose-leaf paper
6. one shelf of a bookcase

Students now apply their understanding to objects that are less than 1 m, equal to 1 m and greater than 1 m which are not available in the classroom. Suggested objects:

1. locker door
2. refrigerator door
3. sidewalk block
4. pillow from your bed
5. lid of the mailbox

B. Perimeter of Special Polygons

Required materials: toothpicks (10 per pair of students), rulers

After explaining the meaning of *equilateral* and reviewing the names of regular polygons—such as square, rhombus, pentagon—have students form each of the following: triangle, square, rhombus, pentagon, hexagon, octagon and decagon.

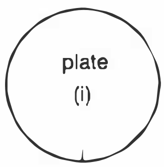
Have students put aside all but one of the toothpicks. Have them measure it and, using the information, calculate the perimeter of each polygon. Have students sketch shapes in their notebooks, writing inside the drawing the length of side, the perimeter and the formula for the shape.

Meaning Activity 4 Circumference

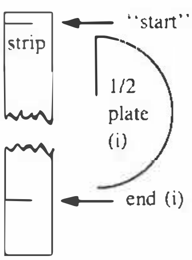
Teacher Sheet

Materials: 3 different-sized paper plates
1 paper strip at least 150 cm in length
1 pair of scissors
1 pencil

Procedures:



A. 1. Mark a start line on one end of the strip.
   2. Make a mark on the edge of a plate.
   3. Match the mark on the plate with the mark on the strip.
   4. Roll the plate through one complete revolution along the strip. Mark the new position of the plate mark on the strip.



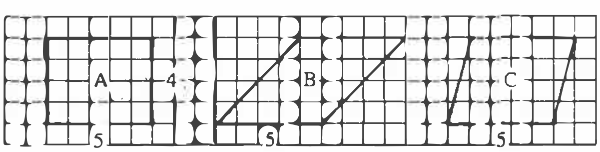
B. 1. Cut the plate in half.
   2. Using the cut edge as a unit of measure, count the number of units needed to measure the line made when the plate was rolled through a complete revolution.

C. Repeat the above procedures for each of the three paper plates.

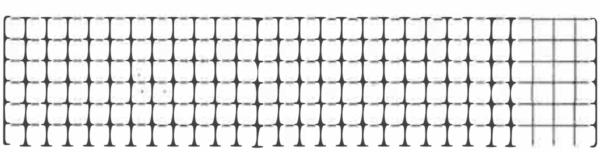
D. Note the constant relationship between the rolled plate edge (circumference) and the cut edge of the half-plate (diameter).

Meaning Activity 5 Parallelograms

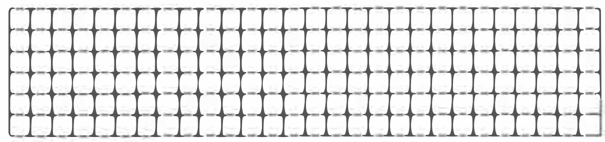
1. Count the squares in each of the shapes below. What is *area*?



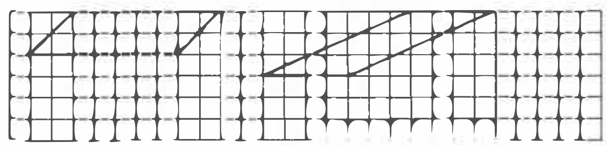
Draw two copies of rectangle A below. Show how parallelograms B and C can be cut so they fit into the rectangles. Count the squares in each of the shapes A, B and C. What is your conclusion? Describe your cutting of parallelograms B and C in words.



Draw another copy of rectangle A below. Show how parallelogram B can be cut and fit into rectangle A in a different way than you showed above.



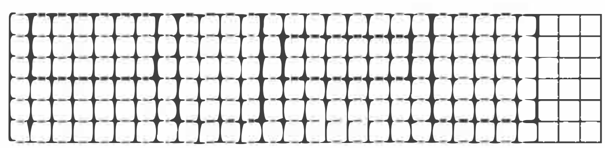
2. For each of these parallelograms, draw a rectangle into which it fits exactly. (Note: A different rectangle is required for each figure.)



3. Every parallelogram can be reconstructed as a rectangle. How does this help in finding area? What is the area of each of the parallelograms in this activity?

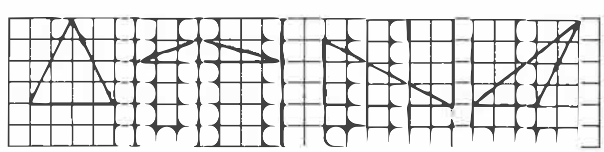
Meaning Activity 6 Triangle Areas

1. In each rectangle below, draw a triangle using a side of the rectangle as a side of the triangle. (Of course, drawing a diagonal is a solution, but find others.)

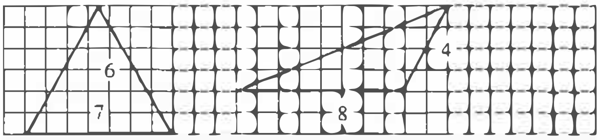


How do the sizes of the triangles compare with the rectangles?

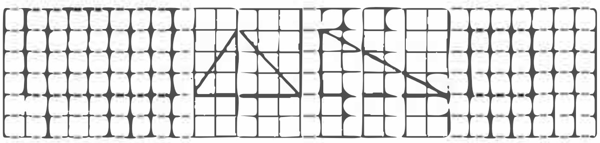
2. Now reverse the process. Given the triangles below, draw the rectangles that are twice as large in area. How does this help us find the areas of the triangles?



3. Find the areas of these two triangles by finding the areas of the related rectangles. (Note: You can do this by counting square units or by using the area formula for the area of a rectangle.)



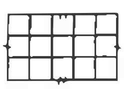
4. With these shapes create nine different shapes and find their areas. Be as original as you can. Pass your work to the person behind you to see if she or he can verify your work.

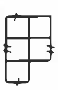


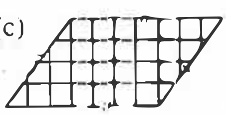
Meaning Activity 7 Squares, Rectangles, Triangles, Parallelograms

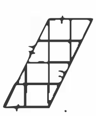
Area: Measurement of the amount of space contained within a shape. It is expressed in square units.

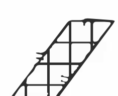
1. Find the area of the following shapes. Express your answers in "grid-paper units."

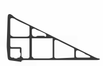
(a)  ____

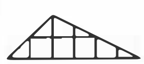
(b)  ____

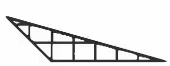
(c)  ____

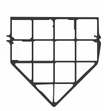
(d)  ____

(e)  ____

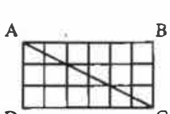
(f)  ____

(g)  ____

(h)  ____

(i)  ____

(j)  ____

2. 

Find the area of shapes

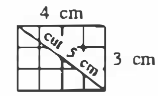
ABCD ______

ABC ______

What do you notice about these areas?

Meaning Activity 8 Squares, Rectangles, Triangles, Parallelograms

1. From grid paper, cut out a 3-cm x 4-cm rectangle. Divide it into two pieces along the diagonal. Join the two triangles to form as many shapes as possible. Find the area of each of these shapes.

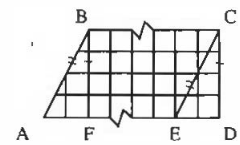


2. Find the area of shapes

ABCE ______

BCDF ______

What do you notice about the two areas? ______________

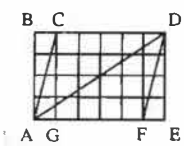


3. Find the areas of and name the shapes.

(a) ABDE _____ ______

(b) ADE _____ ______

What do you notice about the two areas? ______________



Find the areas of and name the shapes.

(c) ACD _____ ______

(d) ACDF _____ ______

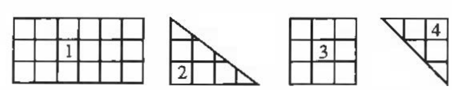
(e) CDEG _____ ______

What do you notice about the areas in:

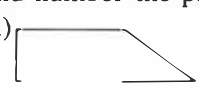
(f) c and d? ______________

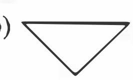
(g) d and e? ______________

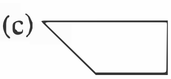
Meaning Activity 9 Area of Polygons

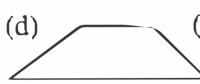
1. 

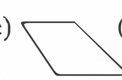
Combine the shapes above to form each of the following polygons. Draw lines to identify the shapes and number the parts.

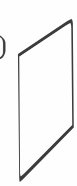
(a) 

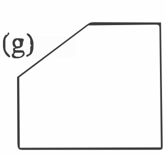
(b) 

(c) 

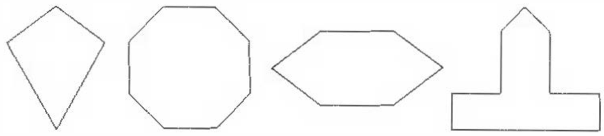
(d) 

(e) 

(f) 

(g) 

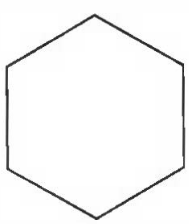
2. Divide each polygon into different types of polygons.



Devise different ways of dividing them so that the total area may be easily found.

Meaning Activity 10 Areas of Regular Polygons

1. One way to find the area of a regular hexagon is to divide it into triangles by drawing lines from the "centre" of the hexagon to the vertices and then finding the areas of the triangles. How many triangles are in a regular hexagon? ________

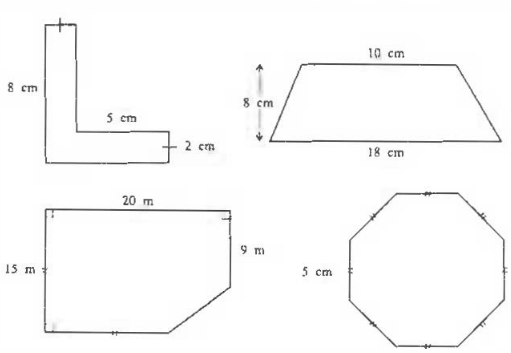


Write a formula for finding the area of a regular hexagon.

How many triangles are in a regular octagon? _____ In a regular decagon? _____

Write a formula for the area of a regular decagon.

2. For each of the shapes below do the following:
   (a) Divide the polygons into polygons for which you can find the area.
   (b) Find the area of each part.
   (c) Find the total area of each shape.



Meaning Activity 11 Areas of Other Polygons

Teacher Sheet

Once the concept of dividing a regular polygon into congruent triangles has been taught, the concept of *area of a circle* can be reinforced. Review regular polygon (Diagram 1) on the board. Then do teacher demonstration using the following steps:

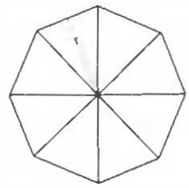


Diagram 1

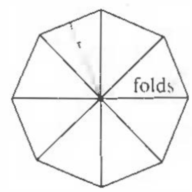


Diagram 2

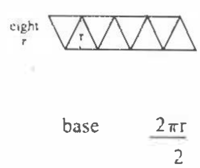


Diagram 3

1. Cut a large circle (minimum 30-cm diameter) into eight congruent triangles by folding in half three times and then cutting.
2. To emphasize the outer edges of the triangles and the radius of the circle, highlight the edges and radius with contrasting colors.
3. Fit the eight triangles together to resemble a parallelogram as shown in Diagram 3.
4. The base of the parallelograms is approximated by one-half the circumference of the circle. The height is approximated by the radius of the circle. Thus:

   Area of parallelogram = base x height

   $$= \frac{\pi \times d}{2} \times r$$

   $$= \frac{\pi \times 2r}{2} \times r$$

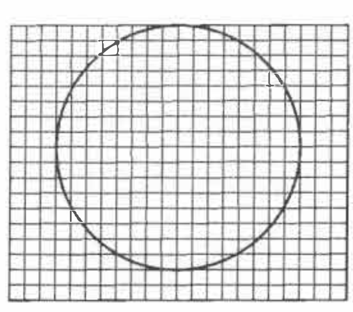
   $$= \pi r^2$$

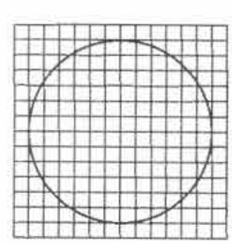
   So the formula for the area is $A = \pi r^2$

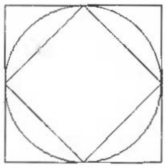
5. Pin your final cutout on the bulletin board.
6. Have students figure out area by finding the area of one triangle and multipying by eight to get the area of the circle. Assume: height equals radius, and a triangle base is $\frac{1}{8}\pi d$.

Meaning Activity 12 Area of Circle

1. Here are two circles. How many units in diameter is each? (Count the squares.)

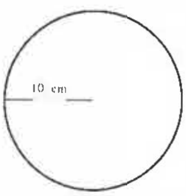






scale: 1 cm = ⊔

2. (a) Mark off the area of the outside square in 0.5-cm units.
   (b) How does the *area* of the circle compare with that of the outside square? By how many square centimetres do they differ?
   (c) How does the *average* of the areas of the inside and outside square compare with the area of the circle?

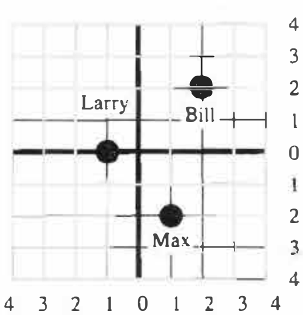


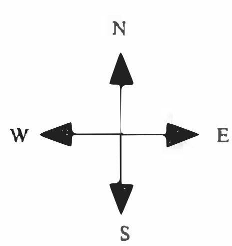
scale: 1 cm = ⊔

3. (a) Mark the edge of the circle in some color.
   (b) What is the circumference? (radius = 10 cm)
   (c) Cut the circle into eight congruent pie shapes.
   (d) Arrange the shapes to form a parallelogram.
   (e) Approximately, what are the dimensions of this parallelogram?
   (f) What is the area of this parallelogram?
   (g) Why is it the same area as the circle?

Extension 1 Linear Measurement

1. The Fall Fun Run started at the school. The participants ran six blocks west, four blocks north, twelve blocks west and sixteen blocks south. How many blocks from the school were they?
2. A grid of Calgary is shown. Bill lives at 2nd Street 2nd Avenue NE; Max lives at 1st Street 2nd Avenue SE; Larry lives at 1st Street 0 Avenue W.

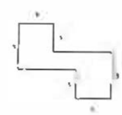




   (a) Which two live the closest to each other? (They must walk along streets or avenues. No shortcuts allowed.)
   (b) Which house(s) should they meet at so that the total walking distance is the smallest possible?

3. (a) Calculate the perimeter of a square having a side of 4 cm. A side of "s" cm.
   (b) Calculate the perimeter of a pentagon having a side of 10 cm. A side of "n" cm.
   (c) The perimeter of a hexagon, "t" cm on a side would be ______ ?
   (d) State a general rule for finding the perimeters of regular polygons.

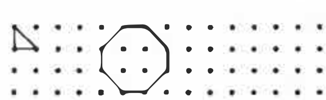
Extension 2 Linear Measurement

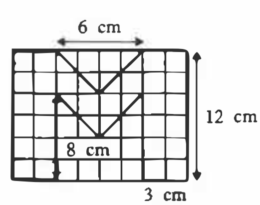


1. If the area of this figure is 108 square units, what is its perimeter?
2. Frames need to be built for the set of a school play. Two door frames with outside dimensions 2 m by 1.5 m, one sign frame with dimensions 1.75 m by 0.67 m and two window frames with dimensions 0.67 m by 0.67 m are needed. If lumber comes in 2.67-m lengths, what is the minimum number of lengths required?
3. How does changing shape while keeping a constant perimeter affect the area of the shape? Using whole number lengths, answer these questions about rectangles that have a perimeter of 48 units.
   (a) What are the dimensions of the rectangle with the largest area?
   (b) What are the dimensions of the rectangle with the smallest area?

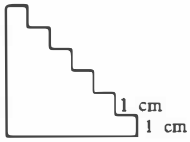
Extension 3 Areas of Other Polygons

1. The area of the triangle is 2 cm^2. What is the area of the octagon? Use grid paper to solve.



2. 

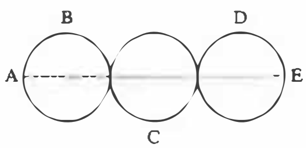
Maggie cut her initial out of a 12 cm by 12 cm piece of felt. How many square centimetres were used? How much felt was not used?

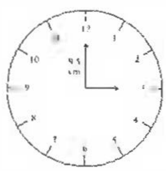
3. 

Find the area of this figure. The line segments of the "steps" meet at right angles and are 1 cm in length.

Extension 4 Circumference

1. A cable company, after having laid a cable on the Earth's surface around the equator, is told that the cable should have been installed 1 m above the surface. How much more cable is needed? (Earth's circumference is approximately 40 000 km.)
2. The three circles shown are identical. The line segment AE is 42 cm long. How long is the curved path ABCDE?

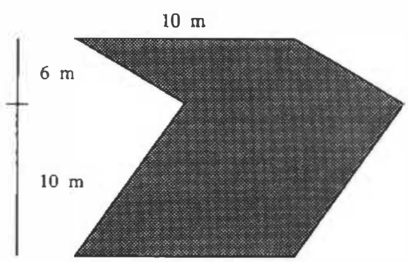




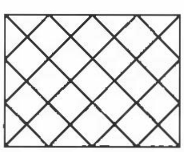
3. The minute hand on the clock is 9.5 cm long. How far does the tip of the minute hand travel in 24 hours?

Extension 5 Area of Rectangle

1. Calculate the area of the shaded figure.



2. This leaded window measures m cm by n cm.



(a) Find the area of a small pane.

(b) If the actual dimensions of the window are 60 cm by 80 cm, find the area of each pane.

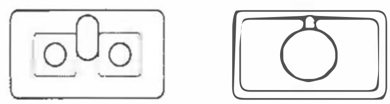
3. (a) What is the greatest area possible for a rectangular carpet which has a perimeter of 26 m? The lengths of the sides in metres are whole numbers.

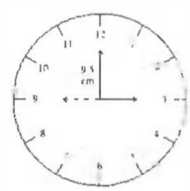
(b) How can a piece of wallboard 1.6 m by 90 cm be cut into two pieces so that it completely covers an area 1.2 m by 1.2 m? (The cut resembles a stairway.)

(c) Remove squares so that the perimeter remains the same, but the area becomes (i) 7 cm^2, (ii) 6 cm^2 and (iii) 5 cm^2.

Extension 6 Area of a Circle

1. Will two 6-cm drains give faster drainage, slower drainage or the same drainage as a 12-cm drain? Explain why.

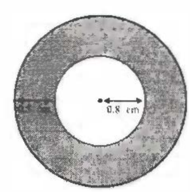


2. 

(a) The minute hand on a clock is n cm long. What would be the area it covered in 45 minutes?

(b) If the length of the minute hand is 9.5 cm, what would the actual area covered be?

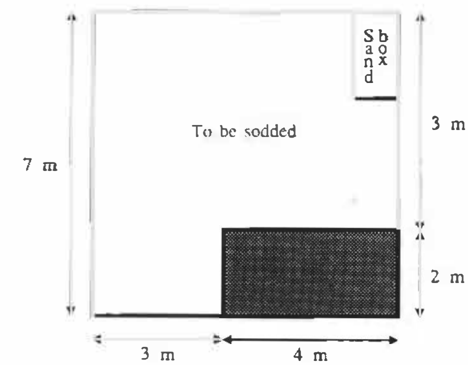
3. What is the area of the shaded region?



Extension 7 Units of Area

1. Mr. Gretzky plans to sod part of his yard. Each piece of sod measures 40 cm by 150 cm and costs \$1.55.*

* Taken from *Journeys in Math* 8, p.91.



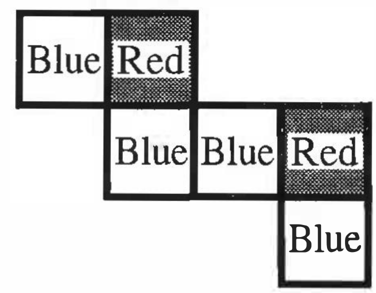
(a) Find the area to be sodded.

(b) Find the area of each piece of sod.

(c) How many pieces of sod are needed?

(d) Find the cost of the sod.

2. This figure consists of six congruent tiles and it has a total area of 294 cm^2. Two tiles are red and four tiles are blue.



(a) What is the length of each tile?

(b) What is the total area covered by blue tile?

(c) What is the total area covered by red tile?