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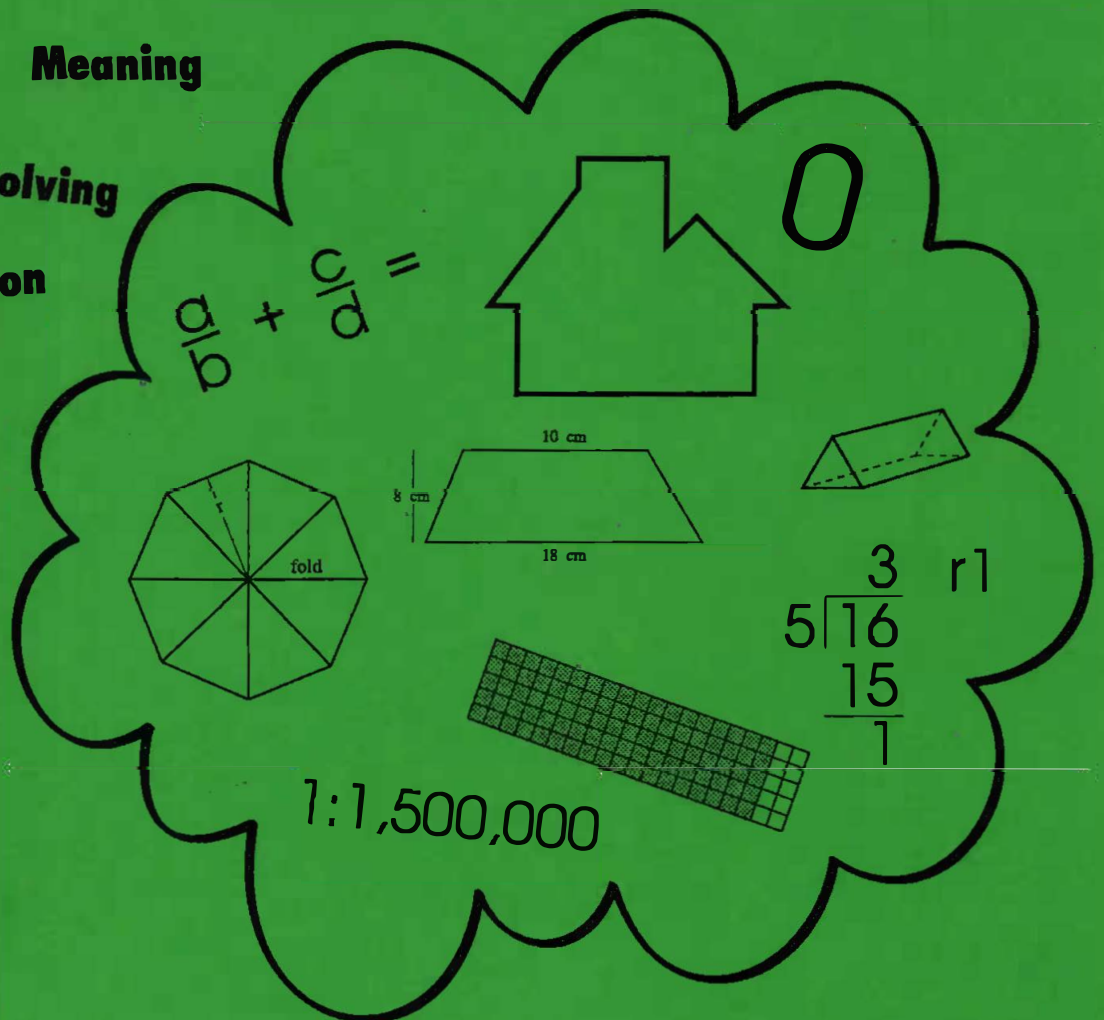
December 1992

## ACTIVITIES FOR ACTIVE MATHEMATICS TEACHING

Meaning

Problem Solving

Mental Computation



## ACTIVITIES FOR GRADE 8 MATHEMATICS

Collected and revised by Sol E. Sigurdson

Edited by Alton T. Olson

## CONTENTS

Comments on Contributors	2	
Editor's Comments	3	<i>A. Craig Loewen</i>
Prologue	4	<i>Alton T. Olson and Sol E. Sigurdson</i>
Summary of Key Instructional Behaviors	4	
Meaning Activities	5	
Problem Solving Questions	6	
Mental Computation	6	
Message to Project Teachers	7	
Unit I: PERIMETER AND AREA		
Teachers' Notes on Meaning Activities	8	
Meaning Activities	8	
Extension	13	
Unit II: SURFACE AREA AND VOLUME		
Teachers' Notes on Meaning Activities	16	
Meaning Activities	16	
Unit III: FRACTIONS		
Teachers' Notes on Meaning Activities	19	
Meaning Activities	20	
Problem Solving Activities	25	
Unit IV: RATIOS AND RATES		
Teachers' Notes on Meaning Activities	28	
Meaning Activities	28	
Problem Solving Activities	31	
Unit V: PERCENT		
Teacher's Notes on Meaning Activities	32	
Meaning Activities	32	
Applications	34	

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## COMMENTS ON CONTRIBUTORS \_\_\_\_\_

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## EDITOR'S COMMENTS

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In the Prologue, Dr. Alton Olson provides the background of the Meaning in Mathematics Teaching (MMT) research project which he and Dr. Sol Sigurdson conducted at the University of Alberta. The MMT involved many Alberta mathematics teachers in the development and delivery of curricular materials.

In addition to the financial assistance noted in the Prologue, the Mathematics Council of The Alberta Teachers' Association (MCATA) provided assistance to facilitate meetings of teachers to critique and to improve the products included in this issue.

MCATA is pleased to present this issue of *delta-K* as "Alberta made," involving your association, Albertan university personnel and, most importantly, Albertan mathematics teachers.

*A. Craig Loewen*

# Prologue

*Alton T. Olson and Sol E. Sigurdson*

The Meaning in Mathematics Teaching (MMT) project was completed just before the National Council of Teachers of Mathematics (NCTM) published the *Curriculum and Evaluation Standards for School Mathematics*. However, the spirit of this project fits very well with the NCTM goals. For example, the following statement is taken from the NCTM standards document and represents the goals of the students:

Educational goals for students must reflect the importance of mathematical literacy. Toward this end, the K-12 standards articulate five general goals for all students: (1) that they learn to value mathematics, (2) that they become confident in their ability to do mathematics, (3) that they become mathematical problem solvers, (4) that they learn to communicate mathematically, and (5) that they learn to reason mathematically.

The MMT project goals, though phrased differently, could be reformed easily to encompass these student goals.

The Meaning in Mathematics Teaching project was funded by the Social Science and Humanities Research Council (SSHRC) and involved 55 Grade 8 mathematics teachers. The study assessed the importance of teaching mathematics “with meaning” within a direct instruction context. The data supports use of meaning in teaching mathematics in that students taught in this way achieved higher marks (than other students) on a test including knowledge, comprehension and problem solving items. This issue of *delta-K* provides a resource manual of meaning activities of the type followed by teachers participating in the teaching-with-meaning group.

Within the project, another group of teachers used daily problem solving activities in addition to meaning. The effect of these activities on student

learning was not clear. At this point, it seems the activities do not add to student achievement, but they do not detract from student learning and can be a motivational element in the classroom. The problem solving activities here were used by project teachers and, for the most part, relate closely to the content of the unit in which they are given.

We did not, within the project, develop common mental computation exercises, but the ones given here in the “Extension” and “Applications” sections will serve as useful illustrations.

In looking at these meaning, problem solving and mental computation activities, the reader should remember that they were used in conjunction with the Missouri mathematics project lesson format outlined in the “Summary of Key Instructional Behaviors” section below.

## Summary of Key Instructional Behaviors

1. Daily review (first 8 minutes, except Mondays)
    - A. Review the concepts and skills associated with the homework
    - B. Collect and deal with homework assignments
    - C. Do several mental computation exercises
  2. Development (about 20 minutes)
    - A. Focus briefly on prerequisite skills and concepts
    - B. Focus on meaning and on promoting student understanding using lively explanations, demonstrations, process explanations, illustrations and so on
- Aids to understanding include
- (1) concrete materials,
  - (2) concrete examples,
  - (3) comparisons and
  - (4) class discussion.

- C. Student Comprehension Assessment
  - 1. Use process/product questions (active interaction)
  - 2. Use controlled practice, correcting misunderstandings
- D. Repetition and elaboration of the meaning portion as necessary
- 3. Seatwork (about 15 minutes)
  - A. Uninterrupted successful practice
  - B. Momentum—keep the ball rolling, get everyone involved and sustain involvement
  - C. Alerting—let students know their work will be checked at the end of the period
  - D. Accountability—check the students' work
- 4. Homework
  - A. Assign homework on a regular basis at the end of each math class (possibly excepting Fridays)
  - B. Include one or two review problems
- 5. Special reviews
  - A. Weekly review/maintenance
    - 1. Conduct during the first 20 minutes each Monday
    - 2. Focus on skills and concepts covered during the previous week
  - B. Monthly review/maintenance
    - 1. Conduct every fourth Monday
    - 2. Focus on skills and concepts covered since the last monthly review

In addition to this format, the Missouri mathematics project recommends highly interactive teaching. Although problem solving is not mentioned in this format, teachers in the problem solving group were asked to do 10 minutes of problem solving activities daily, near the beginning of each lesson.

## Meaning Activities

“Meaning in mathematics teaching” is a fancy label difficult to define. *Meaning* as used in this study includes relationships of formal mathematics to

- 1. other prerequisite mathematical knowledge;
- 2. concrete representations including physical objects and pictures;
- 3. practical uses of mathematics, within the students' world and in wider uses;

- 4. broader mathematical structures and the generality of concepts.

In this way, meaning might include a logical understanding of mathematics but even more importantly connects mathematics to images, the physical world, practical uses and other student knowledge. This view of mathematical learning is supported by the new, cognitive psychological view of learning that the learner continually develops knowledge networks which must connect inevitably to existing knowledge.

A major problem with such a definition is that meaning is not bounded. How is the teacher to know when meaning has been achieved? Indeed, is “true meaning” ever achieved or do knowledge networks continue to grow? These are important theoretical questions. However, in a practical sense for Grade 8 mathematics, the activities here give an appropriate scope for meaning development in mathematics.

In the development part of the lessons, meaning activities were set in an interactive teaching context, followed by seatwork in which students “practised” through assignments. In our particular project, teachers used meaning activities to supplement a textbook which formed the instruction core. The teachers used these activities where they thought the development of meaning was important to students' knowledge. It is probably fair to say that meaning was considered an add-on, as perhaps it must always be.

To illustrate the add-on meaning, suppose you have learned that  $6 + 7 = 13$ . The fact is easy to learn. Then a teacher encourages you to write this as  $(3 + 3) + 7 = 3 + (3 + 7) = 3 + 10$ . Now average 10-year-olds might feel they “understand why”  $6 + 7 = 13$ . They have “added meaning” to their knowledge. It does not follow that all meaning is add-on, but a case can be made that a good part of meaning is aimed at after the fact.

## When to Use Meaning Activities

Where do meaning activities come in the learning sequence? Use your professional judgment. One aspect of this decision became clear during the project. Students in Grade 8 know a lot about what we teach them. Even in the study of percent, not only do they know fractions, decimals and ratio but also they have studied percent before. No mathematics teaching at the junior high level is done on virgin territory. In effect, *all* the teaching we do (and not simply the meaning) is add-on.

Motivational factors may be equally important in considering when to use the meaning activities.

## Meaning Practice

Students must learn to do mathematics by practising it. They also learn meaning the same way. To use the previous example, showing students that  $(3 + 3) + 7 = 3 + (3 + 7) = 3 + 10$  is not enough. They must be encouraged to practise breaking 6 up into its various parts and to see that only one “breaking” works. Students must then practise this process on other numbers to see number breakup as a useful tool to be applied in different situations. Even work with physical models requires not so much *insight* on the students’ part as *practice* in fully understanding the relationship. If “*practice with meaning*” is omitted, a much smaller percentage of students will establish the appropriate relationships.

Unless we are intentional about our meaning implementation, it will not happen. [Our study showed substantial differences in teacher ability to implement a meaning approach.] Teachers must expect students to reproduce meaning learning. It is not enough at the end of a unit to ask  $6 + 7 = \underline{\quad}$ . We want to know if students think of  $6 + 7$  as  $3 + 3 + 7$ . Do they have this particular meaning? If we are interested in meaning, students must be accountable for it. All tests should include meaning items. How else will students come to think of meaning as an essential ingredient in mathematics?

## How to Use Meaning Activities

The meaning activities here can be used as is. Copies can be made and handed out to students. However, none are stand-alone activities; all are meant to be used interactively. Teacher-student interaction should begin every use of these activities. As students become aware of the purpose of the activity and how to do it, they can complete the activity on their own. Every meaning activity presents its own challenges, and each should end with an interactive summation of the activity’s results.

The meaning activities included here are not a complete set but rather the ones project teachers felt were relevant. They have not been perfected, and some may be found wanting. Become familiar with them before you use them. Even then, students will surprise you. Like any useful tool, meaning activities perform best in the hands of a skilled user.

## Problem Solving Questions

The problem solving exercises presented here were project developed. Teachers were urged to include

problem solving in the early stages of a lesson, either to start or after the homework correction and mental computation. Although in our project student achievement was not noticeably enhanced, we think judicious use of these activities can only benefit students.

Theoretically, problem solving is just as difficult to define as meaning. The project took a conservative view, looking at problems that made use of the following ideas:

1. Drawing diagrams
2. Using complex and/or simple numbers
3. Incorporating an overall plan: understand, plan, execute, look back
4. Problems without numbers
5. Estimation of answers
6. Focus on reading skills
7. Two different problems—with the same structure
8. Students making up problems
9. Translating to open sentences

The other consideration we made was to use problems related to unit content. We did not encourage use of classic problems such as “locker” or “checkerboard” problems. A final consideration was to use an interactive approach. Students were not left for the 10-minute period to work on their own. Teachers dealt with problems as a class interaction. Our rationale was to make problem solving processes explicit and up-front in every mathematics period, with the idea that this would influence the remainder of the class. The downside of this approach is that the 10 minutes of problem solving might detract from otherwise valuable time. Although improvement in student learning did not show up on student test scores, many project teachers feel this was an important aspect of the mathematics class.

## Mental Computation

We have included several examples of mental computation. The idea behind mental computation is not so much mental drill as computation without pencil and paper. Given a question like  $1\frac{1}{2} \times 6 = \underline{\quad}$ , on paper one writes  $\frac{3}{2} \times 6 = \frac{3}{1} \times 3 = 9$ . Mentally, one is liable to proceed by thinking of one 6 plus half of 6,  $6 + 3 = 9$ .

By accurate answers achieved mentally, mental computation encourages a number sense in students, not an estimation sense. Although we do not know precisely how this affects student learning, project

teachers identify this as positive classroom activity. The project teachers experimented with how to conduct mental computation. Most teachers eventually had students write down answers for later correction. Otherwise, students who are rapid calculators dominate classroom proceedings. Some teachers wrote the questions on overhead transparencies. What is essential is that students understand the activity is to be done mentally.

## Message to Project Teachers

These activities are laid out according to the several chapters of *Journeys in Math, Grade 8*. Most of them are applicable in any junior high grade. Your using them will help repay you and your students for your much-appreciated efforts during the MMT project.



# Unit I: Perimeter and Area

## Teachers' Notes on Meaning Activities

### Activity 1

These three questions establish perimeter as independent of shape and size. Question 2 shows that removing some squares does not change perimeter. Question 3 relates to formula work.

### Activity 2

This simple activity can be turned into a meaning activity through discussion. The symmetries of these shapes are worth noting. Perimeters of symmetric figures in the grid context are even numbers.

### Activity 3

In the first section, gaining a sense of the length of a metre is essential for interpreting measurement situations. The second section reinforces the meaning of regular polygons. Sketching figures in notebooks formalizes and consolidates student learning.

### Activity 4

In addition to conveying meaning about  $\pi$ , experiments and demonstrations have an "episodic" value. The episode can help students remember the lesson.

### Activity 5

All parallelograms can be divided up and made to fit into the related rectangle. The concept of "rearranging areas" is important for junior high school mathematics. The related rectangle should have the same base and altitude as the parallelogram.

### Activity 6

This activity is similar to Activity 5. This time related rectangles are found for given triangles.

### Activity 7

Here the idea of area as the number of square units is stressed. If areas of parallelograms and triangles

are known, students can check their answers by other means. One of the "definitions" of meaning is having two ways to do things, both correct but useful in different situations.

### Activity 8

This is a reinforcement sheet relating square units to rectangles, triangles and parallelograms. Students can be asked to draw any parallelogram (triangle) on their grid paper and to find its area or to draw a parallelogram (triangle) with area 24 square units.

### Activity 9

Any polygon can be thought of as a combination of simpler shapes. This is an especially useful concept in some of the complex area problems.

### Activity 10

The most interesting polygons are regular. These are usually divided into isosceles triangles. Here students can make use of their isosceles triangle knowledge.

### Activity 11

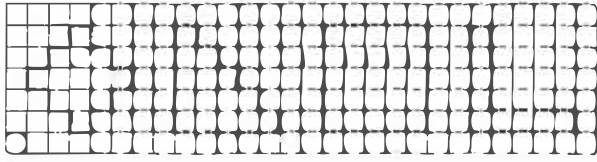
This is the ultimate activity in dividing regular polygons into triangles. Some caution is recommended in this activity as students do not easily see the connections.

### Activity 12

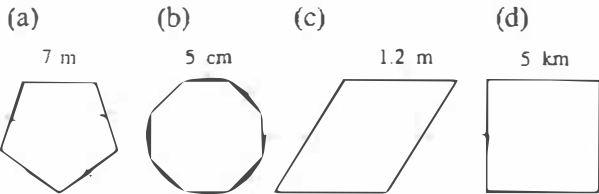
Three interesting and important ways of looking at the area of a circle are dealt with.

## Meaning Activity 1 Perimeter

1. On grid paper, draw five rectangular shapes with perimeters of 24 grid units.
2. Find the shape on the following page that has the largest perimeter. What happens to the perimeter of a shape if a grid square is removed?



3. Show how you would find the perimeter of each figure below in two different ways.



### Meaning Activity 2 Perimeter

#### Perimeter Code

What do you buy if you're dying and only have a dime? To answer this important question, find the perimeters of the shapes below. Find each answer in the code at the bottom of the page. Each time the answer appears in the code, write the letter of that shape above it.

S			F			V														
A																				
E																				
17	34	11	16	12	25	40	21	10	24	20	13	40	28	55	10					

Perimeter Code

What patterns do you notice in these perimeters?

### Meaning Activity 3 Perimeter

Teacher Sheet

#### A. Perimeter (Adaptable to teacher demo)

Required materials: a 1-m length of string per student.

Teacher provides a list of objects available in the classroom. Use the string to determine if a given object has a perimeter of 1 m, less than 1 m or more than 1 m. Suggested objects:

1. desk top
2. overhead projector
3. door
4. math book
5. loose-leaf paper
6. one shelf of a bookcase

Students now apply their understanding to objects that are less than 1 m, equal to 1 m and greater than 1 m which are not available in the classroom. Suggested objects:

1. locker door
2. refrigerator door
3. sidewalk block
4. pillow from your bed
5. lid of the mailbox

#### B. Perimeter of Special Polygons

Required materials: toothpicks (10 per pair of students), rulers

After explaining the meaning of *equilateral* and reviewing the names of regular polygons—such as square, rhombus, pentagon—have students form each of the following: triangle, square, rhombus, pentagon, hexagon, octagon and decagon.

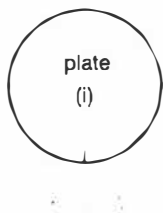
Have students put aside all but one of the toothpicks. Have them measure it and, using the information, calculate the perimeter of each polygon. Have students sketch shapes in their notebooks, writing inside the drawing the length of side, the perimeter and the formula for the shape.

### Meaning Activity 4 Circumference

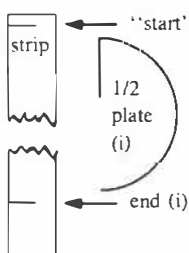
Teacher Sheet

- Materials:
- 3 different-sized paper plates
  - 1 paper strip at least 150 cm in length
  - 1 pair of scissors
  - 1 pencil

## Procedures:



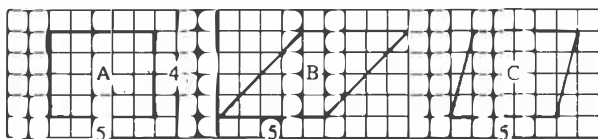
- A. 1. Mark a start line on one end of the strip.
2. Make a mark on the edge of a plate.
3. Match the mark on the plate with the mark on the strip.
4. Roll the plate through one complete revolution along the strip. Mark the new position of the plate mark on the strip.



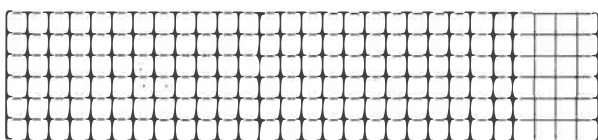
- B. 1. Cut the plate in half.
  2. Using the cut edge as a unit of measure, count the number of units needed to measure the line made when the plate was rolled through a complete revolution.
- C. Repeat the above procedures for each of the three paper plates.
  - D. Note the constant relationship between the rolled plate edge (circumference) and the cut edge of the half-plate (diameter).

## Meaning Activity 5 Parallelograms

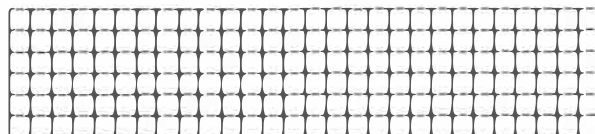
1. Count the squares in each of the shapes below. What is *area*?



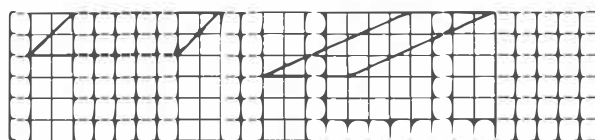
Draw two copies of rectangle A below. Show how parallelograms B and C can be cut so they fit into the rectangles. Count the squares in each of the shapes A, B and C. What is your conclusion? Describe your cutting of parallelograms B and C in words.



Draw another copy of rectangle A below. Show how parallelogram B can be cut and fit into rectangle A in a different way than you showed above.



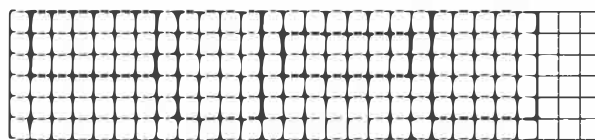
2. For each of these parallelograms, draw a rectangle into which it fits exactly. (Note: A different rectangle is required for each figure.)



3. Every parallelogram can be reconstructed as a rectangle. How does this help in finding area? What is the area of each of the parallelograms in this activity?

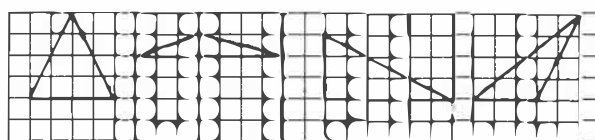
## Meaning Activity 6 Triangle Areas

1. In each rectangle below, draw a triangle using a side of the rectangle as a side of the triangle. (Of course, drawing a diagonal is a solution, but find others.)

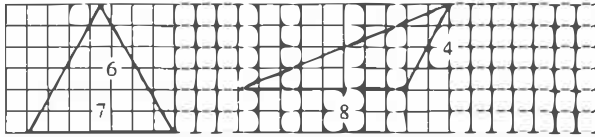


How do the sizes of the triangles compare with the rectangles?

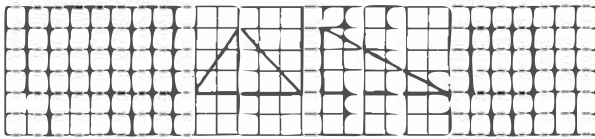
2. Now reverse the process. Given the triangles below, draw the rectangles that are twice as large in area. How does this help us find the areas of the triangles?



3. Find the areas of these two triangles by finding the areas of the related rectangles. (Note: You can do this by counting square units or by using the area formula for the area of a rectangle.)



4. With these shapes create nine different shapes and find their areas. Be as original as you can. Pass your work to the person behind you to see if she or he can verify your work.



### Meaning Activity 7 Squares, Rectangles, Triangles, Parallelograms

Area: Measurement of the amount of space contained within a shape. It is expressed in square units.

1. Find the area of the following shapes. Express your answers in "grid-paper units."

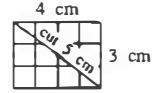
(a)  _____	(f)  _____
(b)  _____	(g)  _____
(c)  _____	(h)  _____
(d)  _____	(i)  _____
(e)  _____	(j)  _____

2. Find the area of shapes  
 ABCD \_\_\_\_\_  
 ABC \_\_\_\_\_

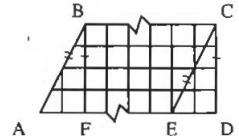
What do you notice about these areas?

### Meaning Activity 8 Squares, Rectangles, Triangles, Parallelograms

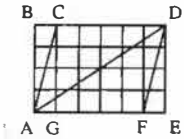
1. From grid paper, cut out a 3-cm x 4-cm rectangle. Divide it into two pieces along the diagonal. Join the two triangles to form as many shapes as possible. Find the area of each of these shapes.



2. Find the area of shapes  
 ABCE \_\_\_\_\_  
 BCDF \_\_\_\_\_  
 What do you notice about the two areas? \_\_\_\_\_



3. Find the areas of and name the shapes.  
 (a) ABDE \_\_\_\_\_  
 (b) ADE \_\_\_\_\_  
 What do you notice about the two areas? \_\_\_\_\_



Find the areas of and name the shapes.

- (c) ACD \_\_\_\_\_  
 (d) ACDF \_\_\_\_\_  
 (e) CDEG \_\_\_\_\_  
 What do you notice about the areas in:  
 (f) c and d? \_\_\_\_\_  
 (g) d and e? \_\_\_\_\_

### Meaning Activity 9 Area of Polygons

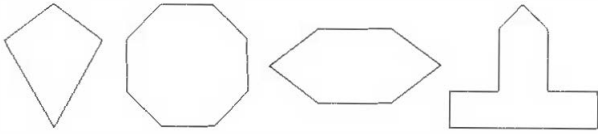
1.

Combine the shapes above to form each of the following polygons. Draw lines to identify the shapes and number the parts.

(a) (b) (c)

(d) (e) (f) (g)

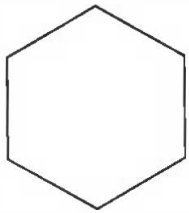
2. Divide each polygon into different types of polygons.



Devise different ways of dividing them so that the total area may be easily found.

### Meaning Activity 10 Areas of Regular Polygons

1. One way to find the area of a regular hexagon is to divide it into triangles by drawing lines from the "centre" of the hexagon to the vertices and then finding the areas of the triangles. How many triangles are in a regular hexagon? \_\_\_\_\_

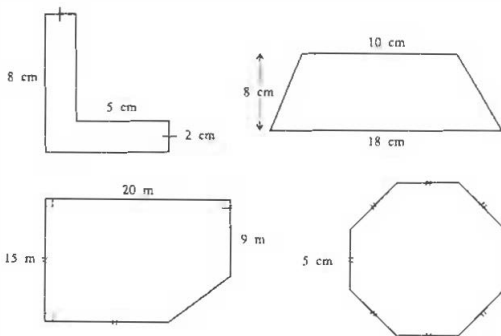


Write a formula for finding the area of a regular hexagon.

How many triangles are in a regular octagon? \_\_\_\_\_ In a regular decagon? \_\_\_\_\_

Write a formula for the area of a regular decagon.

2. For each of the shapes below do the following:  
 (a) Divide the polygons into polygons for which you can find the area.  
 (b) Find the area of each part.  
 (c) Find the total area of each shape.



### Meaning Activity 11 Areas of Other Polygons

#### Teacher Sheet

Once the concept of dividing a regular polygon into congruent triangles has been taught, the concept of *area of a circle* can be reinforced. Review regular

polygons (Diagram 1) on the board. Then do teacher demonstration using the following steps:

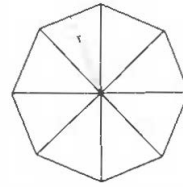


Diagram 1

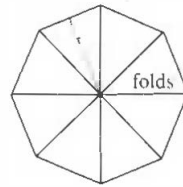
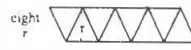


Diagram 2



$$\text{base} = \frac{2\pi r}{2}$$

Diagram 3

1. Cut a large circle (minimum 30-cm diameter) into eight congruent triangles by folding in half three times and then cutting.
2. To emphasize the outer edges of the triangles and the radius of the circle, highlight the edges and radius with contrasting colors.
3. Fit the eight triangles together to resemble a parallelogram as shown in Diagram 3.
4. The base of the parallelograms is approximated by one-half the circumference of the circle. The height is approximated by the radius of the circle. Thus:  
 Area of parallelogram = base x height

$$= \frac{\pi \times d}{2} \times r$$

$$= \frac{\pi \times 2r}{2} \times r$$

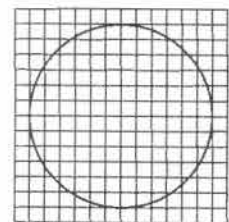
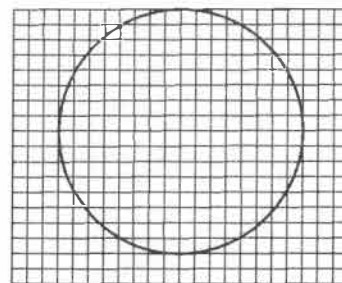
$$= \pi r^2$$

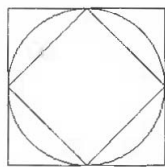
So the formula for the area is  $A = \pi r^2$

5. Pin your final cutout on the bulletin board.
6. Have students figure out area by finding the area of one triangle and multiplying by eight to get the area of the circle. Assume: height equals radius, and a triangle base is  $\frac{1}{8}\pi d$ .

### Meaning Activity 12 Area of Circle

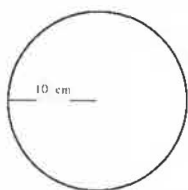
1. Here are two circles. How many units in diameter is each? (Count the squares.)





scale: 1 cm = □

2. (a) Mark off the area of the outside square in 0.5-cm units.
- (b) How does the *area* of the circle compare with that of the outside square? By how many square centimetres do they differ?
- (c) How does the *average* of the areas of the inside and outside square compare with the area of the circle?

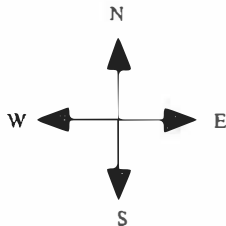
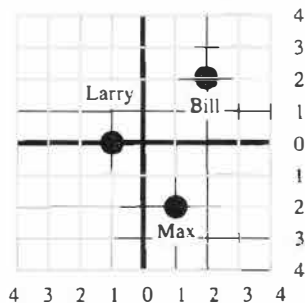


scale: 1 cm = □

3. (a) Mark the edge of the circle in some color.
- (b) What is the circumference? (radius = 10 cm)
- (c) Cut the circle into eight congruent pie shapes.
- (d) Arrange the shapes to form a parallelogram.
- (e) Approximately, what are the dimensions of this parallelogram?
- (f) What is the area of this parallelogram?
- (g) Why is it the same area as the circle?

### Extension 1 Linear Measurement

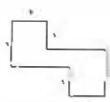
1. The Fall Fun Run started at the school. The participants ran six blocks west, four blocks north, twelve blocks west and sixteen blocks south. How many blocks from the school were they?
2. A grid of Calgary is shown. Bill lives at 2nd Street 2nd Avenue NE; Max lives at 1st Street 2nd Avenue SE; Larry lives at 1st Street 0 Avenue W.



- (a) Which two live the closest to each other? (They must walk along streets or avenues. No shortcuts allowed.)

- (b) Which house(s) should they meet at so that the total walking distance is the smallest possible?
3. (a) Calculate the perimeter of a square having a side of 4 cm. A side of "s" cm.
  - (b) Calculate the perimeter of a pentagon having a side of 10 cm. A side of "n" cm.
  - (c) The perimeter of a hexagon, "t" cm on a side would be \_\_\_\_\_?
  - (d) State a general rule for finding the perimeters of regular polygons.

### Extension 2 Linear Measurement



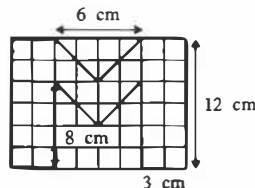
1. If the area of this figure is 108 square units, what is its perimeter?
2. Frames need to be built for the set of a school play. Two door frames with outside dimensions 2 m by 1.5 m, one sign frame with dimensions 1.75 m by 0.67 m and two window frames with dimensions 0.67 m by 0.67 m are needed. If lumber comes in 2.67-m lengths, what is the minimum number of lengths required?
3. How does changing shape while keeping a constant perimeter affect the area of the shape? Using whole number lengths, answer these questions about rectangles that have a perimeter of 48 units.
  - (a) What are the dimensions of the rectangle with the largest area?
  - (b) What are the dimensions of the rectangle with the smallest area?

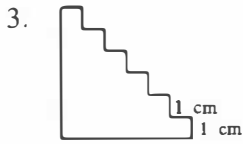
### Extension 3 Areas of Other Polygons

1. The area of the triangle is 2 cm<sup>2</sup>. What is the area of the octagon? Use grid paper to solve.



2. Maggie cut her initial out of a 12 cm by 12 cm piece of felt. How many square centimetres were used? How much felt was not used?

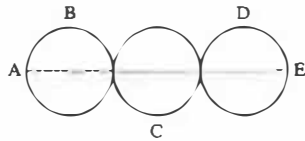




3. Find the area of this figure. The line segments of the "steps" meet at right angles and are 1 cm in length.

### Extension 4 Circumference

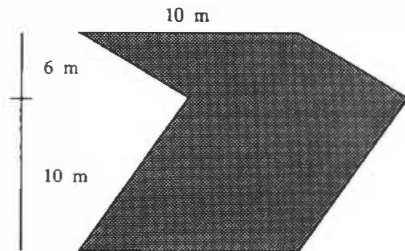
1. A cable company, after having laid a cable on the Earth's surface around the equator, is told that the cable should have been installed 1 m above the surface. How much more cable is needed? (Earth's circumference is approximately 40 000 km.)
2. The three circles shown are identical. The line segment AE is 42 cm long. How long is the curved path ABCDE?



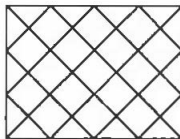
3. The minute hand on the clock is 9.5 cm long. How far does the tip of the minute hand travel in 24 hours?

### Extension 5 Area of Rectangle

1. Calculate the area of the shaded figure.

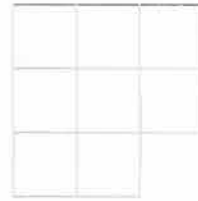


2. This leaded window measures  $m$  cm by  $n$  cm.



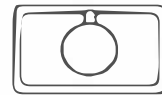
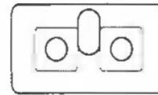
- (a) Find the area of a small pane.
- (b) If the actual dimensions of the window are 60 cm by 80 cm, find the area of each pane.

3. (a) What is the greatest area possible for a rectangular carpet which has a perimeter of 26 m? The lengths of the sides in metres are whole numbers.
- (b) How can a piece of wallboard 1.6 m by 90 cm be cut into two pieces so that it completely covers an area 1.2 m by 1.2 m? (The cut resembles a stairway.)
- (c) Remove squares so that the perimeter remains the same, but the area becomes (i)  $7 \text{ cm}^2$ , (ii)  $6 \text{ cm}^2$  and (iii)  $5 \text{ cm}^2$ .

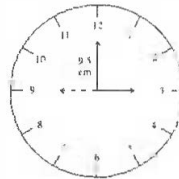


### Extension 6 Area of a Circle

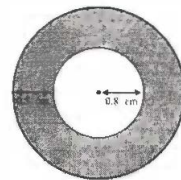
1. Will two 6-cm drains give faster drainage, slower drainage or the same drainage as a 12-cm drain? Explain why.



2. (a) The minute hand on a clock is  $n$  cm long. What would be the area it covered in 45 minutes?
- (b) If the length of the minute hand is 9.5 cm, what would the actual area covered be?



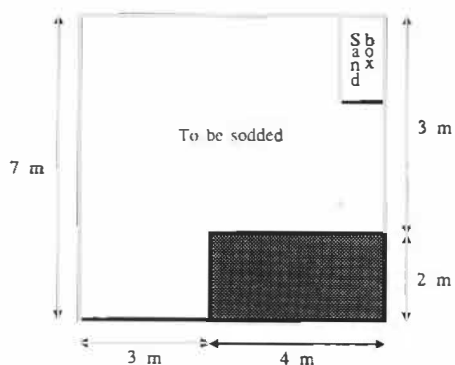
3. What is the area of the shaded region?



### Extension 7 Units of Area

1. Mr. Gretzky plans to sod part of his yard. Each piece of sod measures 40 cm by 150 cm and costs \$1.55.\*

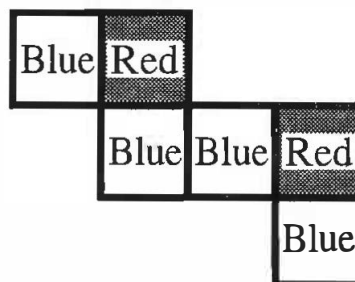
\* Taken from *Journeys in Math 8*, p.91.



- (a) Find the area to be sodded.
- (b) Find the area of each piece of sod.

- (c) How many pieces of sod are needed?
- (d) Find the cost of the sod.

2. This figure consists of six congruent tiles and it has a total area of  $294 \text{ cm}^2$ . Two tiles are red and four tiles are blue.



- (a) What is the length of each tile?
- (b) What is the total area covered by blue tile?
- (c) What is the total area covered by red tile?



# Unit II: Surface Area and Volume

## Teachers' Notes on Meaning Activities

### Activity 1

The realization that several nets are possible is important. A discussion about which possible net is the "most explanatory" would be instructive.

### Activity 2

Detailed breakdown of the surface area of a rectangular prism can result in a formula for its surface area.

### Activity 3

Taking polygons apart that have been constructed from nets is very meaningful. A good net is one that "explains" the polyhedron. Again, keeping track of these activities in a notebook is very helpful for learning.

### Activity 4

This manipulative activity reinforces the concept of volumes consisting of layers which in turn consist of unit cubes. An interesting activity related to this: Given 30 cubes, build a rectangular prism. How many different ones can you build?

Ans.:  $5 \times 3 \times 2$  /  $5 \times 6 \times 1$  /  $30 \times 1 \times 1$  /  $15 \times 2 \times 1$  /  $10 \times 3 \times 1$

### Activity 5

Students will need to be "talked through" this activity because it is dependent on the polyhedron chosen. There are good possibilities for group work and discussion.

### Activity 6

This activity depends on which materials are available. The idea is that volumes calculated through measurements and formulas should yield the same

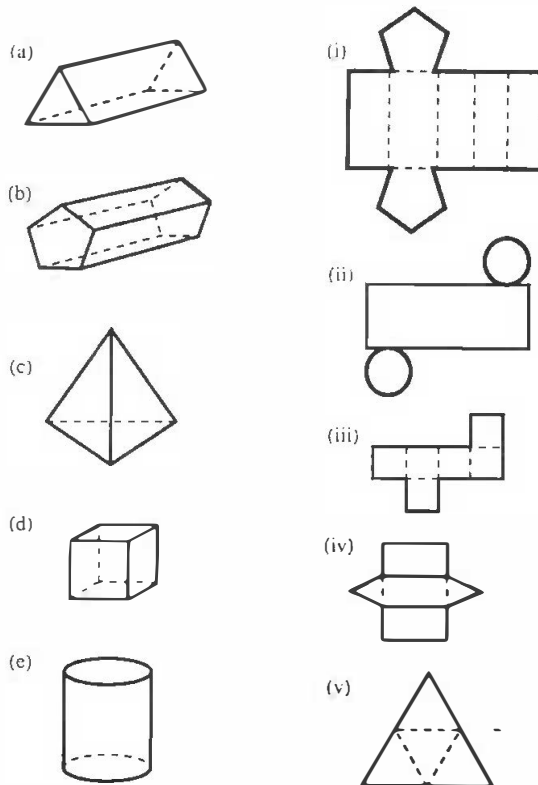
answers as those obtained through experiments. In itself, the episode is an important aid in learning.

### Activity 7

This activity is more difficult because the cutouts do not match the drawings exactly. Once the students have constructed the objects, ask them to reconstruct the square.

## Meaning Activity 1 Polyhedrons

1. Match the polyhedron with its net.



(a) \_\_\_\_\_ (b) \_\_\_\_\_ (c) \_\_\_\_\_ (d) \_\_\_\_\_ (e) \_\_\_\_\_

2. Using the given nets, sketch at least one other net that will produce the same polyhedron.

(i)

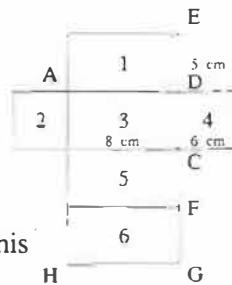
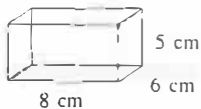
(ii)

(iii)

(iv)

(v)

### Meaning Activity 2 Prisms and Pyramids



- How many faces make up this prism? \_\_\_\_
- What is the length of AD? \_\_\_\_
- What is the length of DE? \_\_\_\_
- What is the length of CF? \_\_\_\_
- What is the length of FG? \_\_\_\_
- What is the length of GH? \_\_\_\_
- Find the area of each face:
  - 1 \_\_\_\_
  - 2 \_\_\_\_
  - 3 \_\_\_\_
  - 4 \_\_\_\_
  - 5 \_\_\_\_
  - 6 \_\_\_\_

- What is the total area? \_\_\_\_\_
- Which faces have the same area? \_\_\_\_\_
- Can you find a shortcut for finding the total surface area?  
Explain: \_\_\_\_\_

### Meaning Activity 3 Surface Area

- Begin with a polyhedron that you have built.
  - Cut it apart to make a "good" net.
  - Find the area of every face. (Mark in altitudes of triangles.)
  - Find the total surface area.
  - Compare the surface area of your polyhedron with that of your neighbor. Which has the longest edge?
- Cylinders:
 

Materials: 1 sheet of ordinary paper, 2 paper clips

Instructions:

  - Fold the paper to make a smaller rectangle.
  - Make a cylinder with your paper, using paper clips to hold it.
  - Find the surface area of your cylinder. Measure with a ruler. Include the top and bottom.
- Find a cylinder using some object from home or school. Find the surface area of the cylinder. This can be done either by cutting out paper to fit your cylinder or by taking measurements. If you cut out shapes, glue them in your notebook.

### Meaning Activity 4 Volume of Rectangular Prisms

- Using 24 blocks (cubes), construct a rectangular shape, using a single layer.
  - What are the dimensions?
  - What is this polyhedron called?
  - What are the units of this shape?
  - What is its volume?
- Build a second layer on top of the first. Answer questions 1a to 1d again.
- What are the answers to 1a to 1d when you build five layers? Ten layers?
- Using five blocks, construct a tower.
  - What are the dimensions?

(b) What is this polyhedron called?

(c) What is its volume?

- Build a second tower beside the first. Answer questions 4a to 4c again.
- If you build 10 towers in two rows of five blocks, what would be the total volume?
- Suppose you had 150 blocks in a rectangular shape 5 blocks high. What may the dimensions of the base (first layer) be?
- If you had an aquarium with dimensions 15 cm by 20 cm by 30 cm, what would its shape be called? How long, wide, high? What is its volume?

### Meaning Activity 5 Polyhedron Construction

Take any polyhedron that you have made and do the following activity.

- Name the polyhedron.
- Number its faces (with large numbers).
- Draw a net of this object (not the same as the original).
- Write formulas for the area of each face and for the total surface area.
- Sketch lines on your polyhedron that will help you understand its volume. Describe to others the volume of the polyhedron.
- With a ruler, measure the polyhedron's dimensions, including its altitude if appropriate. From these measurements, determine its volume. Determine its surface area.
- In a different color pen, draw markings indicating the planes of symmetry of your polyhedron.
- Cut open one face of your polyhedron, and if possible fill it with centimetre cubes. Does this volume compare favorably with the calculated volume (question 6)?
- Cut open your polyhedron and paste it in your notebook.

### Meaning Activity 6 Volume, Capacity and Mass

Teacher Sheet

Materials required:

- 2-L measuring device with marked graduations
- 10-cm<sup>3</sup> container

1-L bottle (soft drink)

3 or 4 square-sided containers (milk, juice cartons)

3 or 4 straight-sided cylindrical containers (glasses, coffee mugs, tins)

3 or 4 submersible blocks

The object of the experiment is to compare calculated volumes with measured capacities.

Have students measure the objects. Have them calculate the volumes. Compare these calculations by seeing how large the object is either by how much water it will hold or by how much water it displaces.

#### Method 1

Use the submersion method to determine the volumes (capacities) of irregularly shaped objects.

#### Method 2

The containers may be weighed, first empty, then full. This may be another way of determining capacity (volume).

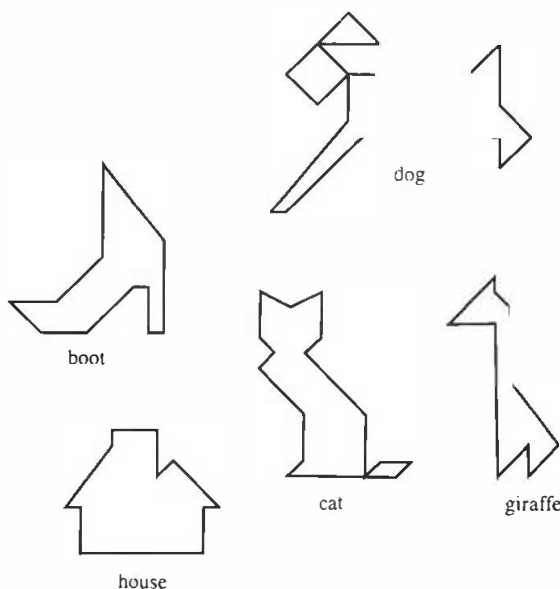
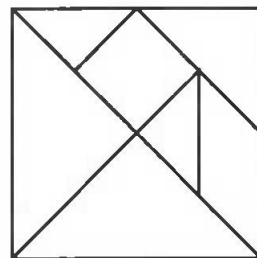
### Meaning Activity 7

Materials: Scissors

Cut out the "tangram square" and cut it into its seven parts.

See if you can duplicate the figures below using the seven shapes.

Can you make some of your own?



# Unit III: Fractions

## Teachers' Notes on Meaning Activities

### Activity 1

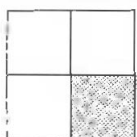
Rectangular dimensions are excellent models for thinking of factors. Of course, a prime number has only one possible and somewhat uninteresting rectangle associated with it.

### Activity 2

Using correct terminology for "cars" and "trains" is essential. This activity requires a high level of teacher participation. Even after students understand what is happening, they should practise to reinforce the connection.

### Activity 3

Students must clearly understand that the shaded part is the representation of the fraction. Other shadings such as  $\frac{1}{4}$  are permitted as shown:



### Activity 4

The fraction bar sheets can be cut up, and they become quite useful. Students can use them to do the fraction bar questions and can draw sketches to find the answers. For example,

$$2 \frac{2}{5} \text{ is } 1 + 1 + \frac{2}{5} = \frac{12}{5}$$



### Activity 5

Teacher help is essential in this activity. This question serves to emphasize what it means to ask how many sixths there are in  $2 \frac{1}{3}$ . In this case,  $2 \frac{1}{3}$  means twice around and one third more.

### Activities 6 and 7

These activities stress the meaning of multiplying fractions as well as a means of finding answers. What is  $\frac{1}{2}$  of  $\frac{2}{3}$ ? The product of the numerators over the product of the denominators is the answer. These activities should help in explaining why that is so.

### Activity 8

This teacher sheet would make a good overhead transparency or blackboard demonstration.

### Activity 9

Understanding division is easier when we have unit fractions such as  $\frac{1}{3}$ .

### Activity 10

Meaning is emphasized by switching from one mode of naming to another. We have many ways of referring to the same numbers.

Activities 11–18 require each student (or student group) to have fraction bar strips. These can be cut from duplicates of the fraction bar sheet. Note that each strip represents one unit. Strips are more durable if duplicated on heavier paper. This fraction bar sheet can be used to make a transparency.

### Activity 11

For two fractions such as  $\frac{1}{2}$  and  $\frac{3}{5}$ , the division lines match the division lines on the  $\frac{1}{10}$  bar. This is the common denominator. The easy visual comparison of  $\frac{1}{2}$  and  $\frac{3}{5}$  with the bars should be integrated with the mathematical idea of common denominators. So,  $\frac{1}{2}$  is  $\frac{5}{10}$  and  $\frac{3}{5}$  is  $\frac{6}{10}$ .

### Activity 12

Note the same bar shows the  $\frac{1}{4}$  and the  $\frac{3}{4}$ . (Some fraction bar packages have strips which distinguish between  $\frac{1}{4}$  and  $\frac{3}{4}$ .)

### Activity 13

Students will need two of the  $\frac{1}{4}$  strips to add  $\frac{1}{4} + \frac{1}{2} = \frac{3}{4}$  because the lowest common denominator is back on the  $\frac{1}{4}$  strip. Demonstrations with the overhead are easier to present if the strips are colored differently.

### Activity 14

To solve  $\frac{2}{3} - \frac{1}{3}$ , you need two  $\frac{1}{3}$  strips. Suggested language use: "How much of the  $\frac{2}{3}$  is left after we take away the  $\frac{1}{3}$ ?" or "How much more is  $\frac{2}{3}$  than  $\frac{1}{3}$ ?"

### Activity 15

Solutions for these exercises should be demonstrated by the teacher. For example,  $\frac{2}{3} + \frac{1}{4}$ . Using the  $\frac{1}{3}$  and  $\frac{1}{4}$  bars, find the bar ( $\frac{1}{6}$ ) whose division lines match the division lines of both the  $\frac{1}{3}$  and the  $\frac{1}{4}$  strips. Find the  $\frac{2}{3}$  point on the  $\frac{1}{3}$  bar and place the  $\frac{1}{4}$  bar to add its length to that of the  $\frac{2}{3}$ . The division line at the addition of  $\frac{2}{3}$  and  $\frac{1}{4}$  matches which division line on the  $\frac{1}{6}$  bar? Answer:  $\frac{5}{6}$ .

### Activity 16

Students should be encouraged to use the fraction bars for these activities even if they know shortcut methods. The use of the bars reinforces the concept of addition and subtraction and the use of the common denominator.

## Meaning Activity 1 Greatest Common Factor

- On grid paper, draw all possible rectangles of area 12 units and area 18 units.
  - From these rectangles, write down the factors of 12 and of 18.
  - Why do factors always occur in pairs?
  - Which are the common factors of 12 and 18?
  - Which of these factors is the greatest?
- Do question 1 again for rectangles of 9 units and 24 units.
- Do question 1 again for rectangles of 12 units and 30 units.
- Find out what a prime factorization is.
- Write out prime factorizations for 12 and 18.
  - Match the prime factors of these two numbers. For example,  
 $12 = 1 \times 2 \times 2 \times 3$   
 $18 = 1 \times 2 \times 3 \times 3$

(b) Find the product of all of the matched numbers. Why does this product give you the greatest common factor (GCF)?

- Do question 5 using 9 and 24.
- Do question 5 using 8 and 18.
- Do question 5 using 15 and 25.
- Do question 5 using 48 and 72.

## Meaning Activity 2 Least Common Multiple

- Make cars of 3-cm and 5-cm lengths using centimetre cubes of two different colors.
- By adding 3-cm and 5-cm cars together, build trains of equal length.
  - How many 3-cm cars did you use?
  - How many 5-cm cars did you use?
- Do question 2 again using 4-cm and 6-cm cars. Now make the shortest train possible using the 4-cm and 6-cm cars.
- Do question 2 again using 6-cm and 8-cm cars. Do not actually make trains, but sketch your answer in your notebook.
- These trains are multiples of the car numbers. Trains of equal length are common multiples. We are usually interested in the least common multiple (LCM). Why?

## Meaning Activity 3 Adding and Subtracting Fractions

Do the following sums using diagrams only. Be sure to express your answers as basic fractions. Example:  $\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$



1. (a)  $\frac{1}{3} + \frac{1}{3} =$



(b)  $\frac{1}{6} + \frac{2}{3} =$



(c)  $\frac{1}{4} + \frac{1}{3} =$



(d)  $\frac{3}{10} + \frac{2}{5} =$



(e)  $\frac{3}{4} + \frac{1}{6} =$



(f)  $\frac{1}{3} + \frac{1}{6} =$



2. (a)  $4/5 - 1/5 =$   -  =
- (b)  $5/6 - 5/12 =$   -  =
- (c)  $5/6 - 1/4 =$   -  =
- (d)  $7/8 - 3/4 =$   -  =
- (e)  $7/10 - 2/5 =$   -  =
- (f)  $4/5 - 1/3 =$   -  =

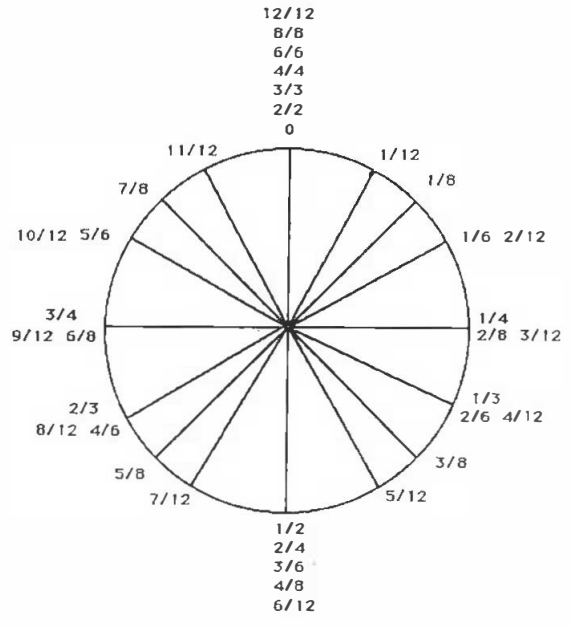
### Meaning Activity 4 Fractions and Mixed Numbers

1. Lay out one whole bar and two thirds.  
   =  $1 \frac{2}{3}$   
 Using thirds, find out how many thirds are in a whole. \_\_\_\_  
 How many thirds are there in all?  
 $3/3 + 2/3 =$   $5/3$ .
2. Lay out two whole bars and three tenths.  
   =  $2 \frac{3}{10}$   
 How many tenths are in each whole bar? \_\_\_\_  
 In two whole bars? \_\_\_\_  
 How many tenths are there in two whole bars and three tenths? \_\_\_\_  
 $(2 \times 10) + 3 =$   $23$   
 $10 \quad 10$

3. Following the pattern set in question 2, change  
 (a)  $3 \frac{2}{7}$  to  $17$ .  
 (b)  $2 \frac{2}{5}$  to  $15$ .  
 (c)  $1 \frac{8}{9}$  to  $19$ .

### Meaning Activity 5 Fractions and Mixed Numbers

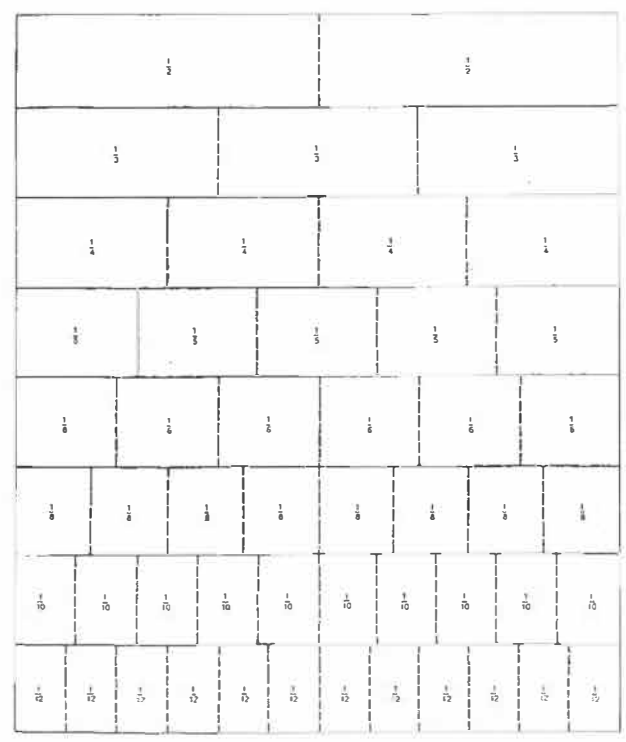
- Use the following circle to answer these questions.
- Using the large circle, how many halves are in a whole?
  - How many thirds are in a whole?
  - $5/2$  is how many wholes?
  - $9/4$  is how many wholes?
  - How many sixths are there in  $2 \frac{1}{3}$ ?
  - How many eighths are there in  $2 \frac{1}{4}$ ?



Use the circle to answer questions 7-9. Sketch your answers.

7.  $2 \frac{1}{2} + 1 \frac{1}{3} =$   
 8.  $2 \frac{3}{4} - 1 \frac{2}{3} =$   
 9.  $2 \frac{1}{2} - 1 \frac{2}{3} =$

### Fraction Bar Sheet



## Meaning Activity 6 Multiplying Fractions

1. Shade in the answer:



$\frac{1}{2}$  of 12    $\frac{1}{3}$  of 12    $\frac{1}{4}$  of 12    $\frac{2}{3}$  of 12    $\frac{3}{4}$  of 12

2. (a) (i) Shade in  $\frac{1}{2}$  of 12.  
(ii) How many did you shade in?
- (b) (i) Shade in  $\frac{3}{4}$  of 20.  
(ii) How many did you shade in?  
(Compare)
- (c) (i) Shade in  $\frac{2}{3}$  of 18.  
(ii) How many did you shade in?  
(Compare)
- (d) (i) Shade in  $\frac{1}{5}$  of 20.  
(ii) How many did you shade in?  
(Compare)

3. (a) (i) Shade in  $\frac{1}{2}$ .  
(ii) Shade in  $\frac{1}{3}$  of the area you just shaded in. What is  $\frac{1}{3}$  of  $\frac{1}{2}$ ?
- (b) (i) Shade in  $\frac{3}{4}$ .  
(ii) Shade in  $\frac{1}{2}$  of the area you just shaded in. What is  $\frac{1}{2}$  of  $\frac{3}{4}$ ?
- (c) (i) Shade in  $\frac{1}{2}$ .  
(ii) Shade in  $\frac{3}{4}$  of the area you just shaded in. What is  $\frac{3}{4}$  of  $\frac{1}{2}$ ?

4. The following represents  $\frac{10}{3}$  or  $3\frac{1}{3}$ . If we took  $\frac{1}{2}$  of  $\frac{10}{3}$ , what do we get?

$$\frac{1}{2} \times \frac{10}{3} = \underline{\hspace{2cm}} \text{ or } \frac{1}{2} \times 3\frac{1}{3} = \underline{\hspace{2cm}}$$



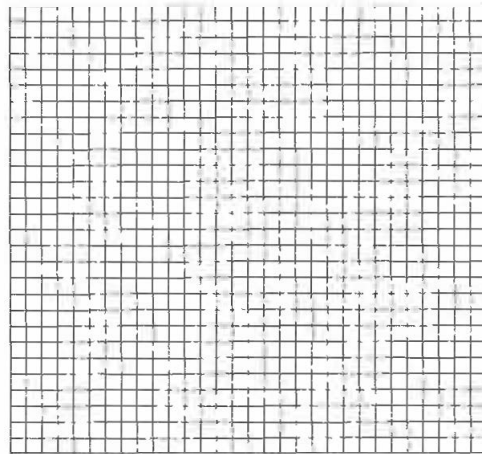
## Meaning Activity 7 Multiplying Fractions

1. Using any size grid paper, show the following multiplication questions. Place the answer after each question.

- (a)  $\frac{1}{2}$  of  $\frac{1}{3} = \underline{\hspace{2cm}}$   
(b)  $\frac{1}{2}$  of  $\frac{1}{4} = \underline{\hspace{2cm}}$   
(c)  $\frac{1}{4}$  of  $\frac{1}{6} = \underline{\hspace{2cm}}$   
(d)  $\frac{3}{4}$  of  $\frac{1}{3} = \underline{\hspace{2cm}}$

(e)  $\frac{3}{4}$  of  $1\frac{1}{2} = \underline{\hspace{2cm}}$

(f)  $\frac{1}{3}$  of  $2\frac{2}{3} = \underline{\hspace{2cm}}$



2. Looking at your answers, how could you obtain the same results by using the given fractions?

## Meaning Activity 8 Dividing by a Unit Fraction

Teacher Sheet

1.  $1 \div \frac{1}{4} = \underline{\hspace{2cm}}$ . How many quarters?
2.  $6 \div \frac{1}{4} = \underline{\hspace{2cm}}$ . How many quarters?
3.  $8 \div \frac{1}{2} = \underline{\hspace{2cm}}$ . How many halves?

This shows that when you divide by a fraction, the result is the same as multiplying by the reciprocal of that fraction.

4.  $4 \div \frac{2}{3} = \underline{\hspace{2cm}}$ . Draw four units divided into thirds. How many groups of two thirds can we get?



## Meaning Activity 9 Dividing Fractions

The teacher will assist you in doing these questions.

1.  $2 \div \frac{1}{2} = \underline{\hspace{2cm}} = 2 \times \underline{\hspace{2cm}}$ . How many halves are there in 2?

2.    $2 \div \frac{1}{3} = \underline{\quad} = 2 \times \underline{\quad}$ . How many thirds are there in 2?
3.    $2 \div \frac{1}{4} = \underline{\quad} = 2 \times \underline{\quad}$ . How many quarters are there in 2?
4.   $1 \div \frac{1}{2} = \underline{\quad} = 1 \times \underline{\quad}$ . How many halves are there in 1?
5.   $\frac{1}{2} \div \frac{1}{2} = \underline{\quad} = \frac{1}{2} \times \underline{\quad}$ . How many halves are there in  $\frac{1}{2}$ ?
6.   $1 \div \frac{1}{4} = \underline{\quad} = 1 \times \underline{\quad}$ . How many quarters are there in 1?
7.   $\frac{1}{2} \div \frac{1}{4} = \underline{\quad} = \frac{1}{2} \times \underline{\quad}$ . How many quarters are there in  $\frac{1}{2}$ ?
8. In your *own* words, describe how to do the following:
- (a)  $1 \div \frac{1}{5} =$
- (b)  $\frac{1}{2} \div \frac{1}{5} =$

### Meaning Activity 10 Fractions and Decimals

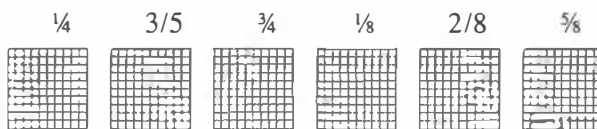
1. Write these decimals as words, fractions, reduced fractions, words and an equation. The first one is done as an example for you.

Decimal	Words	Fraction	Reduced Equation	Words	Equation
0.5	five tenths	5/10	$\frac{1}{2}$	one half	$0.5 = \frac{1}{2}$
0.7					
0.25					
5.8					
0.12					
0.10					
0.04					
3.4					

2. On the hundredths square:
- (a) Shade in  $\frac{1}{5}$ . Now write  $\frac{1}{5}$  as a decimal:
- (b) Shade in  $\frac{1}{2}$ . Now write  $\frac{1}{2}$  as a decimal:



3. Shade in a hundredths square for each fraction below.



### Meaning Activity 11 Comparing and Ordering Fraction Bars

1. Select the  $\frac{1}{3}$ ,  $\frac{2}{3}$ ,  $\frac{2}{6}$ ,  $\frac{4}{6}$ ,  $\frac{4}{12}$  and  $\frac{8}{12}$  fraction bars. Arrange them in ascending order.
2. Select the  $\frac{2}{3}$ ,  $\frac{3}{4}$  and the 12ths bar.
- (a) How many 12ths are in  $\frac{2}{3}$ ?
- (b) How many 12ths are in  $\frac{3}{4}$ ?
- (c) Which is greater numerically?
3. Select the  $\frac{1}{2}$  and the  $\frac{3}{5}$  bars.
- (a) Find another bar with division lines that coincide with the endpoints of each of these bars. Which bar is it?
- (b) Write the fractions  $\frac{1}{2}$  and  $\frac{3}{5}$  as fractions that have the same denominator.
- $\frac{1}{2} = \underline{\quad}$        $\frac{3}{5} = \underline{\quad}$
4. Select the  $\frac{3}{4}$  and the  $\frac{5}{6}$  bars.
- (a) Repeat 3a and 3b above.
- (b) Find a method to help decide which bar to choose.
- (c) What is this bar number called?
5. Select the  $\frac{1}{5}$  and  $\frac{1}{4}$  bars.
- (a) Repeat 4a and 4b above.
- (b) What method did you use for choosing the correct bar?
- (c) What bar number is this?

### Meaning Activity 12 Equivalent Fractions

1. (a) Select the fraction bar for  $\frac{1}{2}$ .
- (b) Find by comparison all those fraction bars that have the same length as the one-half bar.
- (c) Write these as fractions.
- (d) What does the term *equivalent* mean?
- (e) Repeat parts a, b and c for the following bars:
- $\frac{1}{3}$  \_\_\_\_\_       $\frac{2}{3}$  \_\_\_\_\_
- $\frac{1}{4}$  \_\_\_\_\_       $\frac{2}{4}$  \_\_\_\_\_
- $\frac{3}{4}$  \_\_\_\_\_       $\frac{2}{5}$  \_\_\_\_\_
- $\frac{4}{5}$  \_\_\_\_\_       $\frac{1}{6}$  \_\_\_\_\_
- $\frac{5}{6}$  \_\_\_\_\_



2. Write eight equivalent fraction pairs that you can find from your fraction bars. For example,  $2/10 = 1/5$ .

### Meaning Activity 13 Adding Fractions Teacher Sheet

A demonstration with fraction bars.

1. Start with the same denominators.

- (a)  $2/4 + 1/4 = 3/4$   
 (b)  $3/5 + 1/5 = 4/5$   
 (c)  $7/10 + 3/10 = 10/10 = 1$

Observe: In a, b and c, the answer has the same denominator as the sum. When the numerator and the denominator are the same, the result is 1.

2. Next use different denominators (for example,  $1/5 + 1/2 = 7/10$ ). Match the fraction bars with another bar that has a division which lines up with that of the sum.

- (a)  $1/4 + 1/2 = 3/4$   
 (b)  $1/6 + 1/3 = 1/2$   
 (c)  $1/4 + 3/3 = 11/12$   
 (d)  $1/3 + 1/2 = 5/6$

Observe: The answer's denominator might (i) be the same as that of one part of the sum or (ii) not be the same as either part of the sum.

Hence: The idea of finding the lowest common denominator (LCD), which is the same as finding the lowest common multiple (LCM).

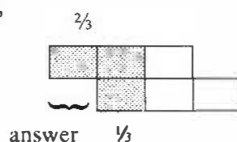
3. Give students several examples of their own to work out on fraction strips.

### Meaning Activity 14 Subtraction of Fractions

#### Teacher Sheet

Use the procedure in Meaning Activity 13 but make the end lines of the two given fractions line up so that one fraction bar connects with the other bar to give the idea of taking away (subtracting) one section from the other section. Find a fraction bar with a division that matches the length of the remaining section.

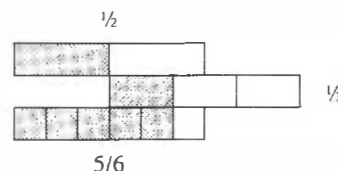
For example,



- (a)  $2/3 - 1/3 = 1/3$   
 (b)  $4/5 - 2/5 = 2/5$   
 (c)  $5/6 - 1/3 = 3/6$   
 (d)  $3/4 - 1/2 = 1/4$   
 (e)  $5/8 - 1/4 = 3/8$   
 (f)  $5/6 - 3/4 = 1/12$   
 (g)  $1/2 - 1/3 = 1/6$

### Meaning Activity 15 Adding and Subtracting Fractions

1. Take a  $1/2$  bar and a  $1/3$  bar. Match along the  $1/4$  and other bars until both match evenly on one of the bar sheets. For example, the  $1/2$  bar will match evenly along the  $1/4$ ,  $1/6$ ,  $1/8$ ,  $1/10$  and  $1/12$  sheet. The  $1/3$  bar will match evenly on the  $1/6$  and  $1/12$  sheet. Thus, use the  $1/6$  bars. Place the  $1/2$  bar alongside the  $1/3$  bar overlapping on a  $1/6$  bar sheet. Count the sections of the  $1/6$  bar sheet covered.



Using the above procedure, try

- (a)  $1/2 + 1/4 = \underline{\quad}$  (d)  $2/3 + 1/4 = \underline{\quad}$   
 (b)  $1/2 + 1/5 = \underline{\quad}$  (e)  $1/5 + 1/10 = \underline{\quad}$   
 (c)  $1/2 + 1/6 = \underline{\quad}$  (f)  $1/2 + 3/4 = \underline{\quad}$

2. Follow the procedure in question 1 to locate the common bar sheet. Now place the  $1/2$  bar along the  $1/6$  bar sheet. Place the  $1/3$  bar below the  $1/2$  bar on the  $1/6$  bar sheet. The amount of the  $1/2$  bar showing beyond the  $1/3$  bar is the difference. Now try the following:

- (a)  $3/4 - 1/2 = \underline{\quad}$  (e)  $7/10 - 1/5 = \underline{\quad}$   
 (b)  $1/2 - 1/6 = \underline{\quad}$  (f)  $2/3 - 1/2 = \underline{\quad}$   
 (c)  $5/6 - 1/2 = \underline{\quad}$  (g)  $5/12 - 1/3 = \underline{\quad}$   
 (d)  $3/8 - 1/4 = \underline{\quad}$  (h)  $7/12 - 1/4 = \underline{\quad}$

3. Follow the procedure from question 1 again. This time write the equivalent fractions, for example,  $1/2$  as  $3/6$ ,  $1/3$  as  $2/6$ , and then add or subtract the numerators as required.

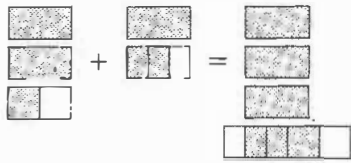
- |     | Equivalent Fractions            | Solution              |
|-----|---------------------------------|-----------------------|
| (a) | $3/8 + 1/2 = \underline{\quad}$ | $= \underline{\quad}$ |
| (b) | $1/4 + 2/3 = \underline{\quad}$ | $= \underline{\quad}$ |

(c)  $\frac{3}{5} - \frac{1}{10} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$   
 (d)  $\frac{3}{4} - \frac{2}{3} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

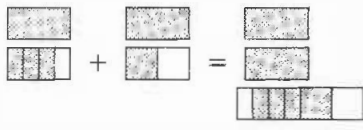
### Meaning Activity 16 Adding and Subtracting Mixed Numbers

Lay your fraction bars in colors on your desk. Do the following with your fraction bars:

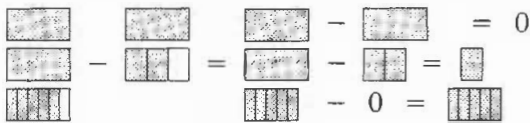
1. Add:  $2\frac{1}{2} + 1\frac{2}{3} = 4$  plus a bit. Find the bar that equals the bit left over.



2. Add:  $1\frac{3}{4} + 1\frac{1}{2} = 3\frac{1}{4}$  or  $3\frac{2}{8}$



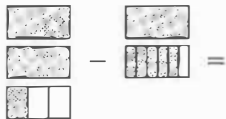
3. Subtract:  $2\frac{5}{6} - 1\frac{2}{3} =$



adding the leftover bits

$\frac{1}{2} + \frac{1}{6} = \frac{7}{6} = 1\frac{1}{6}$

4. Subtract:  $2\frac{1}{3} - 1\frac{5}{6} =$



5. Using the above examples, show the solutions to the following by drawing fraction bars.

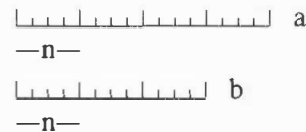
- (a)  $1\frac{1}{2} + 2\frac{1}{3} =$   
 (b)  $2\frac{1}{4} + 2\frac{1}{2} =$   
 (c)  $1\frac{3}{4} + 1\frac{1}{2} =$   
 (d)  $2\frac{2}{3} + 1\frac{1}{2} =$   
 (e)  $3\frac{1}{2} + 1\frac{3}{5} =$   
 (f)  $4\frac{3}{4} - 1\frac{1}{2} =$   
 (g)  $4\frac{2}{3} - 2\frac{1}{2} =$   
 (h)  $4\frac{5}{8} - 3\frac{1}{4} =$   
 (i)  $2\frac{1}{4} - 1\frac{7}{8} =$   
 (j)  $1\frac{1}{5} - \frac{1}{2} =$

### Problem Solving Activity 1 GCF and LCM

- What is meant by the greatest common factor (GCF)?
- What is meant by the lowest common multiple (LCM)?
- What is the GCF of two numbers if one of them is triple the other? What is their LCM?
- Find two numbers that fit the description in question 3.
- What is pictured below? What is the GCF of a and b? The LCM of a and b?



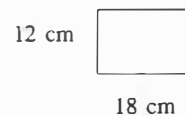
6. What is pictured below?



n is the LCM of a and b  
 n is the GCF of a and b

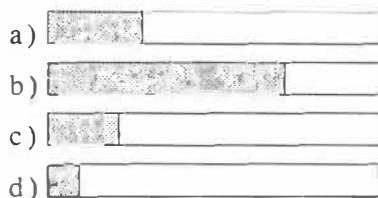
### Problem Solving Activity 2 GCF and LCM

- During the past two years, Laura wore out a pair of runners every 12 months, a pair of socks every 6 months and a t-shirt every 8 months. At these rates, how often will she have to replace all of these items at the same time?
- Bus #32 takes 45 minutes to complete one trip around its route. Bus #36 takes 60 minutes to complete one trip around its route. If they both leave the Southgate terminal at 9:00 a.m., when will they both be there together again?
- Holly went to a card store and bought several packages of stickers. She got 75 stickers altogether. Darlene bought 125 stickers. Each package has the same number of stickers. How many stickers are there in each package?
- Divide the shape into the minimum number of squares which are all the same size. How long is the side of each square?



### Problem Solving Activity 3 Equivalent Fractions

1. Estimate what fractions are represented by the shaded areas:



2. What are equivalent fractions?
3. Make up a problem which has an answer which is a fraction.
4. Make up an equality that fits  $\frac{x}{y} = \frac{4x}{4y}$
5. Find values of  $x$  and  $y$  so that  $\frac{x}{y} = \frac{5x}{5y}$
6. How many quarters are necessary to make  $7/4$  dollars?

### Problem Solving Activity 4 Comparing and Ordering

1. Of the 30 students in Mr. Lee's class, 3 were absent on Monday. On the same day, 2 students were absent out of 25 in Mr. Burton's class. Which class had the smaller fraction of students absent?
2. While playing basketball, Corey sank six out of thirteen shots into the basket. Paul sank five out of eight shots. Who got the greater fraction of shots into the hoop?
3. If  $\frac{a}{b} > \frac{c}{b}$ , what can you conclude about  $a$  and  $c$ ?
4. Give an example that fits  $\frac{a}{b} > \frac{c}{b}$ .
5. If  $\frac{a}{b}$  and  $\frac{c}{d}$ , how do  $b$  and  $c$  compare?
6. Give an example that fits  $\frac{a}{b} < \frac{a}{c}$ .
7. Two fractions,  $\frac{a}{b}$  and  $\frac{c}{d}$ , are between zero and one.  $\frac{a}{b}$  is closer to 1 than  $\frac{c}{d}$ . Which fraction is greater?
8. Make up a problem that requires comparing two fractions.

### Problem Solving Activity 5 Adding and Subtracting Fractions

1. In a hall, one third of the people present are men, one quarter are women and the rest are children. If 1,152 people are in the hall, how many children are present?
2. Michael has two containers, A and B. Container B holds twice as much as container A. A is  $1/2$ -filled and B is  $1/3$ -filled with syrup. Michael fills the rest of each container with water. He then pours the contents of containers A and B into a third container. What fraction of the total content is water?
3. Perform the following operations:

(a)  $\frac{a}{b} + \frac{c}{b} =$

(b)  $\frac{a}{b} - \frac{c}{b} =$

(c)  $\frac{a}{a} + \frac{b}{a} =$

(d)  $\frac{a}{b} + \frac{c}{d} =$

(e)  $\frac{a}{b} - \frac{c}{d} =$

### Problem Solving Activity 6 Fractions and Mixed Numbers

1. After a farmer died, his will was read. He left instructions that his three sons were to divide up 17 cows as follows: "Tom gets  $1/2$ , or  $8\frac{1}{2}$  cows; Dick gets  $1/3$ , or  $5\frac{2}{3}$  cows; Harry gets  $1/9$ , or  $1\frac{8}{9}$  cows." The sons were each able to get their full shares, and all the cows lived. How did they do this? (Hint: Borrow 1 cow from a neighbor, then divide.)
2. Estimate first, then solve the following:
- (a)  $4\frac{2}{3} + 1\frac{3}{4} = (5 + 2 = 7) =$
- (b)  $2\frac{1}{2} + 3\frac{3}{4} =$
- (c)  $20\frac{4}{5} + 3\frac{1}{3} =$
- (d)  $6\frac{7}{8} - 1\frac{1}{3} =$
3. Find four fractions (numbers less than one) whose sum is between two and three.

4. Use long division to create a mixed number; for example:

$$\begin{array}{r} 3 \text{ r } 1 \\ 5 \overline{)16} \\ \underline{15} \\ 1 \end{array} \quad 8 \overline{)47} \quad 6 \overline{)52}$$

5. Given  $n/d$ , what type of fraction do you have if  $n < d$ ,  $n > d$ ,  $n = d$ ? Give an example of each.

### Problem Solving Activity 7 Multiplying Fractions

1. Karen had 15 posters. She sold  $2/5$  of her posters for \$0.50 each and the remaining posters for \$0.35 each. How much money did she get for the 15 posters?
2. Tyler receives \$30 allowance monthly. He spends  $2/5$  of his money on snacks,  $3/10$  on entertainment,  $1/10$  on comics and the remainder is saved. How much money does Tyler save each month? In a year?
3. The Boston Bruins played 80 regular season hockey games. During the first half of the season, they won  $3/4$  of their games. In the second half, they won  $7/8$  of their games. How many games did they win over the season?

### Problem Solving Activity 8 Dividing Fractions

1. On the weekend, Jackie worked  $9 \frac{3}{4}$  hours and earned \$39. What was her hourly wage?
2. Theresa has a part-time job at McDonald's. It takes her  $3/4$  of a minute to assemble a Big Mac. If she worked nonstop for  $7 \frac{1}{2}$  hours, how many Big Macs would Theresa make?
3. Jim bought four rectangular ice cream cakes for his class party. Estimate how many pieces he must cut so that each student receives  $1/12$  of the cakes.

### Problem Solving Activity 9 Problems/ Operations

1. Goalie Grant Fuhr played  $7 \frac{1}{3}$  games without allowing a goal. How many periods did he play?
2. The Edmonton Eskimos played 17 quarters without getting a field goal. How many games did they play?
3. Your teacher wants to give  $1/6$  of a chocolate bar to each member of your class. How many bars will be needed?
4. Your teacher wants to give one and a half chocolate bars to each member of your class. How many bars will be needed?
5. Did you solve questions 3 and 4 the same way? Explain.

# Unit IV: Ratios and Rates

## Teachers' Notes on Meaning Activities

### Activity 1

This is a real problem faced by shoe manufacturers. A discussion of the results from the class should be encouraged.

### Activity 2

Other materials may be substituted for M & Ms if their use is considered unhealthy. If M & Ms are used, then an interesting activity is to find out if the ratios of colors are consistent among packages. When ratios between different colors are considered, many ratios can be found using the same data.

### Activity 3

Unit rate is a fundamental concept in rate problems. It is an alternative to using proportion statements.

### Activity 4

Unit rate is directly applied in the unit-price approach to comparison shopping. Of course, different amounts can be considered units.

### Activity 5

Many excellent applications of the rate/ratio concept are possible once a map is available.

## Meaning Activity 1 Rates and Equivalent Ratios

With the help of your teacher and classmates, fill in the following tables:

Girls' Shoe Size	4	5	6	7	8	9	10	Total
Number of Girls								
Boys' Shoe Size	6	7	8	9	10	11	12	Total
Number of Boys								

- Write the ratios of the numbers of each shoe size to the total number of people for that table. This should provide about six ratios for each table.
- What is the ratio of the number of girls' size 8 to the total number of girls? How does this compare with the ratio for boys' size 8? Which ratio is larger?
- What is the ratio of the number of girls' size 6 to the total number of girls? How does this compare with the ratio for boys' size 6? Which ratio is larger?
- Using the population of Grade 8 students in your school, find the number of girls and the number of boys who wear each size.
- If there are 8,000 Grade 8 students in Calgary and half of them are girls, how many shoes of various sizes would need to be stocked for this market? Calculate this for the boys also.
- Construct other questions that you could answer based on the information from your class.

## Meaning Activity 2 Ratios—Activity and Equivalent Ratios

### Teacher Sheet

- Provide each group of four students with one package of M & Ms. Have students complete the following chart:

Number of M & Ms	
Color	Red
	Brown
	and so on
	Total

2. Compile the information from each group on a transparency representing the entire class. (See Activity 2 transparency.)
3. Have each group determine ratios such as (a) red to brown, (b) brown to green, (c) green to red and so on.
4. From your experience, define ratio.
5. Using information from the transparency, the teacher could ask equivalent ratio questions based on information about M & Ms. For example, if Group 3 had eight brown M & Ms and five yellow M & Ms, how many yellow M & Ms would you expect them to have if they had forty brown ones?
6. Using transparency information, have students answer the following as well as similar problems:
  - (a) Group 3 has 40 M & Ms. They have 8 yellow ones. Group 4 has 7 yellow. How many M & Ms in all will Group 4 need to have the same ratio as Group 3? ( $7/x = 8/40$ )
  - (b) Group 2 has 50 M & Ms. They have 7 orange ones. Group 5 has 48 M & Ms. How many orange ones would they need to have the same ratio as Group 2? ( $x/48 = 7/50$ )

### Meaning Activity 2 Transparency

Color	Groups								Class Total
	1	2	3	4	5	6	7	8	
Number of M&Ms									
Total									

### Meaning Activity 3 Rates

Unit rate is a good way to handle rate problems. When you use this method, you always find the rate of one unit.

Find the unit rates of the following:

1. Six apples cost \$1.50 = \_\_\_\_ dollars per apple.
2. 200 km in two hours = \_\_\_\_ km per hour
3. \$360 for 30 hours work = \_\_\_\_ dollars per hour
4. 200 words typed in five minutes = \_\_\_\_ words per minute.

To solve problems using this method, *always* express the given information as a *unit rate*, then continue to solve the problem.

1. Apples cost \$1.50 for six apples. How much would ten apples cost?  
Ans: Unit rate is \$0.25 per apple. Cost of ten apples:  $10 \times \$0.25 = \$2.50$ .
2. Joe worked 30 hours and made \$240. At this rate, how much would he get for 22 hours?
3. Twelve eggs cost \$1.08. How many eggs could you buy for 75 cents? How much change would you get?
4. Sally can type 350 words in five minutes. How long would it take her to type a 1,000-word essay?

### Meaning Activity 4 Rates

Although all of these problems can be solved by "solving proportions," and in some cases "solving proportions" is faster, be sure that you understand the unit-rate idea. It can be used in many problems.

When we use the unit-rate method in comparison shopping, it becomes the *unit-price* method. In this method, we always find the cost of one unit of the product. Find the *unit price* of the following:

1. Three dozen oranges cost \$4.50. Find *two* unit prices: per dozen, per orange.
2. Three hundred grams of rice cost \$1.75. What is the unit price?
3. For \$12, you get eight cans of tomatoes. What is the unit price?

Using this idea of unit price, do the following questions:

4. Which is a better buy: three bulbs for \$3.60, or five for \$6.00?
5. You are choosing between 100 ml of toothpaste for \$1.29 or 175 ml for \$2.69. Which is the better buy?

In solving question 5, you probably found the unit price of each and compared unit prices. Can you think of a slightly different way to solve these, also using unit price?

6. If you needed 1,000 feet of lumber, what would be the difference in price you would pay at these stores:

Store A: 8-foot boards cost \$2.13

Store B: 25 feet cost \$5.00

Although many comparison shopping questions can be done using different methods, *be sure you can use the unit price method.*

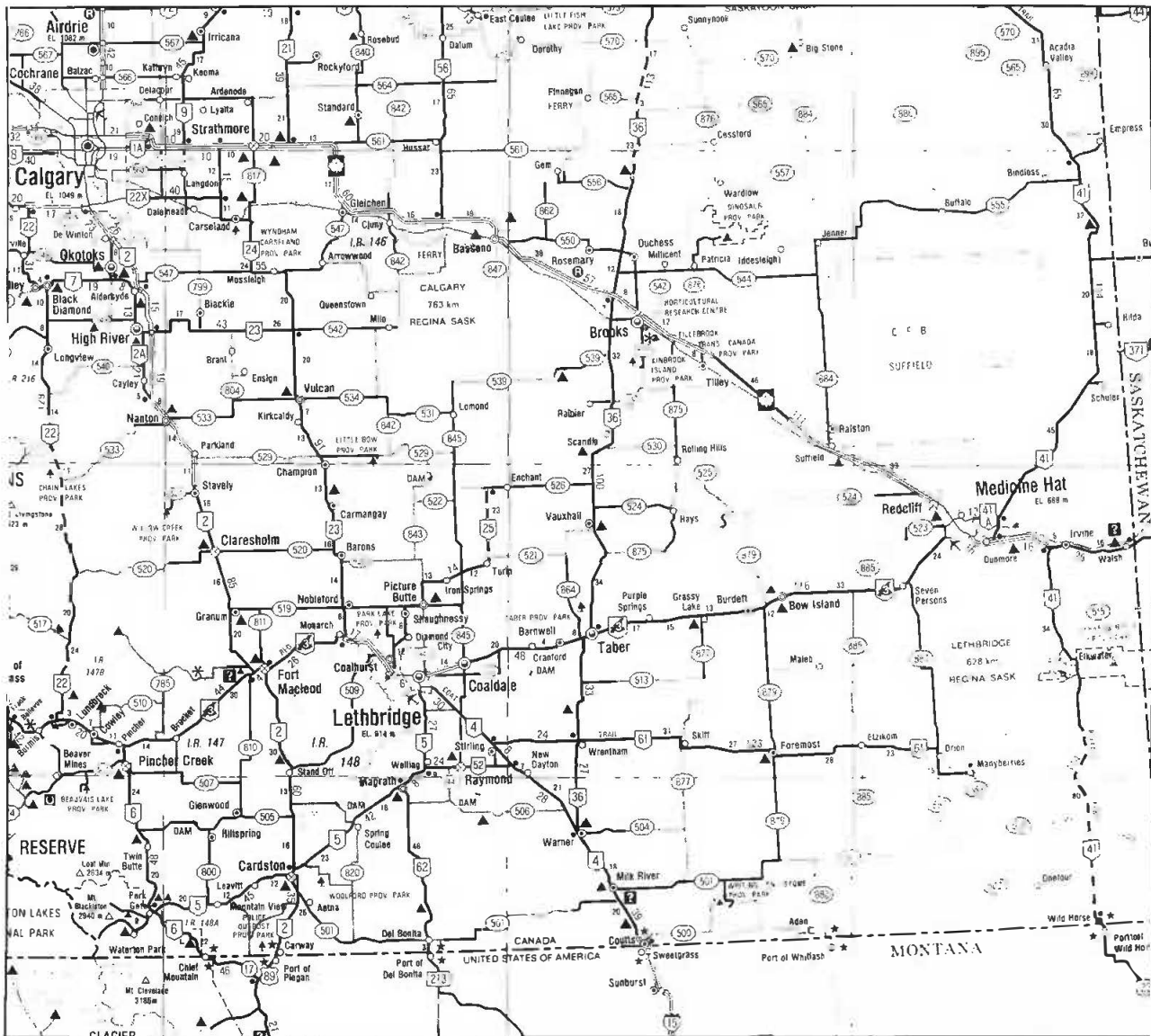
## Meaning Activity 5 Map Scales

A scale for a map is given as Drawing : Actual. The scale for the accompanying map is 1:1,500,000. This means 1 cm is equivalent to 1,500,000 cm.

Figure out how many kilometres are represented by 1 cm on the map. 1 cm = \_\_\_\_ km. (Be sure you have this correct before you answer the following questions.)

1. What is the straight line distance from Calgary (centre of dot) to Lethbridge? \_\_\_\_ How does this compare to road distance?
2. The road distance to Medicine Hat is 293 km. What is the air distance? \_\_\_\_ Why is the air distance shorter?

3. The cost of shipping a large carton of medical supplies from Calgary to Medicine Hat is \$45 per km by truck and \$70 per km by air. Which would be cheaper? \_\_\_\_ Besides cost, what might be some other factors to consider in deciding which method to use?
4. What is the actual length of the Alberta/Montana border?
5. If the rat patrol wants to set up a station every 20 km along the Alberta/Montana border, how many stations would it need?



## Problem Solving Activity 1 Ratio

1. Roll a pair of dice. What are the possible ways to roll the following combinations? (It may help to think of one die as colored red and the other black.)

2: 1 + 1

3: 2 + 1; 1 + 2

4: 1 + 3; 3 + 1; 2 + 2

5: 1 + 4; 4 + 1; 2 + 3; 3 + 2

6:

7:

8:

9:

10:

11:

12:

- (a) What is the total number of possibilities in a roll of two dice?

- (b) What is the ratio of rolling:

- (i) 6 to the total?      (iii) 2 to the total?  
 (ii) 7 to the total?      (iv) 8 to the total?

These ratios are called *probability*.

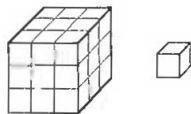
2. During a school election, Shelly got 10 votes for every 3 votes Kari received. Fill in the following information:

(a)

	Shelly	Kari
	10	3
	?	6
	40	?
	210	?

- (b) If 325 students voted, how many votes did each receive? (Hint: find a pattern.)

## Problem Solving Activity 2 Equivalent Ratio



1. You have a cube, measuring 3 cm to a side. The cube is painted red. Cut the 3-cm cube into 1-cm cubes.
- (a) How many cuts are required to do this?
- (b) Find the ratio of cubes with no painted surfaces to cubes with three painted surfaces.
- (c) Find the ratio of cubes with two painted surfaces to cubes with one painted surface.

2. Refer to above, increase the size of the cube by three times and do 1a and 1b again. Compare answers. Do you have equivalent ratios? Why? Why not? Explain.

## Problem Solving Activity 3 Rates

1. An athlete's heart beats about 24 times in 30 seconds.

(a) What is this in heartbeats per minute?

(b) During exercise, the heart rate triples. What does it become?

2. A car travels 60 km in 30 minutes.

(a) What is its rate per second?

(b) What is the unit rate in km per minute?

(c) How many kilometres would be traveled in one hour?

- 3.

(a) How long would it take to go 5 km at 30 km/hr?

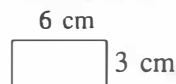
(b) If you had to travel 50 km/hr, how long would it take to make the whole trip of 10 km?

(c) Subtract 3a from 3b to see how much time you have left in which you must travel the remaining 5 km. Quite fast!

(d) Why is this answer such a surprise?

## Problem Solving Activity 4 Ratio, Scale, Map Problems

1. You are asked to draw a scale model of a car, which is 4 m long and 1.5 m wide, so that it fits in this rectangle. What scale would you choose?



2. In a class, there are 10 boys and 15 girls. Three of the boys like classical music, 5 girls like country and western and the remaining students like pop music. How many different ratios can you find? (There are about 15.) Which of these ratios would you use to find the number of students in 2,000 who do not like classical music?
3. On a map with scale 1:1,500,000, the straight-line distance from Edmonton to Calgary measures 20 cm. A man bought a suit in Calgary for \$400 and had it sent to Edmonton. The salesclerk said that it would cost \$0.47 per km to send plus \$10 for wrapping. What was the total cost of the suit?



# Unit V: Percent

## Teachers' Notes on Meaning Activities

### Activity 1

The characteristics you use in this activity should depend on local conditions. As many as 20 yes-no characteristics can be collected.

### Activity 2

After this activity is completed, you will have to provide students with ways to check the accuracy of their answers. Grid paper provides much structure for this activity.

### Activity 3

Translating from fractions to decimals to percents should receive much emphasis. Many teachers believe students should memorize all the numbers on this page.

### Activity 4

These questions centre on a coach and players making decisions about volleyball. Do coaches really use percentages? This activity should reinforce the idea that percentage is a pervasive concept.

### Activity 5

Percent problems require careful reading. This activity focuses on reading and understanding. Students should solve these problems *after* they have made the picture.

### Activity 6

Restating questions is a way for students to internalize the problem. After the restatement, students should solve the problems.

### Activity 7

This activity requires sorting through data and reading carefully. The information in a simpler form might be as follows:

ski	hockey	ice-fishing	stove
50 percent off	30 percent off	20 percent off	regular price
\$400	$3 \times \$35 = \$105$	\$700	\$50

The teacher can provide some guidance but a rigid format should not be required.

## Meaning Activity 1 Meaning of Percent

### Teacher Sheet

Have students construct the following chart(s) in their notebooks:

Characteristic	1	2	3	4
Number with				
Number without				

Characteristic	1	2	3	4
Fraction with				
Fraction without				

Characteristic	1	2	3	4
Percent with				
Percent without				

Which chart gives you the best information? Some possible characteristics to chart are

1. Watch tv
2. Attend hockey games
3. Take a computer class
4. Play musical instruments
5. Plan on attending university
6. Eat bananas
















According to these characteristics, are you a "usual" student? How do you decide this?


### Meaning Activity 2 Meaning of Percent

1. Page 198 of *Journeys in Math 8* has a drawing of a school marked out on a 10 x 10 grid. Percent is easy to calculate using this grid. Why? Be sure you understand this diagram. You are an architect responsible for designing a house floor plan with the following specifications. Use grid paper. (Your house needn't be square.)

- |                        |                      |
|------------------------|----------------------|
| Kitchen 12 percent     | Bedroom A 13 percent |
| Living room 25 percent | Bedroom B 15 percent |
| Dining room 15 percent | Hallway 10 percent   |
| Bathroom 10 percent    |                      |

### Meaning Activity 3

	Fraction	Decimal	Percent	Picture of the Fraction
1			100%	
2		0.9		
3	4/5			
4	5/8			
5			66 2/3%	
6		0.60		
7				
8	2/5			
9		0.375		
10	1/3			
11			30%	
12	1/4			
13	1/8			
14		0.01		
15		0.001		

Try to remember these combinations of numbers. When you see 0.7 or 0.70, your should be able to image 7/10 or 70 percent or 

### Meaning Activity 4 Finding a Percent of a Given Number

- Jane said 50 percent of her volleyball team were playing with injuries. There are 20 members on the team. How many have injuries?
- Jane misses 25 percent of her serves. On average, she makes 80 serves a game. How many does she miss? How many does she make?
- Jane is responsible for 20 percent of the setups for the team in a typical game. The coach wants 120 setups in a game. How many must Jane make?
- Josie spikes 75 percent of all the setups that Jane makes. How many is this?
- The coach has a rule that 60 percent of players with injuries must dress for the game. How many will this be?
- On average, the coach says 10 percent of her players go on to more advanced play. How many members of this team will go on?
- Forty percent of the members have A averages. How many is this?
- On average, about 2 percent of volleyball players at this level go on to play at the national level. How many girls from Jane's team could be expected to make it?

### Meaning Activity 5 Find a Number When the Percent of It Is Known

Read the following questions and draw a simple picture representing the problems.

- (a) Billy's cat is 66 cm long. She was 20 percent of this when Billy got her. How long was she then?  
(b) The veterinarian said the cat is still only 70 percent of her final length. How long will she be?
- John's brother's record collection is 500 percent larger than John's. John has 75 records.
- A cola mix for a birthday party uses 30 percent syrup and the rest carbonated water. Sally's dad bought 5 L of syrup. How much cola could be made? Did Sally have enough for her party?
- The Calgary Flames management estimates that 70 percent of customers are season ticket holders. How many people do they expect if they have 9,500 season ticket holders?

5. At Super Slopes Ski Run, 65 percent of accidents involved hot-doggers. Twelve hot-doggers reported injuries at the end of the day. What is a good estimate of how many average skiers were injured?
6. (a) At the same Ski Run there is a 0.001 percent chance of the ski lift collapsing. If 5,500,000 people use the ski lift in 10 years, how many would you expect would be involved in a collapse in this time?  
(b) How many would this be in one year?

### Meaning Activity 6 Percent Problems

Restate the following questions in your own words:

1. Ninety percent of all students watch at least two hours of tv every week. In your class, how many watch less than two hours?
2. On average, 20 percent of senior hockey players weigh over 95 kg. On the Red River Ramblers, eight of their twenty players were over 95 kg. The coach said half of these were overweight. Is the team above the average in weight? Would they be above the average if half of them lost enough weight to be below 95 kg?
3. Of the tourists traveling to the U.S.A., 99 percent fly or drive their own cars. Of the 3,525,790 who visit the U.S.A. in an average year, how many do not fly or drive?
4. An investor lost 98 percent of his \$500,000 investment. How much was this? What percent did he save? How much was this?
5. (a) Jan's puppy grew from 8 kg to 20 kg in six months. What percent gain was this?  
(b) The puppy became ill and went down to 8 kg. What percent loss was this?  
(c) The veterinarian said that weaned puppies can lose up to 40 percent of their weight. Was Jan's pup within this figure?

### Meaning Activity 7 Discount Problems

Read carefully and write the question in point form below it. Do not answer the question.

1. A sporting goods store is having a sale of all its winter equipment. Angela likes skiing; her brother Manley likes hockey; her parents like ice fishing. All skiing equipment is reduced 50 percent; hockey equipment is reduced 30 percent,

and ice-fishing equipment is reduced 20 percent. Angela is shopping with her parents and her brother. Angela wants the \$250 skis and the \$150 ski pants. Her parents want the \$700 ice auger and the \$50 camp stove. Manley wants three hockey sticks, each priced at \$35. The store clerk says that the advertised discounts apply to all items except the stove because it is already so cheap. How much would the family save by buying now?

Write your version of the problem in simpler form.

### Application 1 Meaning in Percent

1. A cola drink is made by mixing 1 L of syrup to 4 L of water. What percent of a cola drink is syrup?
2. A junior high has 92 Grade 7 students, 96 Grade 8 students and 112 Grade 9 students. What percent of students are in Grade 8?
3. An apple inspector found that Farm A had 200 rotten apples in 1,500; Farm B had 150 rotten apples in 1,000; and Farm C had 280 rotten apples in 2,000. Which farm has the best record? Which has the worst?
4. What percent of each rectangle below is shaded? (Estimate.)



### Application 2 Percent as a Decimal and Fraction

1. Students were asked to name their favorite sport. Of these,  $\frac{1}{5}$  favor hockey,  $\frac{1}{4}$  favor skiing and  $\frac{1}{2}$  like to swim. The remainder said that they had no favorite sport. Of 200 students in the school, how many were in this last category?
2. In a truckload of 4,000 kg of fish, 98 percent were spoiled. Write the number of kilograms of good fish over the total kilograms as a fraction.
3. The fractions below are formed by adding one to the numerator and one to the denominator of the previous fraction. Convert each to a percent. What conclusion can you make?

$$\frac{1}{2} = \frac{\quad}{\quad} \quad \frac{2}{3} = \frac{\quad}{\quad} \quad \frac{3}{4} = \frac{\quad}{\quad} \quad \frac{4}{5} = \frac{\quad}{\quad}$$

$$\frac{5}{6} = \frac{\quad}{\quad}$$

4. A hockey team gets two points for a win, one point for a tie and zero points for a loss. They won 20 games and tied 8 games of the 40 games they played. What percent of the total possible points did they get?

### Application 3 Finding a Percent of a Number

1. Twelve percent of passengers on trains are smokers. In each car, 27 seats are reserved for smokers. In 10 years' time, the railway company expects 4 percent of passengers to be smokers. How many seats will they have to reserve then?
2. Of 16,000 people at Oilers games, 15 percent were children and 25 percent were female. How many people were in each group?
3. When a weather forecaster says there is a 20 percent chance of rain tomorrow, what does that mean?
4. One year, a hockey team scored 450 goals in an 80-game season. The next year, the team's goal production was down by 20 percent. How many goals were scored the next year?

### Application 4 Fractions and Decimals as Percent

1. Which is larger: (a)  $\frac{1}{2}$  of 25 percent of 1,000 kg or (b) 25 percent of  $\frac{1}{2}$  of 2,000 kg? Try to explain this!
2. Divide 1,000 into eight parts. How large is one part? How much are the seven remaining parts? If you put four of these parts together, how much would you have?
3. In a certain dough, the ratio of flour to milk to sugar is 6:3:1. If 12 cups of flour are used, how much milk and sugar are used? Express these amounts in percent.

### Application 5 Finding a Number When a Percent of It Is Known

1. In a light bulb factory, 5 percent of the bulbs are defective. One percent of these defective bulbs are dangerous to use. If a company produces 20,000 bulbs per day for a 20-day period, how many dangerous bulbs will be made?
2. Susan found out that kids her age often spend 25 percent of their allowance on entertainment.

Susan wants to go to two shows at \$3.50 each and two concerts at \$6.50 each. Based on this, what would she want her allowance be?

### Application 6 Discount

1. A certain grapefruit will retain 95 percent of its weight after a month in storage. How much will a dozen grapefruit weigh if each weighed 150 grams when packed?
2. Ninety-eight percent of students who begin a driver-education course pass in three weeks' time. On February 1, 150 students enrolled. How many will graduate on February 22?
3. A ski shop is going to have a sale. The manager is trying to decide if she should offer a 30 percent discount followed by a 20 percent discount or offer the discounts in reverse order. What do you suggest?

### Application 7 Sales Tax

1. How would you solve this problem? "Mr. Murphy bought shoes, socks and a tie. Each had a different price. He also had to pay sales tax which was a percent of the price. How much sales tax did he pay?"
2. A certain number,  $n$ , is 8 percent of 25. What percent of 50 is the same number  $n$ ?
3. Nova Scotia has a 10 percent sales tax. If the cost (original price plus sales tax) of an item is \$0.99, what was the original price?
4. In a province with a 10 percent sales tax, what is saved in sales tax when a \$100 item is discounted by 20 percent?

### Application 8 Gain and Loss

1. Harry got a mark of 70 percent on a test. The teacher increased this by 20 percent because Harry had correctly answered the bonus questions. What was his mark? The teacher decided to decrease this new mark by 20 percent because Harry's bonus answers were messy and did not show all of his work. What was Harry's final mark?
2. Pat had a test score of 55 percent. Because Pat had all her assignments done, the teacher said he would either add 10 points to her score or increase her score by 15 percent of her original score. Which option would you advise Pat to take?

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