Making Sense Out of Number Sense

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In the focus issue of the *Arithmetic Teacher*, Howden (1989, 6) reports that the Standard from the *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989) dealing with number sense "raised the most questions from teachers, parents and administrators. These four questions were most frequently asked: What is number sense? Why is number sense important? How is number sense taught? How is number sense measured?"

The editorial panel of the *Arithmetic Teacher* states that "although mathematics educators agree that the development of number sense is important, no single definition is universally accepted" (Thompson and Rathmell 1989, 2). Hope (1989) agrees when he advises that "number sense is considered a desirable trait to foster, although its meaning as other notions of thinking . . . can be defined only broadly." The same author goes on to explain that "number sense cannot be defined precisely, but situations where it is evidently lacking can easily be recognized" (p. 12).

In a midterm, when asked to "define number sense in your own words" and to "illustrate the definition with an example," one mathematics education student's opinion in a way opposed Hope's. She declared, "I find it very difficult to define number sense, but I would surely recognize its presence when I encountered it." She then described an interview response from an elementary school student to illustrate her declaration.

While taking the above-mentioned mathematics course, student teachers have an opportunity to watch interviews of students from different grades. As part of the follow-up, responses that indicate the absence and presence of number sense are discussed. The student teachers identified several elementary school students who provided responses indicative of the presence of number sense. The excerpts from three different interviews involving students from three different grade levels follow. As you read the excerpts, try to identify indicators of the presence of number sense.

The Grade 3 girl knew that two types of subtraction tasks exist. She knew of more than one strategy to find answers for basic facts. Throughout the interview, she appeared very confident, was more than willing to talk about her knowledge, and expressed herself in a charming and unique way. When asked to find the answer for 56–23, she recorded it vertically, and the following exchange took place (T—teacher; S—student):

T: Why did you write it like that?
S: That's the way I write it.

T: Why?
S: It's an easier way to write it.

T: What makes it easier?
S: Nothing really—just an easier way.

T: So where do you start?
S: With the six and the three.

T: Why do you start on that side?
S: Because it gets sort of complicated if you start there (the tens), because sometimes the three could be here (at the top) and the six could be here and you would have to add it. But you start with this one, then you don't know if you have to add or not (records answer 33).

T: Let's do another one (62 – 27 = is presented). What do you put here (at the end), a box or a line?
S: Nothing, I just put the answer there.

T: Show me how you would find the answer.
S: Do it the same way I did before?

T: Yes.
S: (writes $\begin{array}{r}62\\-27\end{array}$.)

This paper is based on a presentation made at the Mathematics Council Conference in Edmonton in 1991.

T: I would like you to talk to me while you are doing it.

S: OK. Two take away seven you can't do it so you cross the six and put a five above it, and you put a ten there so it gets to twelve. Then you add seven to twelve and that's four, no five (records five), and then you take away the five to the two and that's three (records three in tens place).

T: How did you know this was five (the five in the ones place is pointed to)? How did you figure that out?

S: I just go from seven to twelve.

T: Show me how you do that.

S: I go from seven (seven said with emphasis and holds up one hand and begins to count showing one finger for each of) eight, nine, ten, eleven, twelve (five fingers are held up).

T: So you count up?

S: Yeah. I counted up instead of down. But if it's like 12 take away 1, just . . . or, if it's like 12 take away . . . just pretend I don't know the answer . . . if it's 12 take away 2 . . . and just pretend I don't know (giggles) . . . 12 take away 2 and I take away . . . and I go from 12 down to take "awaying" 2 . . . like 12, 11, 10 . . . but since it's not that I am not going to do it, since it's like a higher number than 3 or 4 I won't do it.

The Grade 5 boy was a learning disabled student. He did not find it easy to talk about his knowledge. Long pauses existed between questions and answers, and between parts of the same answer. During the major part of the interview on multiplication, he kept confusing the terms *multiplied by* and *divided by*. Both verbal interpretations were used for the multiplication symbol.

When the interview dealt with the basic multiplication facts, the student classified the items as easy and difficult. On completion of this task, answers for the facts identified as easy were solicited. All responses were correct, including the answers for 8 x 9 and 6 x 9. The answers for the facts identified as difficult were not known. However, the student was willing to "make an estimate." The following conversation was part of this setting:

T: If you forget an answer or don't know an answer, what would you do (7 x 8 was selected)?

S: I'd work on it.

T: How? How would you work on it?

S: I would go 9 x 7 equals 63 – 8.

T: That's clever. Did you teach that to yourself or did someone show that to you?

S: I teached it to myself.

T: (The flashcard showing 8 x 8 was selected.) What do you think is close to the answer for this?

S: 60.

T: How would you work on this one?

S: Ah, . . . I'd turn that to nine and I'd divide it by seventy-two . . . I mean the answer is seventy-two and I minus it by eight.

T: And you taught this to yourself?

S: Yes.

T: That's fascinating. Let's pretend you forgot the answer for 7 x 6 (selected from the difficult group). You tell me how you would figure it out.

S: How to figure it out?

T: Yes.

S: I'd make that a five (the six) and keep that a seven and I'd . . . and it would be . . . thirty-five, and I'd add a seven onto it.

During the interview dealing with decimal fractions, the boy from Grade 7 appeared confident and was very willing to talk. While the understanding of decimals, and then addition and subtraction were dealt with, part of his mathematical behavior seemed to be rule bound. Reasons were given for most of the students' rules. Other rules simply existed. For example, when the addition of "ragged" decimals (0.86 + 0.4 + 2.0 + 6.125) was discussed, he insisted that "you always write the biggest numbers first. You just do!" As the discussion turned to multiplication, the following dialogue took place:

T: Have you ever multiplied decimals before?

S: Yes.

T: Without finding the answer, try to tell me something about the answer for 0.7 x 0.8.

S: Oh . . . it's just like a normal, just 7 x 8 is 56, but when you multiply decimals . . . you've got to . . . like . . . when you have got the answer . . . you've got to take how many places it is in the whole question . . . two places to the left . . . so 56 is the answer . . . and then you go (begins to write) . . . you go 0.7 and 0.8 . . . you just forget about the decimals . . . and you get

56, but there is 2 places—so you got to go 1 . . . 2 (counts from right) put the decimal there and your answer is 0.56.

T: Is that a rule you were taught?

S: Yes.

T: How would that rule work for 3.05 x 0.9?

S: OK . . . yeah . . . it doesn't matter where the decimal place is. You don't have to line them up. You can just go like (writes 3.05 x 0.9 and multiplies to find the answer, counts decimals places and records 2.745).

The request to place a decimal point into the answer for 15.5 x 8.24 = 12772, resulted in the immediate response, "Right here (12.772) . . . because 8.24 has two decimal places in the number and 15.5 has one place . . . though . . . 8 x 15 = 120 . . . so it would go there—127.72. Oh . . . yeah!"

It was fascinating to note that division of decimals was dealt with by changing every decimal numeral to a fraction numeral. The rationale given was "That's how it is done!"

Why do the student teachers suggest that some of the students' responses indicate the presence of number sense? Do you agree? If so, which responses do you think indicate the presence of number sense? Do any of the responses indicate a lack of number sense?

Number sense is a new expression. Most teachers-in-training would likely not have encountered it as part of their training or by paging through mathematics methods texts (published prior to 1990). That is not to say that these books do not deal with the notion of number sense.

According to the *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989, 39–40),

> number sense is an intuition about numbers that is drawn from all the various meanings of number. It has five components:
>
> 1. Developing number meanings
> 2. Exploring number relationships with manipulatives
> 3. Understanding the relative magnitude of numbers
> 4. Developing intuitions about the relative effect of operating on numbers
> 5. Developing referents for measures of common objects and situations in their environment

Developing number sense is not just an important component of Standard 6 for K to 4: Number Sense and Numeration (ibid., 38–40), but also is part of Standard 12 for K to 4, which states that students should develop number sense for fractions and decimals (p. 57). For Grades 5 to 8 (Standard 5: Number and Number Relationships), it is proposed that "the mathematics curriculum should include the continued development of number and number relationships so that students can develop number sense for whole numbers, fractions, decimals, integers, and rational numbers" (p. 87).

As suggested, the expression *number sense* may be new, but important ideas related to the notion have been dealt with by different authors under different headings, for example, *concept of number*; *understanding numbers*; Howden (1989) talks about Wirtz teaching *friendliness with numbers*; *making numbers come alive* (May 1978; Liedtke 1983); many primary teachers talk about attempting to develop *a feeling for numbers* or they comment that some students lack such a *feeling*.

Whatever number sense or the intuition about numbers and relationships is defined as, authors who have written about it and teachers who attempt to teach it would agree with Thornton and Tucker (1989) who state that it develops over time. The authors propose that for some students number sense develops and matures naturally but "for others it will happen only if the teacher plans ahead to be ready to capture the opportunity of the moment, in the mathematics lesson and beyond" (ibid., 21). This planning by the teacher "should begin in the first grade with appropriate tasks that call on number sense and give children a less mechanical view of mathematics" (Markovits, Hershkowitz and Bruckheimer 1989). As a matter of fact, it would be advantageous if this view of mathematics and a focus on number sense would be part of a child's preschool experience. (This viewpoint will be discussed when a few specific examples and strategies are discussed.)

What about number sense as part of a theoretical framework? Van de Walle (1990a, 12) reminds us that "all theories are just that—theories." The author points out that "if, however, a theory of learning is found to be useful in effectively helping us to be better teachers, then a theory is worthy of consideration." According to Van de Walle, a cognitive theory can heighten teachers' awareness to integrate new ideas with existing knowledge and provides the basis for developmental teaching. The objective of teaching mathematics developmentally is relational understanding. The goal of relational understanding is described in terms of a three-part objective: well-integrated conceptual knowledge (concept, relationships), well-developed procedural knowledge (symbolism, rules, procedures) and clearly developed connections between concepts and procedures.

Any attempt to define Van de Walle's terms *well integrated*, *well developed* and *clearly developed* would likely make it difficult, if not impossible, to do so without referring to number sense. Perhaps relational understanding and number sense are to a degree synonymous, or at least number sense is an important component or subset of relational understanding. The observations that make student teachers suggest that some evidence exists for the presence of number sense for the three students who were featured in the interview excerpts usually fall into a category that includes statements labeled well integrated, well developed and/or clearly developed.

As part of a chapter on "concepts of number," Van de Walle (1990b) presents a list of reasons for an added emphasis on number sense. This list includes the point that "number sense contributes directly to problem-solving abilities and flexible thinking in numerical situations" (p. 63). According to Van de Walle, "without a major commitment by a curriculum to experiences that develop number sense, many children will never understand number sense in any other way than by counting" (p. 64).

What are some general as well as specific teaching-learning strategies and settings that are conducive to the development of number sense? Van de Walle (1990a, 18–19) identifies hallmarks of teaching developmentally. The characteristics of this approach that could be considered especially important for the development of number sense include

> understanding that existing ideas give meaning to new ones; encouraging children to talk about concepts and relationships; using manipulative models as a major tool to create linkage between conceptual and procedural knowledge; capitalizing on children's oral language in the promotion of relational understanding; being careful to see that conceptual knowledge is developed prior to the introduction of symbolism; and avoiding an overemphasis on mindless drill.

The authors who wrote articles for the *Arithmetic Teacher* focus issue on number sense mention many ideas that are related to teaching strategies and classroom settings. These include "doing mathematics" in an environment that fosters curiosity and exploration (Howden 1989, 11); exposing children to "messier" aspects of everyday problem solving and placing more emphasis on thinking about various procedures that can be used to solve a problem and on interpreting the answers that these procedures produce (Hope 1989, 16); promoting discussion through appropriate questioning techniques and planning for the development of number sense for all parts or throughout the lesson (Thornton and Tucker 1989, 19 and 21); having students discuss the application of number-sense concepts to a word problem to contribute to the understanding of the "hows" and "whys" of numbers (Dougherty and Crites 1989, 25); creating a climate that encourages pupils to ask "why" (Whitin 1989, 29); selecting activities that give students the opportunity to verbalize relationships that demonstrate the acquisition of good number sense (Glatzer and Glatzer 1989, 38); revising measurement-based curricular applications to take actual measurement practices into account, encouraging students to work on tasks in groups of three or four, relating the operations of arithmetic to real-world models, making children aware that not every mathematics problem has a single correct answer, enlisting students to help uncover "old-fashioned" or unrealistic material in their textbook, and challenging students to make generalizations (Kastner 1989, 46); provoking each pupil into constructing his or her own knowledge of numbers and the relations among them (Ross 1989, 50); selecting appropriate activities conducive to the development of number sense (Markovits, Hershkowitz and Bruckheimer 1989, 55).

The characteristics of teaching developmentally suggested by Van de Walle and the list of hints from the different authors clearly point to the importance of language or the creation of a teaching environment where students are given an opportunity to talk as they exchange ideas. This role of language is supported in Standard 2: Mathematics as Communication of the *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989). It is advocated that students in Grades K to 4 should be able to "relate their everyday language to mathematical language and symbols" and to "realize discussing, reading, writing and listening to mathematics are a vital part of learning and using mathematics"

(p. 26). Students in Grades 5 to 8 should be able to "discuss mathematical ideas, and make conjectures and convincing arguments" (p. 78). The goal that students can "reflect and clarify their thinking about mathematical ideas and situations" is included for Grades K to 8 (pp. 26 and 78).

Many or most of the teaching strategies that have been advocated and the suggested focus on language can best be accommodated in an environment where students cooperate with each other. As Willoughby (1990, 58) suggests, "Mathematics is not a solitary activity. It should be done and learned with others. . . . This was true ten years ago, it is true today, and it will be true in ten years—whether or not cooperative learning happens to be in vogue."

This emphasis on talk or on students talking does by no means imply increased passivism by a teacher. To get students to clarify or modify their thinking as they are involved in mathematical situations, teachers need to listen carefully as they attempt to accommodate responses at various ability levels. An interactive environment is required where leads change back and forth from student to student as well as from teacher to student. In terms of a two-dimensional teaching model consisting of four quadrants (Calkins 1986), where low and high student input is plotted along the vertical axis and low and high teacher input along the horizontal axis, it is likely that the goals from the *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989) related to language and thinking can best be met in a setting advocated for the quadrant of high-student—high-teacher input.

A synthesis of the different suggestions for teaching ideas that have been made by the various authors points to the importance of appropriate teaching and to the teacher's role as the key to the acquisition and/or development of number sense. Telling and showing may be part of a mathematics lesson, but as Ross (1989, 50) reminds us, "When a teacher shows students something, students do not have to think; they simply follow directions."

Romberg (1990, 472) states that "the notion that mathematics is a set of rules and formalisms invented by experts, which everyone else is to memorize and use to obtain unique, correct answers, must be changed." Teaching about and for problem solving are parts of mathematics lessons; however, teaching via problem solving (Schroeder and Lester 1989) and via thinking is essential when the building of number sense is the goal.

Howden (1989, 7–8) states that "children discover new relationships and properties with numbers when they use concrete materials." Manipulative materials and models by themselves are likely not to contribute to the development of number sense. To design the "mind on" setting that is required, a knowledgeable teacher is essential. To develop a "teaching-via-problem solving" and a "teaching-via-thinking" atmosphere, to "capture the opportunity of the moment" as Thornton and Tucker (1989, 21) suggest, a teacher has to be there to observe, to listen and to phrase as well as to place appropriate questions to students or groups of students. Delicate orchestration by a competent teacher is a necessary element of mathematics learning involving materials and models.

The proposals for appropriate settings these various authors make leave no doubt that teaching for number sense requires careful planning and a lot of effort. Because cooperative settings involve a lot of talk by students, teachers have to learn how to become good listeners. It is difficult to imagine how the development of mathematical power, which is central to the *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989) and includes thinking, talking, connecting and problem solving, could be evaluated without making observations or without conducting interviews. Aligning the curriculum with methods and tasks for assessing students' learning implies that oral questions, oral presentations and student interviews become integral parts of these methods (ibid., 200–201). This alignment is required when the development of number sense and the development of mathematical power become an important part of the mathematics program.

Assessment of development of mathematical power and number sense requires structured interview settings. Because interviews involve probing into how students think as well as how they think about their thinking, these settings represent research at the highest possible level. Teachers will require training and experience to conduct these interviews appropriately and efficiently (Liedtke 1991).*

The last part of this paper describes several activities to illustrate or reinforce a few of the points

*Ten one-hour videotapes entitled "Diagnosis and Intervention in Mathematics" are available from Education Extension, Faculty of Education, University of Victoria, Box 3010, Victoria, B.C. V8W 3N4. Clinical interview tasks and settings are described, illustrated and analyzed. Intervention strategies and activities are also discussed.

that have been raised. A variety of examples are included in the *Arithmetic Teacher* focus issue (February 1989) on number sense.

Readiness for number sense, or the notion that mathematics deals with ideas or problems that do not have to have just one specific answer, can begin in the preschool setting or in the home. For example, when young children are asked to sort objects or pictures of objects (Carson and Lindsay 1972), they are often told the categories of classes that are to be considered such as food or clothing. It is simple to create a problem solving setting by asking children to respond to requests like Which of these do you think are in some way the same? Put them together and tell me how they are the same. Think of other ways of putting things together that are in some way the same. Try to guess how and/or why everyone else (the teacher) puts things that are in some way the same together. The intent would be to get children to find many common characteristics for a given group of objects and to enable them to discover new ways of thinking about and new ways of talking about these familiar objects. This flexibility should make it easier for children to move from a scheme of classes based entirely on perceptions to schemes based on other criteria such as number.

The idea of "there are many ways of *solving problems*," and Willoughby's (1990, 51) reminder that "there is generally more to be gained by solving a single problem in several ways than by solving several problems in a single way," can be permeated while examining patterns or ordered sequences. It would be advantageous for young children to realize that the questions, What comes next? for [red][red][blue][red][red][blue] or for  can be answered in more than one way. Young children need to realize that different number names can be chosen from 1, 2, 3, 4, 5, 6 . . . as a response to, What comes next? for 1, 2, 3, $\square$ as long as a valid reason can be provided. (A teacher or an interviewer may have to judge whether or not the answer, "Six, because I am six years old" is a valid reason!) Listening to others justify their answers can initiate a search for other responses.

An examination or a study of numbers should include activities that lead students to conclude, for example, "Eight—You Are Beautiful!" (Liedtke 1983) or "Six—You Can Be Different." The idea is to present tasks and to initiate discussions to make young children realize that numerousness is independent of color, shape, size and arrangement. They need to realize that "as long as we can match a given number of fingers with an arrangement of objects, the same number name and symbol are assigned, no matter what characteristics these objects display" (ibid., 34).

Finger-flash activities can contribute to the development of number sense. For example, for numbers five and less, children are asked to say the number names (without counting) for displays of any combination of fingers—  ;  ;  ; . . . This can easily be changed into a tactile experience by having a student who has closed his/her eyes attempt to determine (without counting) how many fingers a partner is displaying.

The number of fingers on one hand represents an important benchmark for young children. For each of the possible arrangements of fingers on one hand, students should be able to recognize and state (without counting) the number shown as well as how many more it takes to show five, that is, for  three and two.

The same type of finger-flash activities that have been described for five and fewer fingers can be extended to the fingers on both hands. For any given arrangement of fingers, the students' goal should be to state how many are shown and then how many more fingers it would take to show ten.

After the numbers to 10 have been examined, young children could be faced with an activity where an arrangement of counters, such as 6, is briefly shown, either via an overhead projector or by showing a card of dots. The exposure is too brief to allow children to count. They are asked to indicate in some way, such as thumb up, thumb down (Reys, Suydam and Lindquist 1989) and closed hand, whether they think they saw more than, less than or about the same as a given number (that is, 5), respectively. Reys, Suydam and Lindquist (p. 77) suggest that numbers like 5 and 10 could be used as early benchmarks because they "are internalized from many concrete experiences, often accumulated over many years." A natural extension of this type of activity to benchmarks of 20 or 100 is possible in later years.

Thornton and Tucker (1989) discuss the contribution "lesson warm-ups" can make to the developing of number sense. One Grade 1 teacher begins many of her mathematics classes with a guessing game. For example, "I have recorded a number between five and twenty-five. Guess my number." Several simple ideas can add to the effectiveness of this type of activity:

- Projecting a hundred chart onto the chalkboard allows children to indicate their responses to What number could it be? and What number could it not be? by circling and crossing out numbers, respectively after each guess or response.
- Recording each student's guess, or a few key words of the guess on the chalkboard, will allow students to consider previous information as they attempt to phrase new guesses.
- Introducing the rule for posing questions that you may not say, Is it ______ (name of number)? will force children to think of and use some of the terminology they have encountered in the mathematics classroom.
- The rule that each question has to be of a different type from those asked previously can create a challenge, which initially can best be accommodated by having students in teams of two and then providing a little time to discuss possibilities prior to the posing of the question.

After students have learned to group by 10s and 1s to find the answer to, How many? a group of precounted objects greater than 10 is shown via an overhead projector. The projector is turned off before students can count the objects. They are asked to record 2 responses, an estimate to the nearest 10 (is about ______ 10s) and the other a lucky guess of how many they thought they saw. Although strategies are discussed and compared, students' responses with respect to these strategies are not evaluated or categorized (that is, as good, better or bad strategies). Neither are the estimates evaluated. The correct answer is announced, and the procedure is repeated with a different group of objects. One interesting observation consists of finding out how well some students are able to use the previous task(s) as a benchmark. Initially, a group of 10 could be included as a benchmark in the corner of the overhead display along with the objects to be considered.

Some authors who have written about number sense suggest that estimation contributes to number sense. It can be argued that without the presence of number sense, it would be very difficult, if not impossible, to teach students how to estimate. Without number sense, estimation is likely to be reduced to a procedural skill rather than becoming an important part of thinking. Perhaps it is safe to say that a certain degree of an understanding of number and the relationship between numbers is required before estimation can contribute to the continual development of number sense.

Thornton and Tucker (1989, 19) advise that, "lessons can be created that interweave learning about computation and number sense." As teachers plan, they need to ask themselves how it is possible, for example, to teach the basic facts, the algorithmic procedures for the four operations or measurement keeping in mind the importance of number sense. What kind of activities will contribute to the development of "operation sense" and "measurement sense"? What kind of activities and games can and will contribute to further the development of number sense? I hope that some of the ideas in this article will inspire you to provide an environment with a focus on the development of number sense and to actively search for strategies and activities to reach this goal. Such an environment will require special planning and an atmosphere that encourages children to talk and to take risks. It will require a setting where children feel good about themselves and feel good about the mathematics they know and the mathematics they are learning.

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