# Computer-Based Mathematics Notebooks

*J. Dale Burnett*

The original version of this article was created using the software package Mathematica 2.0 on a Macintosh IIci computer. The article is an example of a Mathematics Notebook. Such notebooks consist of two interrelated components. On the one hand, there are some personal reflections and comments made by the learner (me) as I proceed. On the other hand, there are some mathematical activities, where I use the computing features of the software to "do" some mathematics.

## Technology and Mathematics Education

*It's okay to use a computer/calculator in math class.*

I find the above statement offensive. On the surface it is innocent enough, and it was probably offered in a well-intentioned manner. Furthermore, I am a strong advocate of increased use of technology in most spheres of human inquiry and, in particular, of its use in education. Clearly, I am in favor of computer-augmented mathematics. The difficulty with the statement lies in the word "okay." The implication is that of a minimum incorporation of the technology into the curriculum. It is "okay" to use the technology to relieve us of some of the tedious drudgery associated with complex calculations, or it is "okay" to find the value of a certain trigonometric function. It may even be "okay" to obtain the graph of a new function, or "okay" to do some basic statistical computations with a set of student-collected data.

However, technology has a much greater role to play in education (Forman and Pufall 1988). Technology affects both pedagogy and content. It provides us with totally new methods of representing and exploring mathematical topics. Examples include the use of spreadsheets, curriculum-specific software packages, such as The Geometer's Sketchpad, and programming languages, such as APL and Logo. Such possibilities not only affect our basic problem solving processes but also our pedagogy. Dynamic computer-based demonstrations on a large screen are dramatically different than "chalk on a blackboard" or "markers on an overhead master." To be in mathematics education today means being intimately familiar with a number of new tools, tools that did not exist a few years ago. Even more exciting are the new mathematical topics that are being created by the technology—topics that are not possible without computer support. Topics such as fractals and nonlinear dynamics are now within the range of teenagers, if we open the doors.

Michael Fullan is one of Canada's leading authorities on educational change and innovation. Fullan (1991) distinguishes between first-order change and second-order change. First-order change involves improving the effectiveness of current practice. Second-order change involves altering the fundamental goals and roles of a particular situation. Computer technology will have a substantial impact on mathematics education at both levels. Fullan also notes that all real change involves loss, anxiety and struggle. In a novel about present-day life, Coupland (1991) uses the phrase, "Adventure without risk is Disneyland." The choice for mathematics education is clear—we do not want a Disneyland-curriculum. For mathematics educators, the years ahead will contain pain, and joy.

I recently had the joy of rereading Jerome Bruner's (1979) famous little book *On Knowing: Essays for the Left Hand*. Most practising teachers of today ran across this book during their undergraduate days. I would like to suggest that open-ended computer software that permits, even encourages, the learner to explore the consequences of ideas within a particular domain of inquiry be viewed as Tools for the Left Hand.

Let's now have a brief look at some examples of computer-augmented mathematics, then we will return in the final section with a few suggestions for the future.

# Mathematica Examples

## Arithmetic

**2+3*4**
14

**(2+3)*4**
20

**2^1000**
1071508607186267320948425049060001810561404\
8117055336074437503883703510511249361224931\
983788156958581275946729175531468251871452\
8569231404359845775746985748039345677748\
2423098542107460506237114187795418215304647\
498358194126739876755916554394607706291 4\
5711964776865421676604298316526243868372 0\
5668069376

**1/3 + 4/7**

$$\frac{19}{21}$$

**113/203 - 1/12**

$$\frac{1153}{2436}$$

## Algebra

**Factor [3*x*^2 + 4*x* + 1]**
$(1 + x)\,(1 + 3\,x)$

**Simplify [(*x*^2)^3 / (2*x*)^2]**

$$\frac{x^4}{4}$$

**((*x*–3) / (*x*^2 – 9*x* + 20)) + ((2*x* – 1) / (*x*^2 – 7*x* + 12))**

$$\frac{-3 + x}{20 - 9\,x + x^2} + \frac{-1 + 2\,x}{12 - 7\,x + x^2}$$

**Simplify [%]**

$$\frac{14 - 17\,x + 3\,x^2}{-60 + 47\,x - 12\,x^2 + x^3}$$

**Factor [%]**

$$\frac{(-1 + x)\,(-14 + 3\,x)}{(-5 + x)\,(-4 + x)\,(-3 + x)}$$

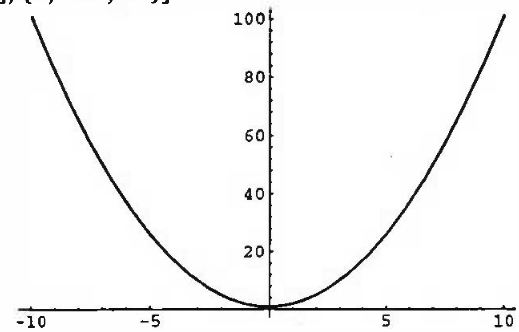
**Expand [4(*x*+1)^3]**
$4 + 12\,x + 12\,x^2 + 4\,x^3$

**Solve [*x*^2 – 2 = = 0, *x*]**
{{*x* –> *Sqrt*[2]}, {*x* –> –*Sqrt*[2]}}

**Solve [{2*x* + 3 = = *y*, *x*^2 + 3*x* + 1 = = *y*}, {*x*,*y*}]**
{{*x* –> –2, *y* –> –1}, {*x* –> 1, *y* –> 5}}

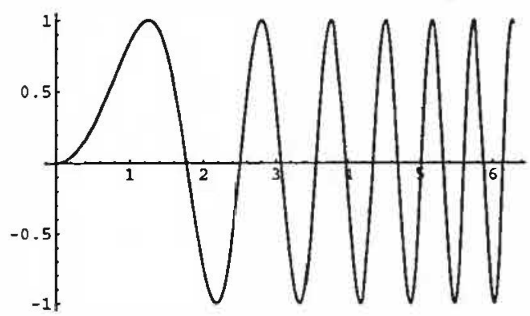
## Functions

***f*[*x*_] := *x*^2 + 1**
**Plot [*f*[*x*], {*x*, –10, 10}]**



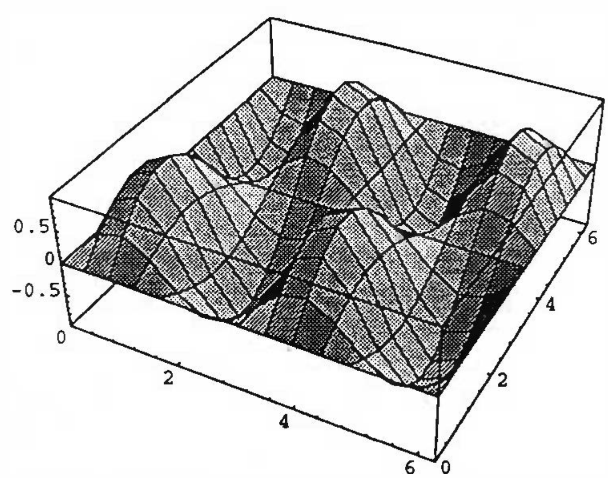

-Graphics-

**Plot[*Sin*[*x*^2], {*x*, 0, 2*Pi*}]**




-Graphics-

**Plot3D [*Sin*[2*x*] *Sin*[*y*], {*x*, 0, 2*Pi*}, {*y*, 0, 2*Pi*}]**




-SurfaceGraphics-

■ Calculus

***D*[*x*^*n*, *x*]**
$n\,x^{-1+n}$

***D*[*ArcTan*[*x*], *x*]**
$\frac{1}{1+x^2}$

***D*[*Cos*[*x*], *x*]**
$-Sin[x]$

***Integrate* [*x*^*n*, *x*]**
$\frac{x^{1+n}}{1+n}$

***Integrate* [1/(1+*x*^2), *x*]**
*ArcTan* [*x*]

## Thoughts for the Future

It is no longer considered appropriate to learn how to compute square roots by hand, but a review of the preceding examples should make it increasingly obvious that much of what we currently attempt to do in mathematics education is also likely to have little import in the future. "Machines compute, people think" (Smith 1992), yet a significant percentage of our curriculum is aimed at producing inferior "computing machines."

The examples of the previous section allow two different perspectives. From a negative viewpoint, they demonstrate the futility of having students learn procedures that are already much more efficiently handled via technology. A positive perspective suggests that when computing power is combined with human reasoning, as in the construction of mathematics notebooks where personal annotation accompanies the results, then we are likely to have a clearer picture of the "why" underlying the learner's efforts.

Fullan (1991, 19) says in the preface to his book, "If we know one thing about innovation and reform, it is that it cannot be done successfully to others." This implies that mathematics educators must assume responsibility for their own future. The Educational Testing Service (ETS) (1992, 19), a major American educational research organization, in a recent review of mathematics education, identified the need for long-term, long-range planned training: "People will ask for a three-day workshop for teachers to train them in the new material, but that is not even in the right ball park. Our people have worked with some teachers for seven years now." Canadians are familiar with the idea of prolonged commitment—witness our efforts at constitutional reform.

In the same report, ETS addresses the issue of teacher expertise: "The teachers will be learning too, but in doing so, they can show their students that learning is not just a child's activity, it's every person's activity. I think if students realized the teachers are learning, just as they are trying to learn, it would set a different notion of what teaching and learning is" (p. 20). Ashton and Webb (1986) report on a multidisciplinary study of teacher efficacy. They document the "uncertainty, isolation, and a sense of powerlessness" that affects a teacher's sense of self-esteem. In many ways, teachers are reactors rather than pro-actors.

Let me make a soft suggestion. Perhaps a new orientation toward a more proactive attitude, where mathematics teachers assert more control over what *they* learn, is required—an orientation with an increased sense of professionalism. And I would like to further suggest that becoming a tool-user, particularly a left-handed tool-user, might be worthy for such attention. Finally, the construction, *by teachers*, of mathematics notebooks could be a first step along this new path.

Let me illustrate with two possibilities. I am currently playing my way, using Mathematica, through an introductory book on the mathematics of chaos theory (Devaney 1990). I have created about a dozen mathematical notebooks, each between 10 and 20 pages in length, as I explore various possibilities while examining a variety of mathematical functions under repeated iteration. I am learning both some features of Mathematica and a relatively new topic in mathematics: nonlinear dynamics. I see no reason why junior and senior high school students could not handle both topics. A second possibility is the mathematical modeling of various physical and economic situations. The sequel to *The Limits of Growth* (Meadows et al. 1972), titled appropriately, *Beyond the Limits* (Meadows, Meadows and Randers 1992), shows what is possible while constructing world models on the computer. Once again, these are topics that could be candidates for the school curriculum. But first we teachers need to become familiar with the possibilities.

There is much that is new, much to change. Developments will continue not only in technology but also in all other domains of human interest and expertise—psychology, philosophy, the arts, the humanities, politics, medicine, ecology, economics, sports and religion. Not only is planet Earth hurtling through space at a dizzying speed, but so are our mental constructions. Mathematics education must shoulder its share of obligations.

## References


Ashton, P.T., and R.B. Webb. *Making a Difference.* New York: Longman, 1986.

Bruner, J. *On Knowing: Essays for the Left Hand.* Cambridge, Mass.: Harvard University Press, 1979.

Coupland, D. *Generation X.* New York: St. Martin's, 1991.

Devaney, R.L. *Chaos, Fractals, and Dynamics.* Menlo Park, Calif.: Addison-Wesley, 1990.

Educational Testing Service. *The Metamorphosis of Mathematics Education.* Focus 27. Princeton, N.J.: ETS, 1992.

Forman, G., and P.B. Pufall. *Constructivism in the Computer Age.* Hillsdale, N.J.: Lawrence Erlbaum, 1988.

Fullan, M.G. *The New Meaning of Educational Change.* 2d ed. Toronto: OISE, 1991.

Meadows, D.H., D.L. Meadows and J. Randers. *Beyond the Limits.* Toronto: McClelland and Stewart, 1992.

Meadows, D.H., D.L. Meadows, J. Randers and W.W. Behrens. *The Limits of Growth.* London: Pan Books, 1972.

Smith, D.A. "Questions for the Future: What About the Horse?" In *Symbolic Computation in Undergraduate Mathematics Education*, edited by Z.A. Karian. Washington, D.C.: The Mathematical Association of America, 1992.