delta-k

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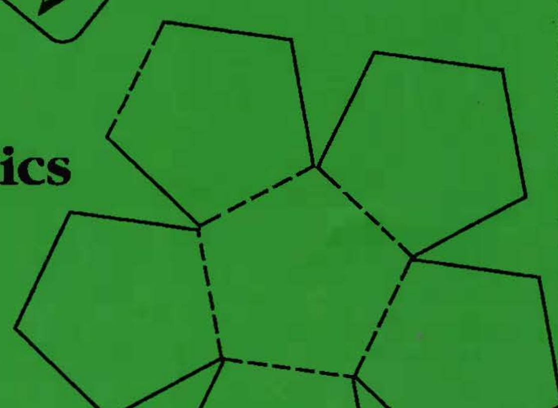
In-Between Hits

- **Computers in the Mathematics Classroom**
- **Recreational Mathematics**
- **Teaching Ideas**



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CONTENTS

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EDITORIAL

By the time you receive this issue of *delta-K*, you will undoubtedly be back at school after a well-earned summer break. I hope your summer was enjoyable and restful, leaving you ready for an exciting and challenging new school year!

This issue of *delta-K* focuses on using computers and calculators in the mathematics classroom. At times, it seems difficult to know how to use these technological devices effectively in our classrooms. Because they are so common in our general society, they will inevitably become part of our teaching reality. But this realization does not help us learn more about how to employ them effectively. After all, tallying grades using a computer seems distant from designing and developing learning opportunities for students using the same computer. And, merely amusing children with computer or calculator games does not guarantee that they will benefit from the experience. These issues and many more are pursued in the pages ahead.

Our leadoff article was contributed by Dr. Marlow Ediger who investigates several issues surrounding implementing computers in mathematics classrooms. His suggestions may form a helpful mind-set for teachers interested in incorporating computers more fully into their instruction.

Craig M. Findlay talks about computers from the perspective of one highly familiar with these machines. He describes some surprising research findings that contradict his prior perceptions.

J. Dale Burnett has contributed a summary of his presentation at the 1992 Mathematics Council annual conference in Medicine Hat. In his article, he introduces us to the Mathematica program and to the implications that this program and others like it have for the mathematics curriculum.

John G. Heuver leads us through two investigations of interesting mathematical problems, using the computer (and the Pascal programming language) as his context for exploration.

This issue has two articles in the recreational mathematics section. The first article by Bonnie H. Litwiller and David R. Duncan investigates the probability of certain card combinations in a dealt hand. In the second article, Sandra M. Pulver describes a summer-time activity designed for investigating familiar polyhedra.

The final section deals with some teaching ideas. The first article by Geri Crossman explains an application in which students are encouraged to calculate the cost per serving of foods in the supermarket. She also lists several ways the home economics and mathematics curricula may be integrated. In the final article, three authors present a collection of three games. Each game centres on using calculators to teach estimation and mental computation.

Enjoy!

A. Craig Loewen

Computer Use and the Mathematics Curriculum

Marlow Ediger

The number of computers in the classroom setting has increased in the past few years. Reviews of software in mathematics reveal numerous deficiencies. Much needs to be done to increase the effectiveness of software and computer use in the teaching of mathematics. This article describes weaknesses in software for the mathematics curriculum and proposes remedies.

Diagnosis of Problems

Software emphasizing mathematics objectives should not stray from significant ends in this important academic area. If an integrated program is emphasized with another discipline, such as political science, significant learning in mathematics may be greatly minimized. Producers need to realize that software emphasizing content in mathematics needs to stress quality, scope and sequence. The student may lose sight of valuable goals in mathematics if an integrated curriculum is emphasized for its own sake.

Recommendations

1. Software stressing mathematics needs to contain vital learning for students. If an integrated curriculum is inherent, adequate emphasis must be placed on mathematics as being the core of the subject matter presented.
2. Other subject matter areas may then reflect the mathematics core learning. Appropriate breadth and depth of content in mathematics must be emphasized in the software.

A second deficiency in software content pertains to emphasizing trivia. When students are asked to find the value of 5/7 of 93 = _____, the viewer wonders if the programmer considered the concept of relevancy in program development. Many major goals and objectives can be selected for students to attain. Would 5/7 of 93 = _____ be of these? No. With the explosion of knowledge, it behooves the programmer to be highly knowledgeable of subject matter for students to acquire in mathematics. The age-old question arises, "What knowledge is most worth?" This problem still exists as it did with Herbert Spencer when in 1859 he wrote an essay on that exact title. Mathematicians and mathematics educators are stimulated to seek and evaluate the most significant ends for learners to achieve.

Recommendations

1. Engage in research to select vital goals and objectives for learner attainment. Basal textbooks comprising reputable series, filmstrips, films and research study results might well provide background information for the ensuing research proposals developed by teachers and supervisors.
2. Appraise current materials used in ongoing lessons and units. Objectives, learning activities and evaluation procedures need to be assessed in terms of desired criteria.

A third weakness in computer programs is that students must respond correctly the first time to a multiple-choice item pertaining to subject matter presented on the monitor. Certainly, students should have a second opportunity to enter the correct command on the keyboard. To be sure, students may merely guess at the first chance to respond. Also, the second opportunity to respond might involve a random guess. However, the printout or the monitor should show at the end of the program how many first-response items, as well as second-chance answers, a learner got correct or if a correct response was obtained rarely.

Recommendations

Software should

1. allow for a second opportunity for learners to respond correctly within each specific program involving drill and practice, as well as tutorial learning. Mathematics can be too technical to respond correctly initially on any given program. For example, in programmed items pertaining to decimal points, the 0.1, 0.01, 0.001 and 0.0001 demand precision and exactness. Even highly responsible students can make a mistake. When estimating is involved, a learner may well do more in-depth reasoning when given a second chance, as compared with only a single response;
2. emphasize clear subject matter in deductive or inductive presentations on the monitor, prior to learners responding to receive feedback on the response; and
3. generate new questions to present content in diverse ways rather than the same subject matter and the same questions asked in drill and practice or tutorial programs.

Excess loading time can be frustrating for students. Mathematics teachers need to evaluate if programs take too much time to load, whereby the students' time is wasted and subject matter lacks sequence. Also, how much time is wasted waiting for a program to present subject matter on the screen so that appropriate learner responses can be made?

Recommendations

1. Subject matter in basal textbooks, workbooks, work sheets and a laboratory approach in achieving may be more effective, as compared with programs with the loading problem in mathematics software.
2. Time on task research may well say that software must emphasize continual progress and achievement.

Weak software either fails to reward students for correct responses or the rewards may be repetitious. Rewards need to be adequate and different and should stimulate students to achieve at a more optimal rate. Rewards reinforce a correct response to encourage learners to achieve, attain and progress.

Recommendations

Rewards should be

1. encouraging and motivational. Loud, distracting, long and lavish rewards using peripherals should be discouraged; and
2. appropriate pertaining to the involved program. The rewards must be ample and sequential. They must be related to program content. A lavish display of clowns for each correct response is disruptive, time-consuming and unrelated to the task at hand.

Software that has timed tasks have inherent problems. Slow achievers may have difficulty responding because of the extremely limited amount of time. A program should emphasize what a student can achieve. If too little time is given to read the content on the monitor and to respond, the software is self-defeating. Reasonable specific time limits must be available for students' responses.

Recommendations

Software must be

1. field-tested adequately before it is marketed, and
2. judicious in time provided for learner responses. Let the student determine the time needed to respond on the keyboard.

Software must contain interesting subject matter. All things being equal in stated goals, the more interesting the program, the more likely students will achieve at a more optimal level in goal attainment. Interest is a powerful factor in learning. Boring content has no place in the mathematics curriculum. Subject matter needs to be stimulating and dynamic.

Recommendations

1. Programmers must be aware of the principles of learning from educational psychology and incorporate desired criteria therefrom, such as interest in student learning.
2. Software must have prior testing in classrooms to determine if learners' interest has been secured. A psychological curriculum in mathematics is

evident if, from the learner's (not the programmer's) viewpoint, a program provides set establishment.

Criteria for Selecting Software

Principles of learning from educational psychology have much to offer in guiding teachers to select objectives, learning activities and appraisal procedures. These criteria may well be used for choosing software in the mathematics curriculum.

As a first principle of learning, students need to experience interesting subject matter in computer activities. Software needs to secure the learners' attention. Boring content will not facilitate the student in attaining desired mathematics objectives. Establishing the set or getting learners to attend to ongoing lessons and units is vital. Mathematics teachers need to try out software, prior to purchasing it, to notice if involved students are interested and to achieve vital goals.

A second principle of learning advocates that students be actively involved in a program. Each student needs involvement in making sequential responses to a stimulus. If learners merely absorb information from the monitor, passivity in learning is involved. Rather, students individually need to respond to subject matter presented on the monitor. After acquiring content, the learner must answer questions pertaining to ideas attained. Feedback may then be inherent in providing students with information about the correctness of the response. Thus, learners need to respond frequently to subject matter presented in each program. Based on the response, feedback to the learner is a must.

Meaningful content needs to be presented to students. With meaning attached to subject matter being pursued, students understand what has been taught. It is indeed unfortunate if a student does not attach meaning to content being read on a monitor. Certainly, to be useful, subject matter must be on the understanding levels of students. Success in learning comes about when the learner understands what has been learned and is able to achieve sequentially.

Success on the part of each student is important when pursuing a software program. With carefully prepared programs tested in pilot studies, learners should succeed in approximately 90 percent of the program's responses. Quality attitudes within learners may well be enhanced with successful experiences in the mathematics curriculum. Developing an adequate self-concept is important on the part of each student.

Students also need to perceive a purpose for learning. Reasons should be inherent when pursuing a program. A lack of motivation for achieving may accrue when a student fails to sense reasons for participating in a mathematics program. Reasons for students to participate need to be stressed in any one of the following kinds of programs:

- Drill and practice. Reasons for experiencing drill and practice need to be explained to the learner. A deductive approach is then emphasized. Or, the teacher may wish to use an inductive procedure to have learners perceive values in experiencing drill and practice programs.
- Tutorial. New sequential subject matter in mathematics is emphasized with tutorial programs. Success in learning here is enhanced when content is based on previously acquired subject matter.
- Games. Selected students may be stimulated to achieve more optimally in mathematics through enjoyment. Two to four pupils generally can be involved in computerized games. Wholesome competition needs to be evident among participants. Easier items in mathematics to respond to earn fewer points per item as compared with increasingly difficult questions. Thus, four levels of complexity of responses to questions pertaining to mathematics content could exist. Easy items answered correctly could receive five points. Increasing complex items may receive 10, 15 and 20 points sequentially. If students are evenly matched for the game, much learning can accrue within a quality learning environment.
- Simulations. With life-like experiences, problem solving and higher levels of cognition can truly be evident within the framework of simulated content in software. Several learners generally will be involved in simulation or role-play activities. Feedback to each decision made by a learner must appear on the monitor.
- Diagnosis and remediation. Quality software in diagnosis should specifically pinpoint the kind of errors a student makes. Models on the screen should show what the correct procedure would be to remedy the identified deficiency.
- Computer-managed instruction (CMI). Checking answers on computerized answer sheets is a useful, time-saving approach for teachers to use to appraise learner progress. The printout should clearly point out how many students missed each test item. Feedback might then be given to the teacher in terms of the quality of each test item,

as well as success in learning by pupils. Students' grades can also be stored on a computer. CMI has many practical uses for the mathematics teacher.

In Closing

Software and microcomputers have a significant role to play in assisting students to achieve in mathematics. Weaknesses in software and computers need to be identified and remedied. Technology must assist learners to achieve optimally on an individual basis. Computers and software, as audiovisual aids, should be used to guide each student to achieve as much as possible in mathematics.

Positive headway has been made in attempting to develop quality software for students in mathematics. Long strides still have to be made to analyze and remedy the identified deficiencies. Mathematics educators, educational psychologists as well as programmers must harmonize efforts to secure the best programs for learners. Software should not be developed for the sake of doing so. Rather, each program must assist students to achieve mathematics proficiency. Drill and practice, tutorial, games, simulations or remedial programs need field testing and necessary modifications prior to their use in the classroom. Problem solving in school and in society needs to be an ultimate goal in the teaching of mathematics. Life consists of identifying and solving problems.

Learning About Computers and Mathematics: A Student Perspective

Craig M. Findlay

We are living in an "information age," and it seems that the technology of computers is here to stay. For many, including me, the computer has become an essential tool for information storage, manipulation and output. In the last couple of years, following the purchase of my own computer, I too have been caught up in our technological era. As an educator of the future, the question then becomes, How can I integrate this technology into my profession so that it can aid in my teaching endeavors? In my research, I quickly discovered that many people before me have pondered this question. Winzer (1990, 112) says, "Computers cannot replace classroom teachers, but they are patient, consistent and accurate teaching tools that possess unlimited appeal and motivational value for students." These ideas first surfaced over 20 years ago when the computer was being billed as the educational utensil of tomorrow. In this light, the technology was proposed largely in the field of mathematics.

In our modern society, literacy refers to language, as well as mathematics (Mendoza 1989 in Winzer 1989). Bangs (1982, in Winzer 1989) reveals very specifically that mathematics is indeed itself a language. In comparison with instruction in language, mathematics has received little attention when it comes to diagnosis, instruction and remediation (Winzer 1989). Mathematics has been a domain in which I have not had a great deal of success. Therefore, when I saw a chance to learn about new ways to teach and explore mathematics in an area where I do have a great interest, computers, I obviously accepted the challenge. This becomes increasingly important when one realizes that in our rapidly changing technocratic society, people will use their arithmetic skills more than ever (Winzer 1989). Those of us lacking in this burgeoning domain will find survival even more difficult than it already is and inevitably will be left behind.

Inherent in my discussion will be the use of computers in mathematics instruction for "students" in general, although I will point out where the computer can enhance the learning of exceptional students. Lerner (1981, in Winzer 1989, 319) describes certain principles that are applicable to all forms of mathematical learning and are key ingredients to effective teaching:

> Students understand concepts best when they move from the concrete to the abstract. They need plenty of drill and practice to develop automaticity about facts and operations. Finally, they need the opportunity to see mathematics as part of the real world.

Mendoza (1989, in Winzer 1989) describes mathematics as hierarchical in nature. Thus, gaps in learners' backgrounds will go on to hinder their future successes. Drill and practice therefore become important components to mathematics teaching to promote the acquisition of fundamental facts and concepts. Many contemporaries in mathematics education would no doubt debate this, but this point still holds true for some, especially when dealing with learning disabled students. In attempting to gain a certain functional level of mathematics savvy, this traditional approach seems essential for special learners (Winzer 1989). One benefit of computers emerges with their ability to perform repetitive tasks with immediate user feedback. This can help students, especially in mathematics, who require repetition of facts and concepts. Modern computers also allow for other minor alterations that will assist special learners. For example, font sizes can be enlarged, and braille printouts can be made for visually impaired students. The speed of the presentation of material can also be altered to meet learners' needs. Some computer programs are based on "real-life" situations, making the content more functional than otherwise possible. Making education functional is vitally important to effective teaching. The graphics that modern computers offer enable users to manipulate seemingly concrete objects, making learning more genuine. On the other side of the spectrum, computer technology can help gifted students who wish to pursue more complex learning. Gifted students need expanded and enriched curricula that will stimulate higher-level thinking and will allow them to apply their skills in a variety of contexts (National Council 1986 in Winzer 1989). With this in mind, modern software is moving toward allowing the user to simulate certain ideas and concepts; opening new avenues of trial and error, exploration and higher-level learning.

In trying to understand computers and their use as an educational tool, I wanted to obtain a certain breadth of research. I chose to look at 11 different journal articles. In doing so, I obtained work from a variety of publications and from different time periods to represent as many perspectives as possible. The first article goes back to when computers were just being explored and their potential was only beginning to be forecast. The rest of the articles reflect more modern ideas and represent a transition from the computer "boom" of the 1980s to the present. The articles reflect several standpoints and highlight the computer as an increasingly important, if not controversial, instructional tool in education.

My bibliography also includes research that I have done outside of the 11 chosen articles. The concluding portion of the paper discusses the pieces in a more comparative light, recognizing that each article represents a certain aspect of computers in mathematics instruction. Finally, and in a much broader context, I have addressed whether or not the computer has lived up to mathematics teachers' expectations and to the expectations of educators as a whole.

Discussion

The research that I have read constitutes somewhat of a "jarring" experience to my previous conceptions. My appreciation of the computer had been pedestaled largely because of my own perceptions of the technology. Despite the area of mathematics benefiting most from the advent of the computer, it too has not lived up to the early expectations beset on it in the field of education.

The computer was first conceived in terms of its value to educators in the late 1960s (Zinn 1969). From that time, the technology has advanced and experienced a large amount of growth through the 1980s to the present. The focus on computer education has itself seen a shift, one I have experienced. When I was in junior high school in the early 1980s, the emphasis in computing science, as the subject was called, was on programming. We focused on learning how to program the computer to meet our problem solving needs. Today, the computer is used as a practical tool, where large innovative software designers provide us with the programs. In these modern software packages, for the most part, we are limited within the confines of the program. Demarin (1991) also sees this transgression, but from a feminist perspective. She argues that the software is somewhat limiting and suggests how software designed from the feminist standpoint, based on certain "feminine" characteristics, could eliminate many problems associated with present-day computer software.

Contradictory to the previous paragraph, proponents to certain software packages are out there. Within certain software applications, users can manipulate programs in a variety of ways and, unknowingly or not, emerse themselves in the traditional parameters of academia, including mathematics. Burnett (1987, 1988), Hoyles and Noss (1987), and Parker and Widmer (1989) have found computer applications to meet their own and, more important, their students' educational needs. These programs are the most useful and yet the most simple. Seymour Papert's Logo language as described in Burnett (1987), Land and Turner (1988), and Hoyles and Noss (1987) and the development and use of the spreadsheet as highlighted in Burnett (1987, 1988) and in Parker and Widmer (1989), are two such programs. Logo is said to be an environment that promotes "mathematizing," while the spreadsheet is billed as a notational system for exploring ideas. These authors are perhaps more optimistic about the technology than the other researchers and have worked to find feasible uses for what is available.

Johnson (1988) claims that the research is too general and that it does not reflect the problems that students encounter in their work with computers. Other research proposes remedies for the situation. Zehavi (1988) argues that we need to design software for our students' specific needs. Backing this point up, MacGregor and Shapiro (1988) reveal that we must concentrate on individual learning and cognitive styles. This is something that most computer and software technology has failed to do. Land and Turner (1988) conclude that using certain programs only reveals that they help students with higher cognitive levels. In other words, students who do well in most areas are also going to succeed in the computer environment. Researchers also discovered that low-achieving students eventually reach a certain plateau in understanding mathematical concepts with a computer program helping them. This research would support the evidence that technology can help some students, but certain people need more than just "fancy" technology. The computer can be an effective tool for some students in specific situations, but to tap into its true effectiveness, more emphasis needs to be placed on computer use with "individual" needs in mind. Computers and computer programs cannot be seen as generic, no more so than can individual students in any given classroom. Parker and Widmer (1989) stress the importance of the teacher in the computer equation. They argue that the teacher must be responsible to students by identifying and selecting appropriate applications to be used in the classroom. Johnson (1988) outlines an additional concern about the use of computers by pointing out a situation where he saw the computer become an educational crutch to a student. Some educators are really concerned that students might become dependent on the technology, robbing them of their own intuitions and problem solving abilities (Zehavi 1988; Demana and Waits 1992).

Using computers does not come without costs. Zinn (1969) forecasted problems surrounding the cost of computer technology. Demana and Waits (1992) highlight similar modern-day concerns. They argue that there is too much pressure on students and educators alike to purchase and implement expensive computer systems. They go on to suggest that other forms of technology are much cheaper while still meeting the same instructional needs. For example, graphing calculators can aid secondary students with more complex mathematical concepts and related exercises. Today's students live in a society filled with innovation and technical gadgetry. Most students are engulfed in worlds of multimedia presentation (for example, television and videos) and video games. I fear that the novelty of the computer and computer software will eventually fade in the eyes of students. Many students gain motivation from using technology, and it is therefore up to the teacher, not the computer, to keep student interest and involvement (Johnson 1988; Demarin 1991 and Zehavi 1988).

Duguet (1989) discusses the problem that the education field has faced with computer applicability; an obvious gap has existed between the hardware and the software. The main argument is that educators do not know enough about how students learn or exactly what they learn when they interact with computer-based materials. A review board or an organization needs to be established to study and screen software. The market is flooded with computer technology, and teachers cannot be expected to keep on top of it all. The international Organization for Cooperation and Economic Development (OCED) has started to set up such educational review centres. Statistics presented by OCED reveal that in mathematics only 49 percent of the software was recommended for use by teachers. Of the 457 software packages reviewed, only 223 were recommended (Duguet 1989). This presents an obvious problem for teachers *and* their students.

Two other articles of interest relate directly to the use of computers and computer software for special learners. Eiser (1986) discovered that few, if any, software titles are labeled as special education. This does not mean that the technology cannot be used for this portion of the population, but rather, modifications need to be made. Special educators need to look for two things in computer software. First, the programs need to be flexible and modifiable, and second, the software needs to have a record keeping option so that teachers can monitor student progress. These software attributes are a good indicator of software effectiveness in all realms, not just for special learners. Perhaps the most encouraging research that I discovered, in terms of special education, came from Divoky (1987). The Apple Computer Company announced the establishment of a National Special Education Alliance (NSEA). This organization provides resources and information about computers and other technology to the disabled population. Apple has also established an awareness program in its development of hardware and software. Serious efforts are being made to eliminate any obstacles to special learners. Little things like making the repeat key optional with an on-off switch, which will help students with motor skill disabilities. Divoky (1987) lists the standard and special features offered to computer buyers.

Three main points contribute to the apparent dilemma that educators face regarding the use of computers in education:

1. Computers are a rapidly changing area of technology. Today's hardware and software will almost inevitably be obsolete in five years. This begs the question, Why get involved in an obviously unstable situation?
2. The expense of computer technology is staggering, especially in light of the rapidly changing nature of the industry. Personal and/or school involvement demands a great deal of time and money, in terms of training and in hardware and software purchases.

3. Computers pose that threat of the unknown and symbolize "change," which many veteran professionals and laymen alike are weary of. Not understanding something can make people avoid and ignore it, creating "computer anxiety." The computer is another stepping stone we have yet to conquer in everyday life, as well as in education.

Computers are indeed going to be part of my educational career. Too much valuable technology exists out there that has yet to reach its full potential. There are of course concerns as with anything innovative, especially in such an important facet of society. We must remember though that education is the pathway to our future. Technology has began to take over and navigate our journey. In 15 or 20 years, I will look back and laugh at the archaism of the instrument on which I composed this article. Change is inevitable; the real choice is whether or not you decide to jump on and enjoy the ride.

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Computer-Based Mathematics Notebooks

J. Dale Burnett

The original version of this article was created using the software package Mathematica 2.0 on a Macintosh IIci computer. The article is an example of a Mathematics Notebook. Such notebooks consist of two interrelated components. On the one hand, there are some personal reflections and comments made by the learner (me) as I proceed. On the other hand, there are some mathematical activities, where I use the computing features of the software to ''do'' some mathematics.

Technology and Mathematics Education

It's okay to use a computer/calculator in math class.

I find the above statement offensive. On the surface it is innocent enough, and it was probably offered in a well-intentioned manner. Furthermore, I am a strong advocate of increased use of technology in most spheres of human inquiry and, in particular, of its use in education. Clearly, I am in favor of computer-augmented mathematics. The difficulty with the statement lies in the word ''okay.'' The implication is that of a minimum incorporation of the technology into the curriculum. It is ''okay'' to use the technology to relieve us of some of the tedious drudgery associated with complex calculations, or it is ''okay'' to find the value of a certain trigonometric function. It may even be ''okay'' to obtain the graph of a new function, or ''okay'' to do some basic statistical computations with a set of student-collected data.

However, technology has a much greater role to play in education (Forman and Pufall 1988). Technology affects both pedagogy and content. It provides us with totally new methods of representing and exploring mathematical topics. Examples include the use of spreadsheets, curriculum-specific software packages, such as The Geometer's Sketchpad, and programming languages, such as APL and Logo. Such possibilities not only affect our basic problem solving processes but also our pedagogy. Dynamic computer-based demonstrations on a large screen are dramatically different than ''chalk on a blackboard'' or ''markers on an overhead master.'' To be in mathematics education today means being intimately familiar with a number of new tools, tools that did not exist a few years ago. Even more exciting are the new mathematical topics that are being created by the technology—topics that are not possible without computer support. Topics such as fractals and nonlinear dynamics are now within the range of teenagers, if we open the doors.

Michael Fullan is one of Canada's leading authorities on educational change and innovation. Fullan (1991) distinguishes between first-order change and second-order change. First-order change involves improving the effectiveness of current practice. Second-order change involves altering the fundamental goals and roles of a particular situation. Computer technology will have a substantial impact on mathematics education at both levels. Fullan also notes that all real change involves loss, anxiety and struggle. In a novel about present-day life, Coupland (1991) uses the phrase, ''Adventure without risk is Disneyland.'' The choice for mathematics education is clear—we do not want a Disneyland-curriculum. For mathematics educators, the years ahead will contain pain, and joy.

I recently had the joy of rereading Jerome Bruner's (1979) famous little book *On Knowing: Essays for the Left Hand*. Most practising teachers of today ran across this book during their undergraduate days. I would like to suggest that open-ended computer software that permits, even encourages, the learner to explore the consequences of ideas within a particular domain of inquiry be viewed as Tools for the Left Hand.

Let's now have a brief look at some examples of computer-augmented mathematics, then we will return in the final section with a few suggestions for the future.

Mathematica Examples

Arithmetic

2+3*4
14

(2+3)*4
20

2^1000
1071508607186267320948425049060001810561404\
8117055336074437503883703510511249361224931\
9837881569585812759467291755314682518714528\
5692314043598457757469857480393456777482\
4230985421074605062371141877954182153046474\
9835819412673987675591655439460770629145\
7119647768654216766042983165262438683720\
5668069376

1/3 + 4/7
$\frac{19}{21}$

113/203 - 1/12
$\frac{1153}{2436}$

Algebra

Factor [3*x*^2 + 4*x* + 1]
(1 + *x*) (1 + 3 *x*)

Simplify [(*x*^2)^3 / (2*x*)^2]
$\frac{x^4}{4}$

((*x*–3) / (*x*^2 – 9*x* + 20)) + ((2*x* – 1) / (*x*^2 – 7*x* + 12))
$\frac{-3 + x}{20 - 9\,x + x^2} + \frac{-1 + 2\,x}{12 - 7\,x + x^2}$

Simplify [%]
$\frac{14 - 17\,x + 3\,x^2}{-60 + 47\,x - 12\,x^2 + x^3}$

Factor [%]
$\frac{(-1 + x)\,(-14 + 3\,x)}{(-5 + x)\,(-4 + x)\,(-3 + x)}$

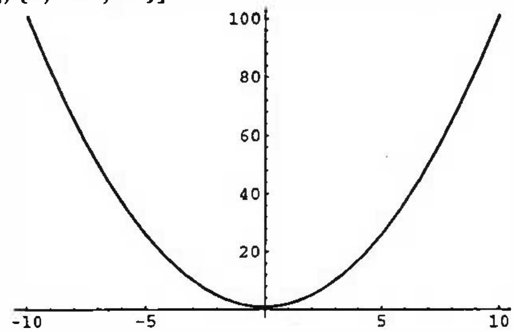
Expand [4(*x*+1)^3]
$4 + 12\,x + 12\,x^2 + 4\,x^3$

Solve [*x*^2 – 2 = = 0, *x*]
{{*x* –> *Sqrt*[2]}, {*x* –> –*Sqrt*[2]}}

Solve [{2*x* + 3 = = *y*, *x*^2 + 3*x* + 1 = = *y*}, {*x*,*y*}]
{{*x* –> –2, *y* –> –1}, {*x* –> 1, *y* –> 5}}

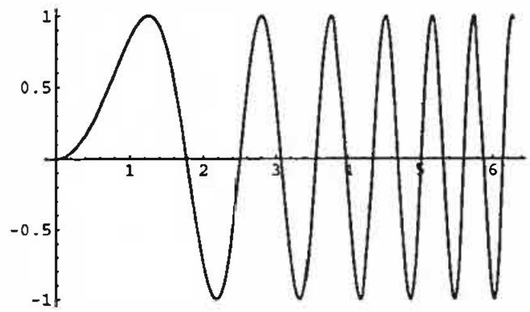
Functions

***f*[*x*_] := *x*^2 + 1**
Plot [*f*[*x*], {*x*, –10, 10}]



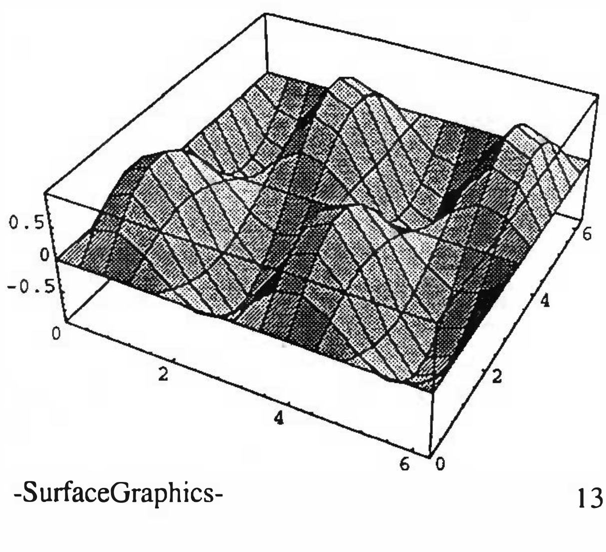
-Graphics-

Plot[*Sin*[*x*^2], {*x*, 0, 2*Pi*}]



-Graphics-

Plot3D [*Sin*[2*x*] *Sin*[*y*], {*x*, 0, 2*Pi*}, {*y*, 0, 2*Pi*}]



-SurfaceGraphics-

■ Calculus

***D*[*x*^*n*, *x*]**
$n\, x^{-1+n}$

***D*[*ArcTan*[*x*], *x*]**
$\frac{1}{1+x^2}$

***D*[*Cos*[*x*], *x*]**
$-Sin[x]$

***Integrate* [*x*^*n*, *x*]**
$\frac{x^{1+n}}{1+n}$

***Integrate* [1/(1+*x*^2), *x*]**
ArcTan [*x*]

Thoughts for the Future

It is no longer considered appropriate to learn how to compute square roots by hand, but a review of the preceding examples should make it increasingly obvious that much of what we currently attempt to do in mathematics education is also likely to have little import in the future. "Machines compute, people think" (Smith 1992), yet a significant percentage of our curriculum is aimed at producing inferior "computing machines."

The examples of the previous section allow two different perspectives. From a negative viewpoint, they demonstrate the futility of having students learn procedures that are already much more efficiently handled via technology. A positive perspective suggests that when computing power is combined with human reasoning, as in the construction of mathematics notebooks where personal annotation accompanies the results, then we are likely to have a clearer picture of the "why" underlying the learner's efforts.

Fullan (1991, 19) says in the preface to his book, "If we know one thing about innovation and reform, it is that it cannot be done successfully to others." This implies that mathematics educators must assume responsibility for their own future. The Educational Testing Service (ETS) (1992, 19), a major American educational research organization, in a recent review of mathematics education, identified the need for long-term, long-range planned training: "People will ask for a three-day workshop for teachers to train them in the new material, but that is not even in the right ball park. Our people have worked with some teachers for seven years now." Canadians are familiar with the idea of prolonged commitment—witness our efforts at constitutional reform.

In the same report, ETS addresses the issue of teacher expertise: "The teachers will be learning too, but in doing so, they can show their students that learning is not just a child's activity, it's every person's activity. I think if students realized the teachers are learning, just as they are trying to learn, it would set a different notion of what teaching and learning is" (p. 20). Ashton and Webb (1986) report on a multidisciplinary study of teacher efficacy. They document the "uncertainty, isolation, and a sense of powerlessness" that affects a teacher's sense of self-esteem. In many ways, teachers are reactors rather than pro-actors.

Let me make a soft suggestion. Perhaps a new orientation toward a more proactive attitude, where mathematics teachers assert more control over what *they* learn, is required—an orientation with an increased sense of professionalism. And I would like to further suggest that becoming a tool-user, particularly a left-handed tool-user, might be worthy for such attention. Finally, the construction, *by teachers*, of mathematics notebooks could be a first step along this new path.

Let me illustrate with two possibilities. I am currently playing my way, using Mathematica, through an introductory book on the mathematics of chaos theory (Devaney 1990). I have created about a dozen mathematical notebooks, each between 10 and 20 pages in length, as I explore various possibilities while examining a variety of mathematical functions under repeated iteration. I am learning both some features of Mathematica and a relatively new topic in mathematics: nonlinear dynamics. I see no reason why junior and senior high school students could not handle both topics. A second possibility is the mathematical modeling of various physical and economic situations. The sequel to *The Limits of Growth* (Meadows et al. 1972), titled appropriately, *Beyond the Limits* (Meadows, Meadows and Randers 1992), shows what is possible while constructing world models on the computer. Once again, these are topics that could be candidates for the school curriculum. But first we teachers need to become familiar with the possibilities.

There is much that is new, much to change. Developments will continue not only in technology but also in all other domains of human interest and expertise—psychology, philosophy, the arts, the humanities, politics, medicine, ecology, economics, sports and religion. Not only is planet Earth hurtling through space at a dizzying speed, but so are our mental constructions. Mathematics education must shoulder its share of obligations.

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Two Computing Exercises with Mathematical Overtones

John G. Heuver

It is sometimes necessary to have exercises that demonstrate the need for a sound knowledge of mathematics while teaching the intricacies of computer programming. The programming language I have chosen is Pascal, which was developed by Niklaus Wirth in the 1970s for the specific purpose of teaching programming. Pascal is being maintained and upgraded as a suitable language that works well on present-day computers. It is not overburdened with unnecessary detail. Version 7 is apparently the latest by Borland. One has to keep in mind that in general a high correlation exists between the maintenance of software and its effectiveness. The programming language C is more flexible in the sense that it allows close control of the hardware and the interaction with assembler is easier. The use of pointers in C makes programming more agile. However, to get a good insight in basic programming, Pascal is simpler because it has fewer trappings and is less cumbersome in more instances. I have chosen the following two examples simply because I am personally intrigued by the questions they raise.

The $3x + 1$ Conjecture

The $3x + 1$ problem has been attributed to Lothar Collatz. Pick a positive integer; when it is odd, multiply it by three and add one, and if it is even, divide by two, continuing the process until you end up with one. The sequence 3, 10, 5, 16, 8, 4, 2, 1 is an example. The program that follows performs this task for a number entered from the keyboard. The solution to the question of why the sequence ends with 1 or, simply put, why it ends in finite time is still beyond our present-day knowledge. Note that the entire sequence is not uniquely reversible while this is the case in the second example.

```
program COLLATZ;
var
    a:integer;
    begin
    writeln('Enter a number.');
    readln(a);
    repeat
    if odd(a) then
    a:= 3*a + 1
else
    a:=a div 2;
    write(a:5);
    until a = 1;
    readln;
end.
```

The Problem of Cups and Stones

Barry Cipra (1992, 1993) proposed the following problem. Suppose we have n cups arranged in a circle and k unmarked stones are placed in each cup. Mark these cups 1, 2, 3 . . . , n, clockwise. Pick all the stones out of the first cup, and put one in every subsequent cup moving clockwise, leaving your hand by the cup in which you dropped the last stone. Pick up all the stones in that cup, and start over again. This process is simulated in the program that follows with four cups and one stone per cup. Depending on one's type of computer, the number of cups and stones cannot be increased much beyond 10 and 4, respectively. Otherwise, it takes a long time to watch the stream of output. The questions that this problem raises are (1) why does the process end in accumulating all the stones piled up in the first cup? and (2) how many moves accomplish this process? The first question was answered and the second only solved for the case of two cups. This is not the place to consider the proofs.

```
program CIPRA;
type
container = array[1..20] of integer;
var
    cups:container;
stones, num, a, b, c, startcup, stonesincup:integer;
begin
    b:= 0;
    num:= 4;
    stones:= 1;
for a := 1 to num do
```

```
begin
    cups[a] :=stones;
end;
    startcup:= 1;
repeat
    stonesincup:= cups[startcup];
    cups[startcup] := 0;
for c:= 1 to stonesincup do
begin
    startcup := startcup + 1;
if startcup num then
    startcup := startcup - num;
    cups[startcup]:= cups[startcup] + 1;
end;
    b:=b + 1;
for c:= 1 to num do
    write(cups[c]:5);
    writeln;
until cups[1] = num * stones;
    writeln(b:5,'iterations');
    readln;
end.
```

1111;0211;0022;1120;1030;2101;
2011;2002;3100;3010;3001;4000;
11 iterations

Conclusion

The problems are meant as exercises in programming techniques while focusing on some of the more subtle questions about the mathematics in the background. The programs can also be used as exercises for designing a file for printable output. In the $3x + 1$ case, a loop can be built to obtain the number of iterations, for example, from 2 to 500, while suppressing the output of the actual sequences. Programs for graphs of mathematical functions are less suitable because they are easier to do with Maple V, the mathematics software package developed at the University of Waterloo, or equivalent software.

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Counting Card Combinations

Bonnie H. Litwiller and David R. Duncan

A poker player Jeanne notices that most of her five-card hands contain at least one heart. What is the probability that this result occurs; that is, that a given poker hand contains at least one heart?

We shall solve this problem in two ways. Each method uses the fact that the total number of possible five-card poker hands is

$$C(52,5) = \frac{52!}{5!47!} = 2{,}598{,}960.$$

Each of these approximately 2.6 million poker hands is equally likely to be dealt to Jeanne.

Method 1

We will count directly the number of hands having exactly one, two, three, four and five hearts.

Exactly One Heart

This type of hand would contain one heart and four cards chosen from the remaining 39. The number of ways in which the single heart can be chosen is $C(13,1) = 13$, while the number of ways in which the four non-hearts can be chosen is $C(39,4) = 82{,}351$.

By the Fundamental Principle of Counting, the number of poker hands that contain exactly one heart is then $(13)(82{,}251) = 1{,}069{,}263$.

Exactly Two Hearts

The number of ways of selecting the two hearts is $C(13,2) = 78$ while the number of ways of selecting the three non-hearts is $C(39,3) = 9{,}139$. Thus the total number of hands of this type is $(78)(9{,}139) = 712{,}842$.

Exactly Three Hearts

The number of ways of selecting the three hearts is $C(13,3) = 286$.

The number of ways of selecting the two non-hearts is $C(39,2) = 741$.

The total number of hands of this type is $(286)(741) = 211{,}926$.

Exactly Four Hearts

The number of ways of selecting the four hearts is $C(13,4) = 715$.

The number of ways of selecting the non-hearts is $C(39,1) = 39$.

The total number of hands of this type is $(39)(715) = 27{,}885$.

Exactly Five Hearts

The total number of ways of selecting five hearts and zero non-hearts is $C(13,5) \bullet C(39,0) = (1287)(1) = 1287$.

Altogether, the number of poker hands that contain at least one heart is $1{,}069{,}263 + 712{,}842 + 211{,}926 + 27{,}885 + 1{,}287 = 2{,}023{,}203$. The probability that Jeanne is dealt a hand containing at least one heart is then $\frac{2{,}023{,}204}{2{,}598{,}960} \approx 77.8\%$.

Method 2

The calculations of Method 1 involved many steps. A shorter and more elegant approach is to first consider the number of five-card poker hands that contain no hearts. Once this number is determined, the remaining hands can be found by subtracting from 2,598,960.

The number of poker hands containing *no* hearts is determined as follows:

1. Select the five cards from the 39 non-hearts; this can be done in C(39,5) = 575,757 ways.
2. Select the empty set from the 13 hearts. This can be done in (13,0) = 1 way.
3. The product of these two combinatorial results is 575,757; this is the total number of hands that contain no hearts.

Recall that there are 2,598,960 possible poker hands. If 575,757 hands contain no hearts, then the other hands must contain at least one heart. Therefore, 2,598,960 - 575,757 or 2,023,203 poker hands contain hearts. This is the same result as achieved by Method 1.

Note that exactly the same methods would apply if any of the other three suits (clubs, diamonds, spades) had been used instead of hearts. This leads to the following results:

P (at least one heart) = 77.8%
P (at least one diamond) = 77.8%
P (at least one club) = 77.8%
P (at least one spade) = 77.8%

Because one of these four cases must occur, why is the sum of the four probabilities not equal to 100 percent? This question could lead to an interesting class discussion. The teacher can reemphasize that probabilities can be summed only if the events in question are mutually exclusive.

Challenges for the reader and his/her students:

1. Redo the problem of this article (finding the probability that a poker hand contains at least one card from a certain category) for the categories of kings, face cards and red cards.
2. Redo the original problem (at least one heart) and the three additional problems of challenge 1 for a 13-card bridge hand. The reader should be aware that the size of the numbers involved in the computations may cause their calculators to round and thus yield only approximate answers. Alternative computing algorithms may be used to eliminate this rounding problem.

Solids Construction

Sandra M. Pulver

While searching for some recreational mathematics for my own children to work on this summer, I found some constructions that I thought they (and other elementary or junior high school children) would enjoy—a mobile using the five Platonic Solids and other "space" figures made from regular polygons. Although these solids can be constructed in many ways, I will demonstrate the simplest.

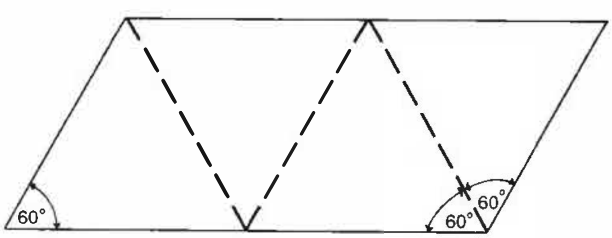
The solids can be made with heavy construction paper and tape. They can then be strung up as a mobile or used as a display.

Through this exercise, Grades 5 to 7 students will learn to use a ruler and protractor correctly. They will learn new terminology such as equilateral triangle, regular pentagon, regular hexagon, and of course, the names of all the new solids they construct. A good idea is to have them label each of the sides of the solid with the solid's name before taping the sides together. You may also informally define the congruent regular polygons of which the solids are made and have them count the number of faces, edges and vertices on each solid. Have them try to guess formula relationships between what they have found.

Vertices + Faces = Edges + 2

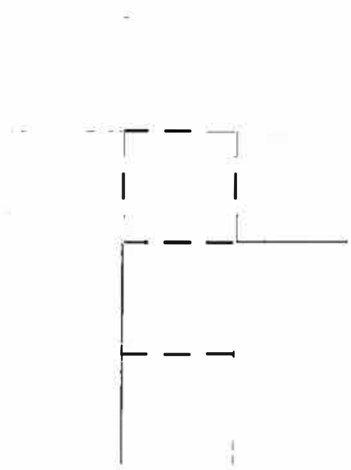
The five regular polyhedrons and their constructions are as follows:

1. Tetrahedron

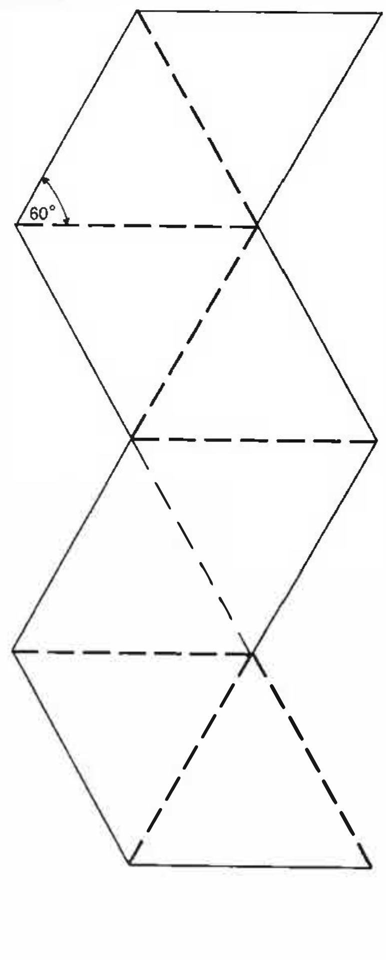


Fold on dotted lines.

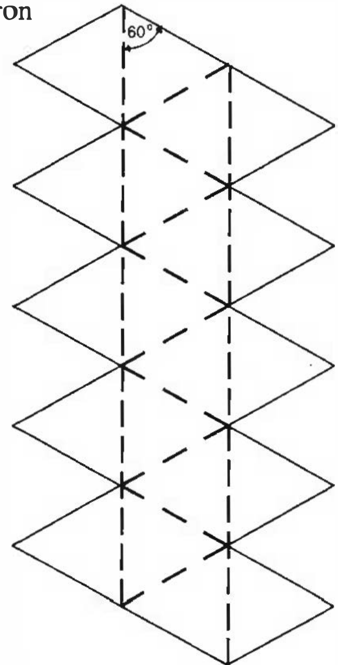
2. Hexahedron (cube)



3. Octahedron

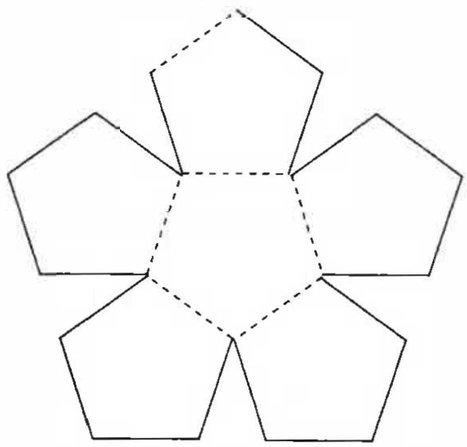


4. Icosahedron



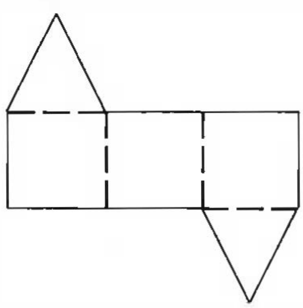
5. Dodecahedron

Make two of these figures. Attach the second one along the outside dotted lines here.

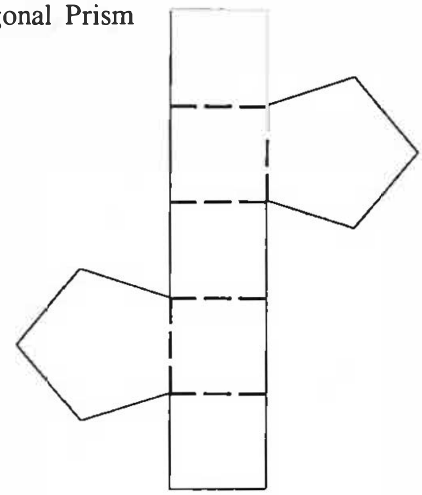


Each face of a regular polyhedron is a regular polygon that is congruent to every other face. The following are polyhedrons that are not regular:

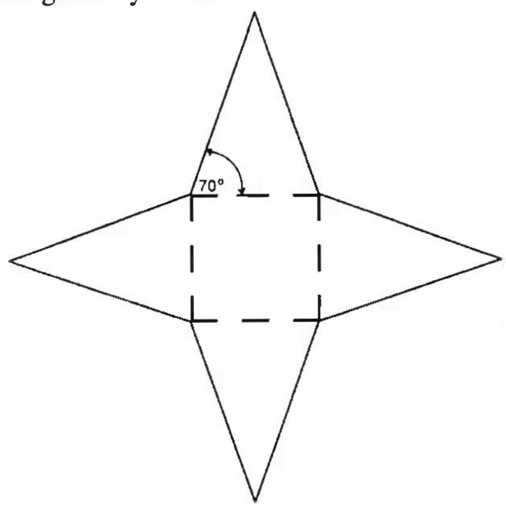
1. Triangular Prism



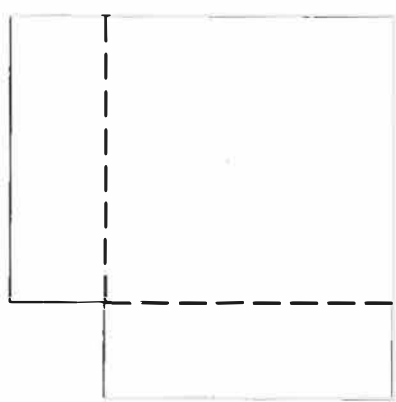
2. Pentagonal Prism



3. Rectangular Pyramid

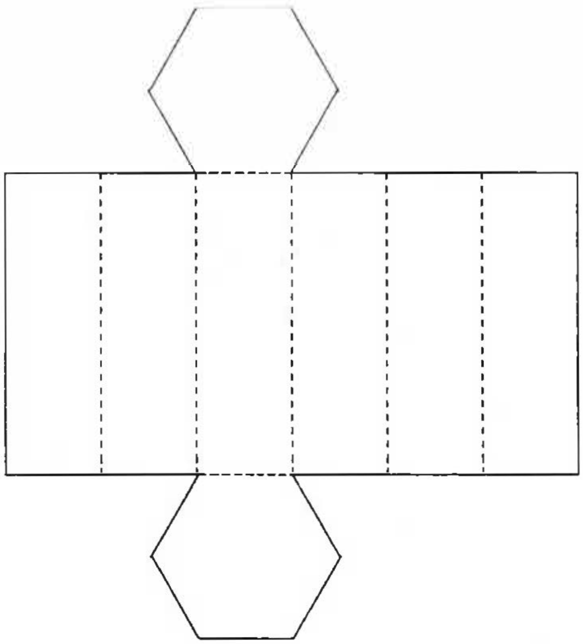


4. Rectangular Prism



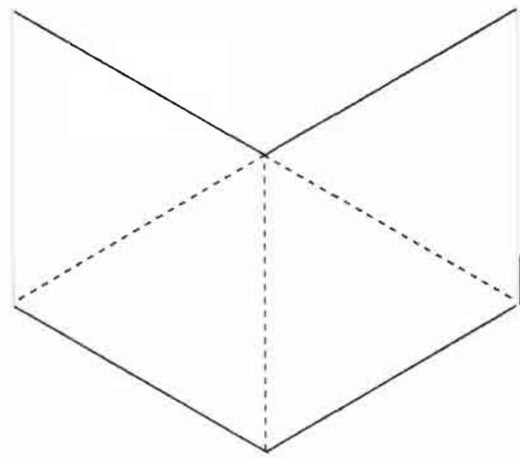
Make two of these.

5. Hexagonal Prism



Each angle at 120°, each side same length.

6. Octahedron
Make two of these.



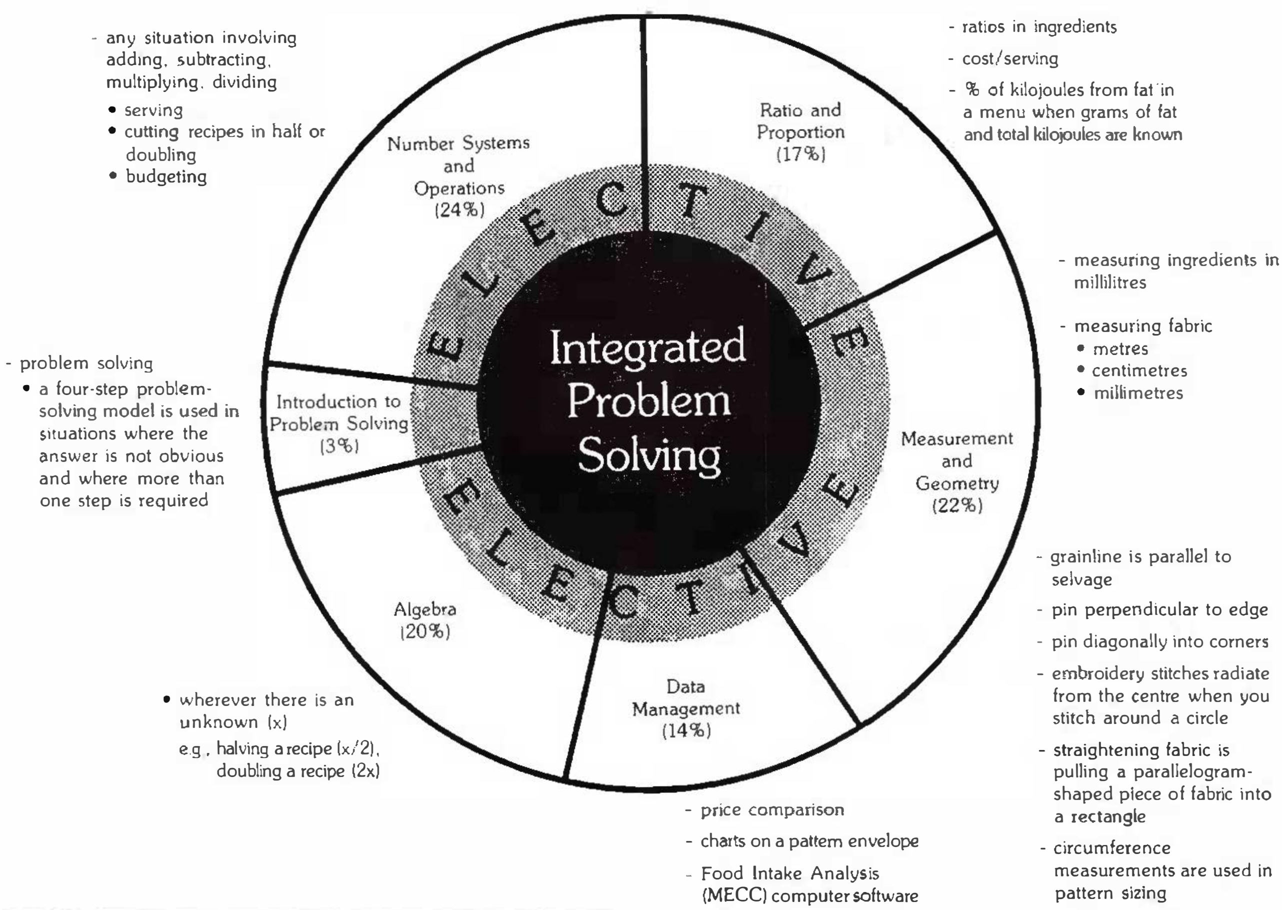
Curriculum Connections

Geri Crossman

The opportunities for learning within the home economics curriculum are almost endless. I was therefore interested in being a part of a group of Calgary home economics teachers that was examining subject curricula in looking for "curriculum connections." An examination of the junior high math curriculum yielded even more connections than I expected. I found opportunities to incorporate each of the math strands in the home economics program, which in turn supported and developed the rationale and philosophy of the math program. The home economics program provides many relevant and concrete opportunities for students to apply math concepts and, therefore, generate positive attitudes toward using math daily.

The following diagram shows the six strands of the math program, with examples from the home economics program.

Grade 9 Mathematics



This article previously appeared in the Journal of Home Economics Education, *Vol. XXX, No. 2, December 1991.*

The following activity from the module "A New View of Food" (Foods III, Unit V) requires the student to use five of the six math strands. Example 2 similarly involves five of the six math strands plus the use of the MECC computer program Food Intake Analysis to help students organize and display data and eliminate computations.

Example 1

Visit a supermarket to compare food prices. Record your findings on this chart.

Food Item	Size	Total Cost per Package	Cost per Serving (1 serving = 35 g)
Corn Flakes	350 g		
Corn Flakes	575 g		
Corn Flakes	675 g		
Corn Flakes (generic)	500 g		

What would be the best buy for your family? Explain here.

1. Problem solving because the answer is not obvious
2. Ratio and proportion: $350/\$ = 35/x$
3. Data management: collecting, interpreting, organizing, making predictions
4. Algebra because there is an unknown
5. Number systems and operations because you have to multiply and divide

Example 2

According to Jesse's Food Intake Analysis, he consumed 2,400 calories on Tuesday. The menu contained 80 grams of fat. A healthy diet should not contain more than 30 percent fat calories. What percentage of the calories in this diet comes from fat?

Additional examples of the application of mathematics skills are also found in "Challenges and Choices" and "Personal Money Management" (both in Family Studies III module).

This is the vital question: Is there a better way to help students interpret and understand their world in relation to math than to use math to develop consumer skills, clothing construction skills or other life simulation activities that are part of the home economics program?

My familiarity with the math curriculum has added another dimension to my program. I can relate math situations in home economics to specific concepts in the math program using terms with which students are familiar.

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Calculators, Baseball and Mathematics: A Winning Team

A. Craig Loewen, Dino Pasquotti and Lon Bosch

In honor of the recent triumph of the Toronto Blue Jays, we have constructed a small collection of estimation and calculator games built around a baseball theme.

Most people have well-formed opinions about the use of calculators in the mathematics classroom, which tend to stem from personal experiences while learning math. For example, people who were allowed to use the calculator (or some other computing device such as a slide rule) during class instruction believe that the application of the calculator is desirable, or at least acceptable. Those of us who were not allowed to use such computing devices are somewhat more hesitant to embrace the calculator.

Some more common arguments put forward by those of us reluctant to use the calculator in the classroom are (1) "students must learn how to add, subtract, multiply and divide. How will they ever develop these abilities if they are permitted to use the calculator in class?" and (2) "the calculator quickly becomes a crutch. Students will become too dependent on it and forget their number facts." The first argument is quite incorrect in its assumptions. The first argument assumes that the major goal of mathematics instruction is the development of fluent arithmetic computation skills. The learning of mathematics is far more comprehensive than these skills—mathematics involves the development of number sense, estimation and mental computation skills, reading and writing skills, generalized thinking and problem solving skills and abilities, not to mention a broad host of other important concepts, relationships, algorithms, communication skills and exploratory talents. In short, mathematics involves much more than simple arithmetic computation skills, and the calculator can play a role developing and retaining these important skills and abilities.

The second argument, that the calculator is a crutch, is also fraught with difficulties. No current research evidence supports the claim that using the calculator diminishes retention of number facts (see Hembree and Dessart 1992 for a nice summary of research). It is also important to realize that where and how the calculator is used is a negotiable topic in any classroom. The students and teacher in every mathematics classroom should take the opportunity to discuss appropriate applications of the calculator. For example, it may not be sensible to turn to the calculator to complete computation such as

- simple addition or subtraction, for example, $2 + 3 = 5$ or $9 - 7 = 2$;
- multiplication or division by powers of 10, for example, $3 \times 10 = 30$ or $540 \div 100 = 5.4$;
- finding certain percents of a number, for example, 10% of 920 is 92 or 1% of 3,000 is 30.

In short, it is fair to say that the calculator is not necessary for instructing each and every arithmetic concept in the mathematics curriculum: its limitations and appropriate applications need to be discussed and explored through honest communication between the teacher and students.

The best application of the calculator is in the teaching of estimation and mental computation skills. The problem with teaching these concepts is that students need a means to check their estimates and their mental computations for accuracy and reasonability. The question is, If students are asked to estimate the product of 289 and 21, how would they know when a reasonable estimate has been found? However, if the students complete the estimate ($300 \times 20 = 16{,}000$) and then are allowed to compare the result with the actual product computed with the aid of the computer (6,069), the reasonability of the estimate is quickly determined. In this sense, the calculator's speed and accuracy make it a useful tool for providing effective and immediate feedback.

The application of the calculator to the instruction of estimation skills can be housed within a game format. The following three games each employ the calculator in a problem solving, gaming situation based on estimation skills and a baseball context. The games are offered as examples of how calculators can be used in an effective and enjoyable manner in the junior high mathematics classroom.

Game 1: Calculator Baseball

Objective

- Given the product, player identifies probable multiplicands from the range of numbers provided.

Goal

- To maximize the number of correct estimates while minimizing the number of strikes.

Number of Players

- Players work as individuals, but any number of people can play the game at one time.

How to Play

- Each player needs his or her own game board (see Figure 1).
- Beginning with Line 1, the player tries to identify the correct multiplicands for each product shown in that line. Multiplicands are selected from the Pitching List at the top of the game board. The player works from left to right across each line.
- The player records the two multiplicands in the boxes underneath the product, and then using a calculator multiplies these two numbers.
- If the player has correctly identified the two multiplicands for a given product, a check mark is placed next to the product. If the multiplicands the player has chosen do not produce the correct product, an "X" is placed under a strike at the end of the line.
- If a player reaches the end of a line before receiving three strikes, then he or she simply continues on to the next line.
- If a player gets three strikes before reaching the end of a line, then he or she must leave that line and proceed to the next line.

Rules

- Final score is calculated by counting the number of check marks in each line.

Example

- The target products in the first line are 273, 1,638, 756, 1,248 and 3,024. Assume the player correctly decides that 13 and 21 will provide a product of 273. After recording the numbers 13 and 21 as shown, and confirming his or her choices with the aid of a calculator, the player places a check mark next to 273. The player now proceeds to find the multiplicands for 1,638.

273 √	1,638	756	1,248	3,024	strike	strike	strike
13 | 21							

Adaptations

- The game can be easily adapted by changing the range of numbers from which players must select and by changing the target products in each line.

Game 2: In-Between Hits

Objective

- Player estimates the missing multiplicand given one multiplicand and the product and orders four digit numbers.

Goal

- To score the greatest number of runs by correctly estimating the missing multiplicand.

Number of Players

- Two teams of one or more players each.

How to Play

- Each team needs one copy of the game board (see Figure 2), a calculator and small markers such as buttons, coins or paper squares.
- The team having the oldest player is designated as the home team and will have last "at bats."
- The game begins with each team listing its players in batting order on the game board. The reverse order is known as the pitching order.
- The pitching team sends their first pitcher forward while the batting team sends their first batter forward. The pitcher places a number of his or her choice in the calculator (for example, 101.32) and passes the calculator to the batter.
- The batter now multiplies the number in the calculator by any number that he or she chooses trying to obtain a product that lies in one of the ranges specified as a hit on the Hit Chart.
- If a hit is scored, the batter may move his or her marker to the appropriate space on the team's game board. If the batter fails to score a hit, then a tally mark is recorded as an out.
- Each team now sends forward a new pitcher and a new batter, and the process is repeated until the batting team has three outs. The teams now reverse roles, and the pitching team becomes the batting team for the second half of the inning.
- The team that scores the greatest number of runs is the winner.

Rules

- A game has six complete innings.
- Once a number is entered into the calculator, it may not be changed by either the pitcher or batter.

- When a hit is recorded, all runners on bases move forward the specified number of bases, that is, each runner would move forward one base on a single, two bases on a double and three bases on a triple. All on-base runners advance home in case of a home run.
- The umpire (teacher) decides if a batter is taking too long to enter the second multiplicand. If found to be taking too long, the batter is considered to have struck out.

Example

- Assume a pitcher punches the number 325 into the calculator before passing it to the batter.
- Assume the batter multiplies this number by 17. The product of 325 and 17 is 5,525, and the batter will have "flied out."
- Assume instead the batter multiplied the pitch (325) by 15.2. The product of 325 and 15.2 is 4,940, and the batter will have recorded a double hit.

Adaptations

- The game may be made less difficult by limiting the range of numbers the pitcher uses, or by reducing the magnitude of the products in the Hit Chart.
- The game may be made more difficult by constructing several different Hit Charts. When a batter comes forward, he or she must randomly select a Hit Chart after the pitch.

Game 3: Three Strikes and You're Out

Objective

- Given the product, player identifies probable multiplicands from the range of numbers provided.

Goal

- To correctly identify combinations of multiplicands that give specified products thus avoiding strikes. To be the last player to record three strikes.

Number of Players

- Two players.

How to Play

- The two players will share one game board (see Figure 3) and a calculator. Players also need pencils.
- The youngest player gets to go first.
- This player selects two numbers from the Number Chart at the top of the game board. The product of these two numbers is found with the aid of the calculator.
- If the product of these two numbers is found on the game board, then that product is crossed out and play is passed to the other player.
- If the product of the two selected numbers is not found on the game board, then that player must score a strike by placing an "X" in one of the strike boxes.
- Play continues to pass between the two players until one player has scored three strikes. This player is out of the game, and the remaining player is the winner.

Rules

- If a player chooses two numbers that when multiplied have a product that has already been crossed off the game board, then that player must score a strike.

Example

- Assume a player chooses 9 and 44 that have a product of 396. The product 396 is found on the game board, so it is crossed off and play is passed to the opponent.
- Assume the player chooses 87 and 91 that have a product of 7,917. The product 7,917 is not found on the game board, so the player must score a strike.

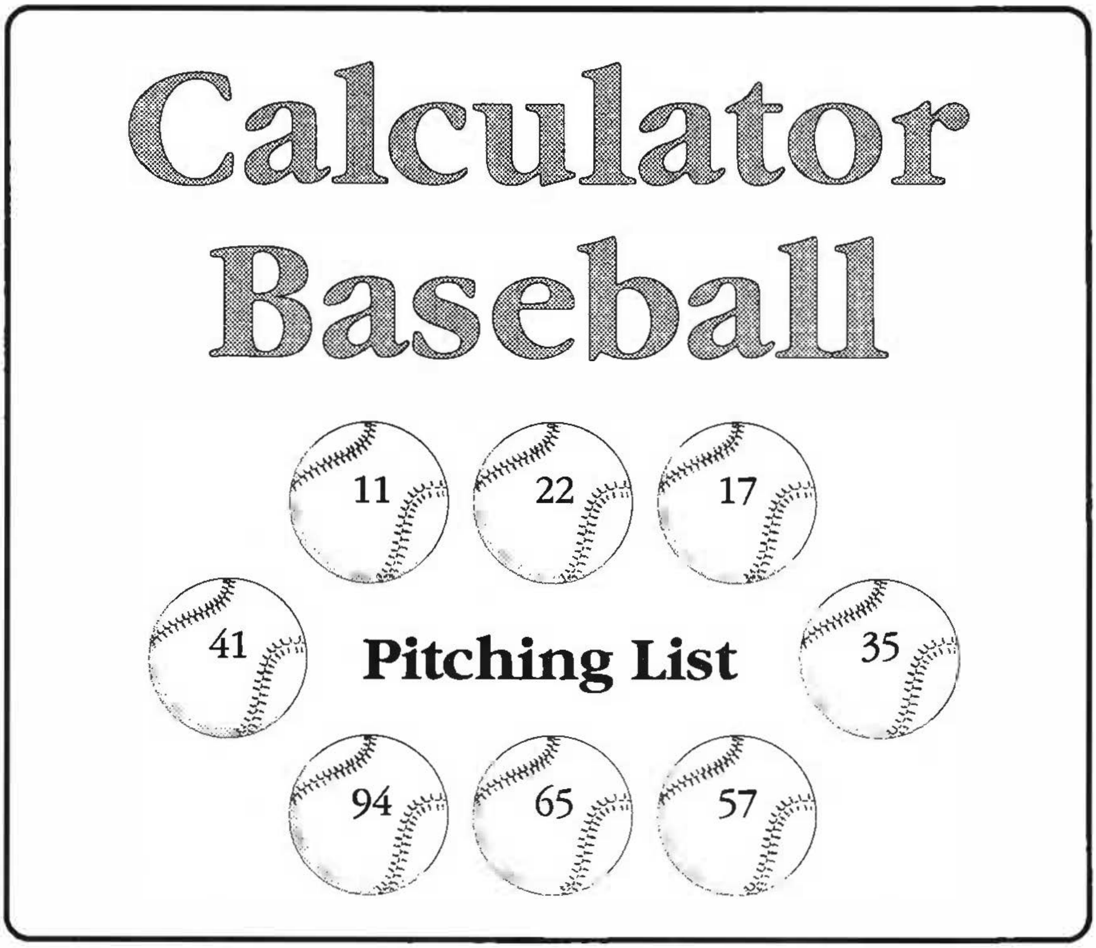
Adaptations

- Change the game so that students must estimate sums instead of products.
- Allow more than two players to play the same game as individuals or as teams. If more than two teams or two individual players play, then they will have to keep track of their strikes on a separate piece of paper.
- To increase the difficulty of the game, have players identify the product they are trying to achieve prior to selecting two numbers from the Number Chart. If the two numbers do not have the specified product, the player must score a strike.

Reference

Hembree, R., and D.J. Dessart. "Research on Calculators in Mathematics Education." In *Calculators in Mathematics Education* (1992 Yearbook), edited by J.T. Fey and C.R. Hirsch. Reston, Va.: National Council of Teachers of Mathematics, 1992.

Figure 1: Calculator Baseball Game Board



Line 1	273	1638	756	1248	3024	strike	strike	strike
Line 2	2268	234	8736	1728	648	strike	strike	strike
Line 3	5733	624	378	3456	1183	strike	strike	strike
Line 4	468	4608	234	756	819	strike	strike	strike
Line 5	2304	441	1134	1008	6048	strike	strike	strike

Figure 2: In-Between Hits Game Board

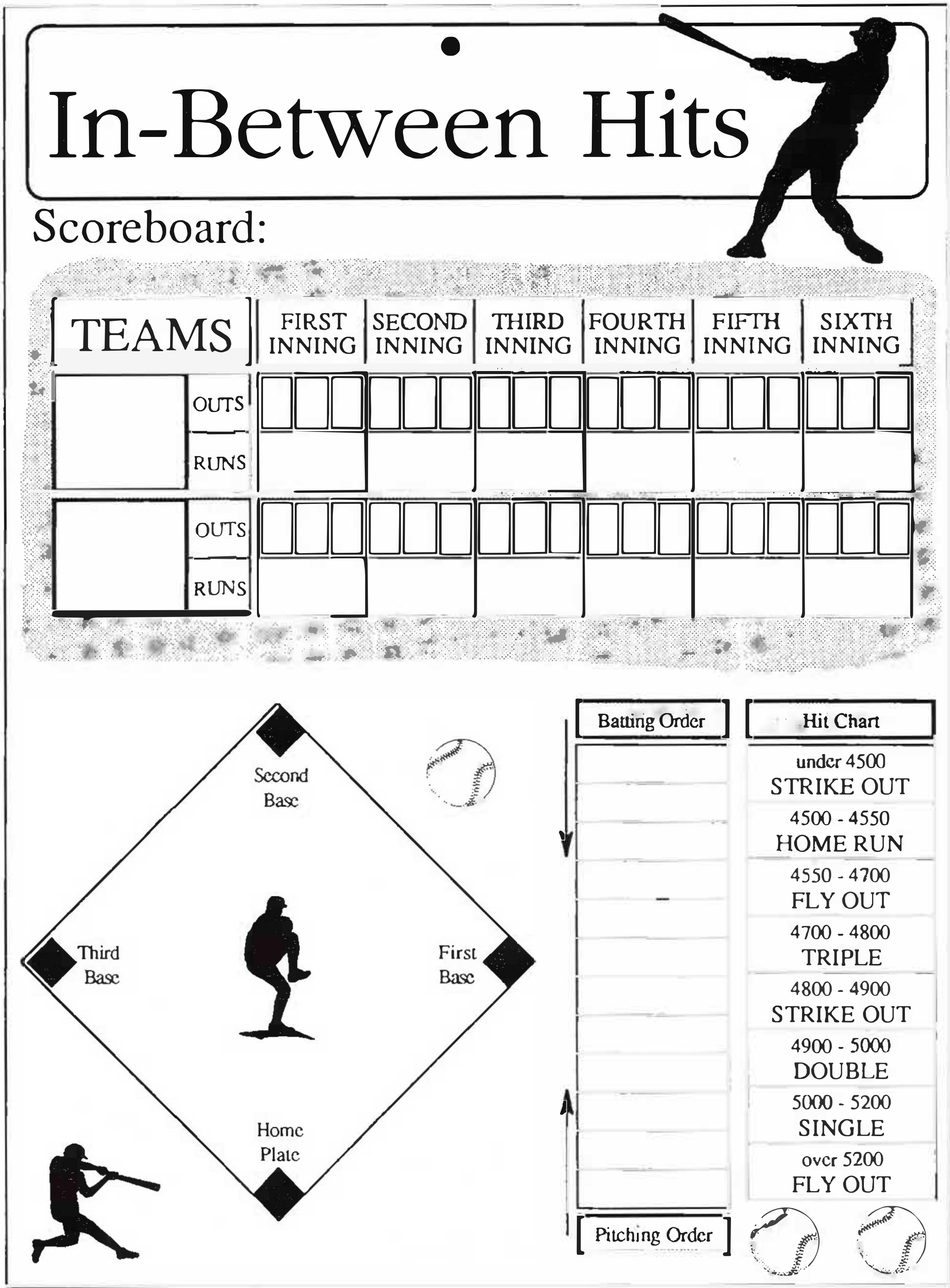
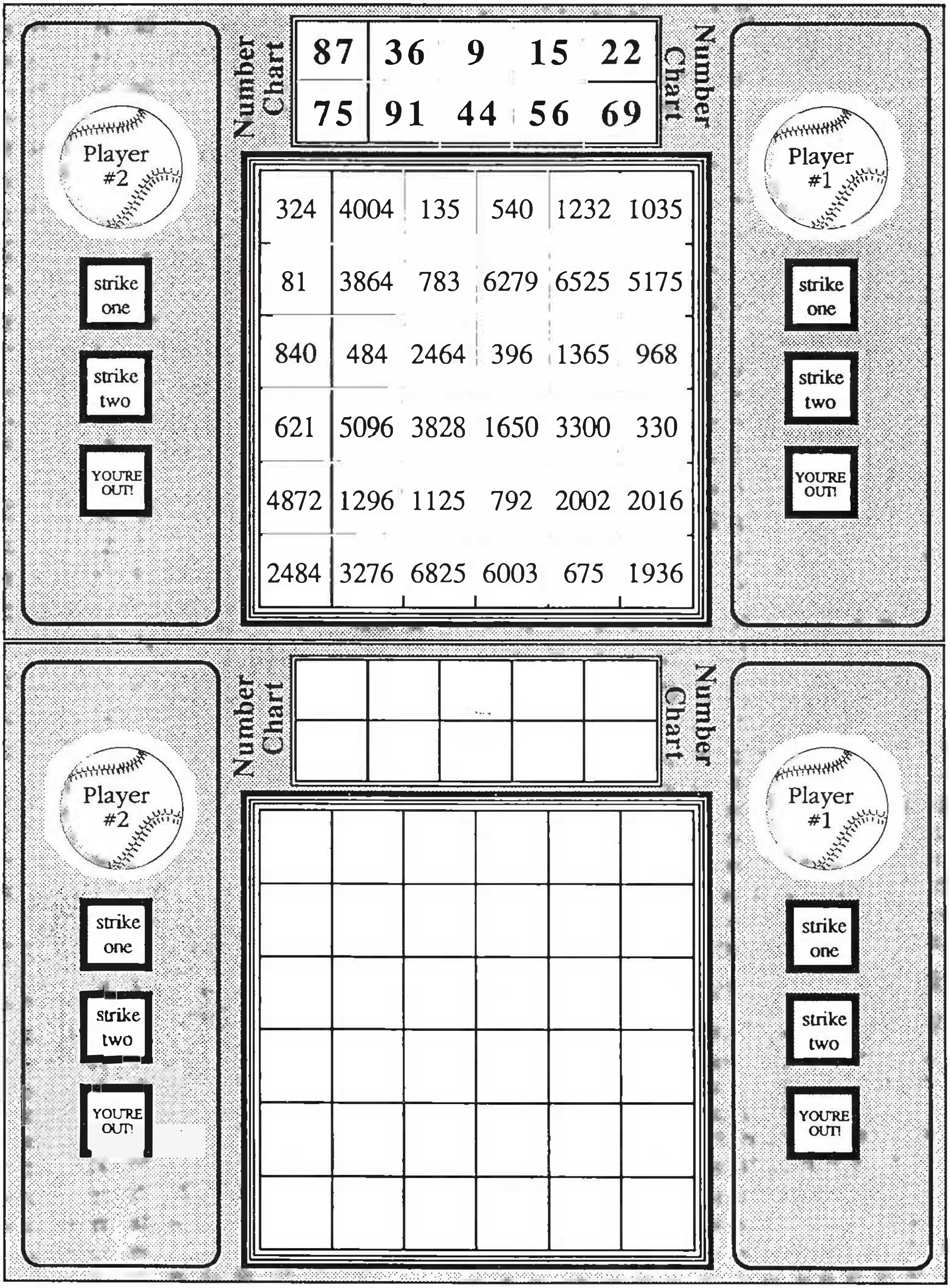


Figure 3: Three Strikes and You're Out Game Boards



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