Calculator Explorations in Junior High Mathematics

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The fact that calculators are inappropriate for presenting or discussing all mathematical topics is probably true; like any good instructional tool, they have their limitations. However, we must be careful not to rule out all instructional applications of calculators due to our reluctance to employ them in specific situations.

It is becoming difficult to argue that calculators should not be a part of mathematics instruction: calculators are simply too common in our society to be denied—it almost seems impossible or even unreasonable to expect or ask students *not* to use them. Our task as teachers is to decide when and how calculators are to be used, not if they should be used. Teachers must learn to use the calculator as a tool for facilitating instruction and experimentation. The calculator can be a useful tool for helping us all explore the nature and limitations of formulas and other mathematical relationships addressed in the junior high mathematics curriculum. This article will present four such explorations.

What Is a Calculator Exploration?

In a calculator exploration, the teacher introduces and monitors an experiment whereby students can derive or validate a mathematical idea through many exemplars. A calculator exploration activity involves the following steps: (a) a topic, idea, formula, property or other mathematical relationship is briefly introduced to the students, (b) the brief discussion is followed by a period of investigation where the limitations and plausibility of the idea or relationship are explored, and (c) the exploration concludes with a summary and generalization of the main idea, formula, fact or relationship.

A Familiar Example

Our first example of a calculator exploration involves an activity designed to determine the approximate value of pi (π). In this exploration, the students will use a piece of string to measure the circumference and diameter of a variety of cylindrical objects. The students will construct a list of the measurements for each cylinder and organize the information in a table, comparing circumference, diameter and the quotient of circumference divided by diameter (Figure 1). The students should find that circumference for each cylinder is a little more than three times the diameter of that cylinder. If students combine their results with those of other students, they may be able to compute an approximation of the value of π.

To engage the exploration, the teacher could start by rolling a tin can one rotation along the surface of a table or blackboard—the path of the can may be shown by marking the starting and ending points and connecting the two with a straight line (it is important that students realize the length of the line drawn represents the can's circumference). The distance the tin can has rolled may now be compared with the diameter of the can by tracing the can, measuring the diameter and the length of the path or marking diameter lengths along the path (Figure 2). The teacher may wish to repeat this process with another can of a different size to show that in both cases the circumference is approximately three times the diameter (or conversely that the diameter is 1/3 the circumference). The students are now ready to enter into an exploration of the value of π.

As the students measure the diameter and circumference of each cylinder, they should be sure to record the values in a table. A great deal of information is typically collected, analyzed and summarized in calculator explorations, so one must employ a mechanism for keeping account of this information (the need to generate and/or keep a table is a powerful problem-solving strategy and should not be de-emphasized in the investigative process).

A large number of examples will be needed to attain a reasonable approximation of the value of π. The examples may be collected in various ways such as having each student in the class measure one object and report the results to the entire class, or by having students work in small groups where each group is responsible for measuring five to ten different objects. Once all the data have been collected, students can compute the ratio of circumference to diameter for each object as well as compute an arithmetic mean of these quotients. This mean should be relatively close to the actual value of π (the teacher may wish or need to engage a discussion of experimental error). Students may enjoy calculating a whole class mean, comparing this mean with the results of individual groups (who was closest?) and to the actual value of π (3.14159. . .). Conducting calculator explorations using group structures is not difficult—calculator explorations and cooperative learning activities are extremely compatible.

After the calculations are finished, students will quickly realize that few or no wildly divergent ratios between the circumference and diameter exist—if the activity is completed carefully, each ratio is approximately three to one. Students should be encouraged to summarize the relationship in an equation such as

Circumference $\approx 3 \times$ diameter or Circumference $= \pi \times$ diameter

The large number of consistent examples encourages a sense of confidence and believability in an otherwise extremely abstract relationship. The sense of believability enhanced by this physical experience will encourage retention. In essence, the variety of examples combined with the concrete exploration has encouraged a form of informal inductive proof.

Explorations, such as the one described above, are possible only through the use of the calculator. Students would be unwilling to complete the requisite number of computations (and probably incapable of accurately completing them) without the aid of the calculator. In calculator explorations, students are working with real data which are collected firsthand. Real data are rarely whole numbers (students often describe these as "messy numbers"). However, students need to learn to work with "messy" numbers drawn from real objects, and such experiences may serve to provide the concept with a sense of relevance.

In calculator explorations, the calculator serves as a tool to free the student's mind to focus on a relationship without the frustration of repetitive computations. Generally, in calculator explorations, the relationship or idea of interest takes precedence over the computations. In any classroom activity where this generalization is true, the calculator may be effectively employed as a learning/teaching tool. Three other calculator explorations are provided for the reader's consideration.

Exploration 1: Summing the Interior Angles of a Triangle

Objective

Determines that the sum of the interior angles in a triangle is 180°.

Materials

Protractor, Figure 3 (duplicated as student handout), pencil, ruler, calculator.

Process

To participate in this activity, students must be able to use a protractor to find the measure of a given angle. Once this skill has been established, the teacher can distribute the handout (Figure 3) and explain how the chart at the bottom of the page is to be completed. The students should be encouraged to begin with the given triangles but then branch out to include other triangles they have constructed.

Generalization

In this activity, students will quickly learn that regardless of the shape of the triangle (or the type of angles used to build the triangle, that is, acute, obtuse or right), the three angles always sum to a constant value (180°). The teacher may wish to adapt and extend the activity to address the sum of the interior angles of any quadrilateral. In the extended activity, students should realize that any quadrilateral can be divided into two triangles by connecting diagonally opposite corners (Figure 4), and that this realization can be used to explain why the interior angles of any quadrilateral sum to 360°. The activity can be further extended to five- (and more) sided figures.

Exploration 2: The Pythagorean Theorem

Objective

Recognizes the relationship between the lengths of the sides of any right-angled triangle, that is, that the length of the hypotenuse squared is equal to the sums of the squares of the lengths of the remaining two sides.

Materials

Protractor, Figure 5 (duplicated as student handout), pencil, ruler, calculator.

Process

To begin this activity, the teacher will need to emphasize that we are working with right-angled triangles, and that one side of the right-angled triangle has a special name: hypotenuse. The exploration is conducted by asking students to measure the lengths of each of the sides of the given right triangles and to record these lengths in the chart shown at the bottom of Figure 5. After measuring the given triangles, students are again encouraged to construct one or more of their own right-angled triangles and include the measurements of these triangles in their charts.

Generalization

Through this activity, students should generalize that $a^2 + b^2 = c^2$ where c is the length of the hypotenuse (the side opposite the right angle) and a and b represent the lengths of the remaining two sides for *any* triangle containing a right angle.

Exploration 3: Area of a Circle

Objective

Determines the value of pi (π) through exploring the relationship between the radius and area of a given circle.

Materials

Figures 6 and 7 (duplicated as handouts), pencil, millimetre ruler, calculator, protractor (or photocopied sheets of various-sized circles).

Process

Before undertaking this activity, the student must first review the process of determining the area of a rectangle. The student must also be able to identify and measure the radius of a circle.

To begin the activity, students draw the largest rectangular shape they can inside the circle (Figure 7). The area of this rectangle is calculated after measuring its length and width. This value should be entered into the chart in Figure 6. The process of constructing and measuring rectangles should continue as long as the rectangles are sufficiently large for reasonably accurate measurement (a completed example is shown in Figure 8). After the last rectangle has been drawn and measured, the areas of all rectangles are summed with the aid of the calculator. The final computation involves dividing the experimental area by the square of the radius of the circle. The students should be encouraged to repeat the process and computations beginning with a different circle. Each time the process is completed, the students should find that the ratio of area to radius squared is a value a little larger than three.

This relationship between area and radius can also be seen through considering Figure 9. The shaded area within the square may be described as having an area equal to the radius of the circle squared (r^2). The entire area of the circle then could be described as something less than $4r^2$ (and through visual inspection) probably approximately $3r^2$.

Some questions to ask:

- How does this process approximate the area of the circle?
- Assuming we have stayed within the boundaries of the circles with all our rectangles, how should our estimate compare with the actual area of the circle?

Generalization

While the value π is the constant relating circumference of a circle to its radius, π similarly relates area of a circle to its radius squared. The area of a circle is approximately equal to a little more than three times the radius squared.

A final note: The teacher may wish to allow students to develop their own experimental process, that is, to devise their own activity to test a given relationship or formula. Further, the teacher may wish to encourage this process when students suggest or even adopt inappropriate relationships. Though it is not true that these explorations constitute valid proofs of mathematical ideas, the process of revealing situations where a relationship does not hold may be useful in breaking students' error patterns.

Figure 1: Calculating the value of pi (π) through investigation of circumference of circles (cylindrical objects)

OBJECT	CIRCUMFERENCE (C)	DIAMETER (d)	CIRCUM ÷ DIAM ($C \div d$)
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

INSTRUCTIONS:

[a] Measure the circumference and diameter of 10 cylindrical objects and record the values in the table above.

[b] For each object calculate the value of the circumference divided by the diameter.

[c] Answer the questions below.

QUESTIONS:

[a] What do you notice about the values in the right most column of your table? What does it mean?

[b] Calculate the average of all of the values in the right most column. These values should average to approximately 3.14. How close is your average? Why is your average not exact?

[c] Compare your average with that of a classmate. Who was closest? Calculate the average for the entire class. How close is this average to 3.14?

Figure 2: Approximating the value of pi (π) through investigation of circumference of circles.

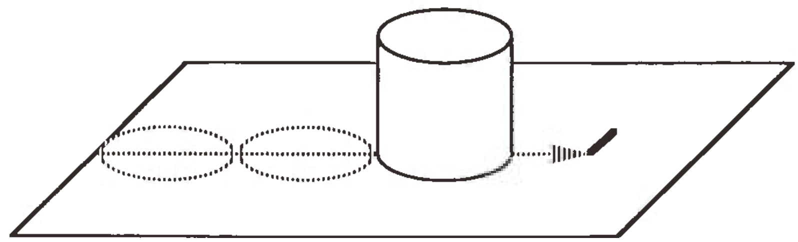
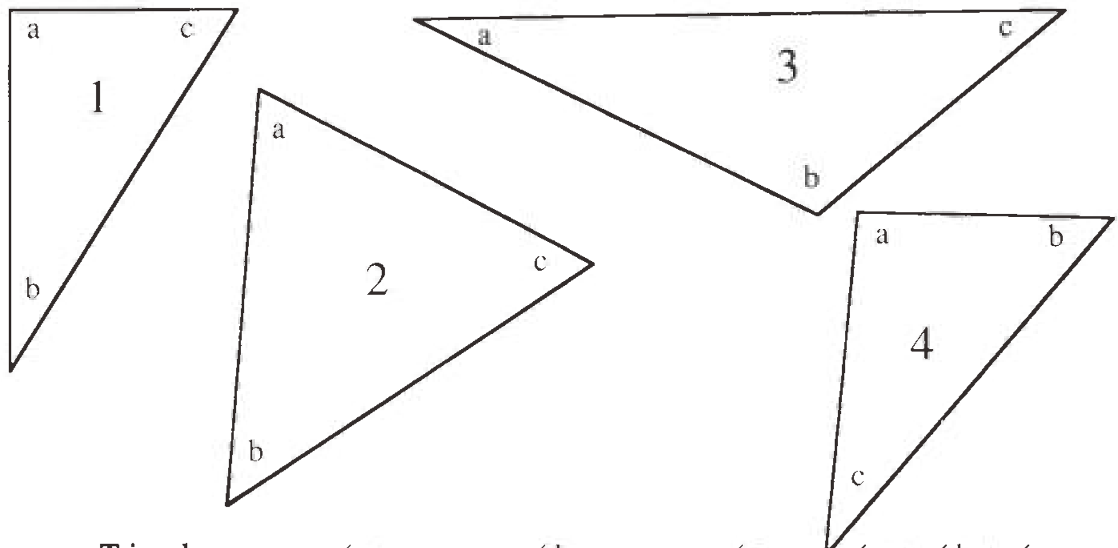


Figure 3: Calculating the sum of the interior angles of a triangle.



Triangle	$\angle a$	$\angle b$	$\angle c$	$\angle a + \angle b + \angle c$
1				
2				
3				
4				
5				

Instructions:

[a] Using a protractor, measure each of the angles in each triangle above. Record the measure of each angle in the table above.
[b] Sum the measures of the angles to complete the fifth column of the chart.
[c] Draw one more triangle on the back of this page to complete the fifth row.
[d] Answer the questions below.

Questions:

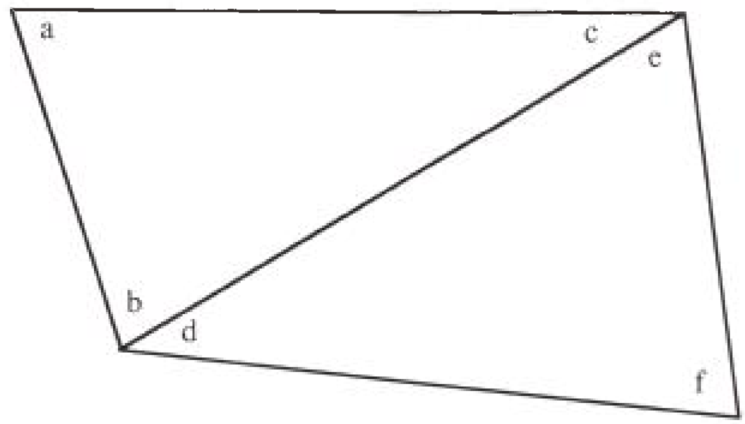
[a] What do you notice about the values in the last column of your table? Generalize to a rule about all triangles.

[b] Construct a triangle with one very large angle. Measure. Does the rule still hold?

[c] Hypothesize: what would the interior angles of any four sided figure sum to? Construct an experiment to test your hypothesis.

[d] Does it matter which angle of these triangles is labeled a, b, or c? Why?

Figure 4: Demonstrating that the measure of the interior angles of a quadrilateral sum to 360°.

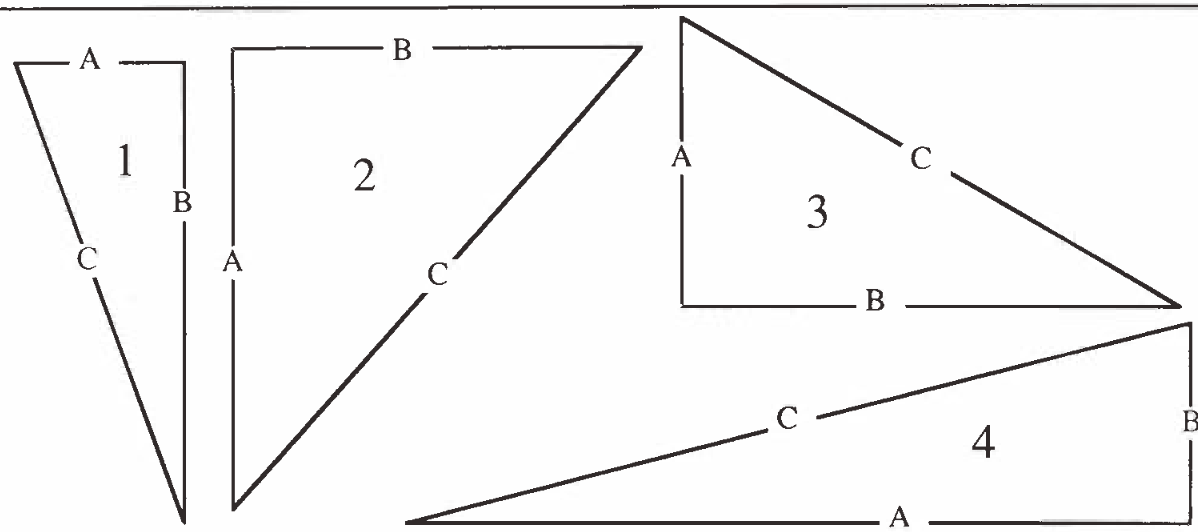


$\angle a + \angle b + \angle c = 180^\circ$

$\angle d + \angle e + \angle f = 180^\circ$

$\angle a + \angle b + \angle c + \angle d + \angle e + \angle f = 360^\circ$

Figure 5: Exploring the Pythagorean relationship.



Triangle	Length of A	Length of B	Length of C	A^2	B^2	$A^2 + B^2$	C^2
1							
2							
3							
4							
5							

Instructions:

[a] Using a ruler, measure the length of each side of each triangle above. Record the length of each side in the table above.

[b] Using these measurements, complete the remaining columns in the chart.

[c] Draw one more triangle with a right angle on the back of this page to complete the fifth row.

[d] Answer the questions below.

Questions:

[a] What do you notice about the two last columns in the chart above? Summarize this result to a rule about right angled triangles.

[b] Does it matter which of the sides of triangle are labeled A, B or C? Why?

[c] Does this relationship hold true for triangles that do not have one right angle? Draw two or more such triangles and test your hypothesis.

Figure 6: Exploring the relationship between area and radius of a circle.

Radius of Circle:

Rectangle	Length	Width	Area
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			

Rectangle	Length	Width	Area
13			
14			
15			
16			
17			
18			
19			
20			
21			
22			
23			
24			

Area of Circle:

Instructions:

[a] Measure and record the radius of the circle.
[b] Construct a rectangle inside the circle.
[c] Measure the length and width of rectangle and calculate its area.
[d] Continue constructing and measuring rectangles until the circle is virtually filled.
[e] Sum all of the areas of all of the rectangles to approximate the area of the circle.
[f] Repeat with the second circle. Try again with a large circle you draw.

Questions:

[a] Calculate the approximate area of the circle given.

[b] Divide the area of the circle by the square of the radius of that circle.

[c] Do you think dividing the area by the square of the radius would give a similar value for all circles? Why or why not?

Figure 7: Exploring the relationship between area and radius of a circle.

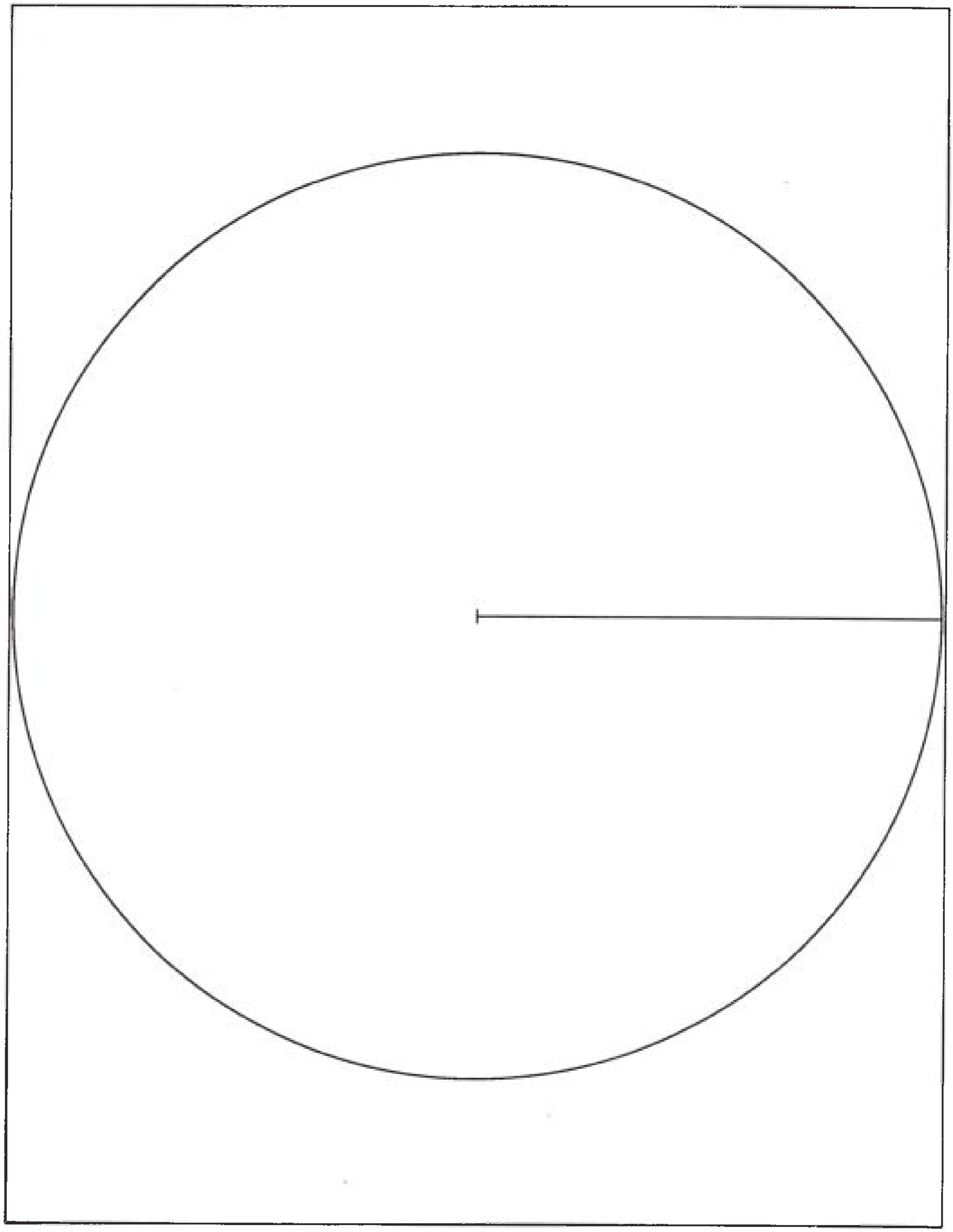
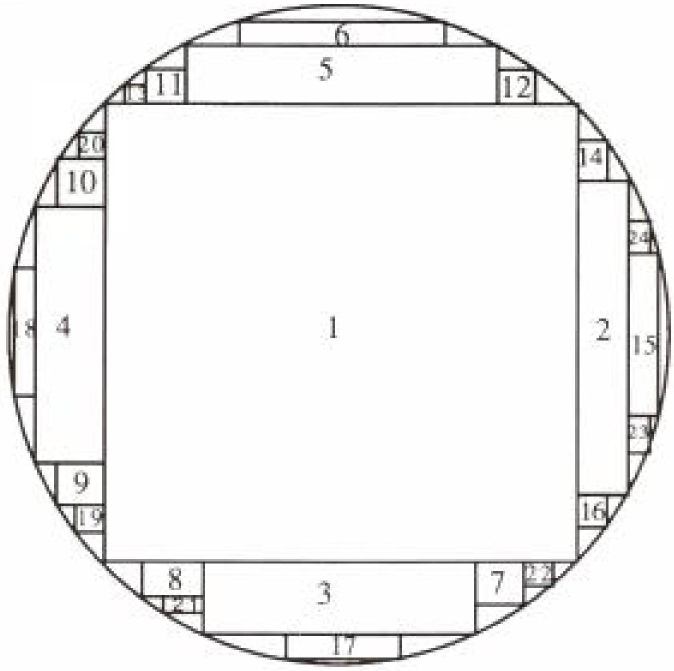


Figure 8: Completed example of Exploration 3.



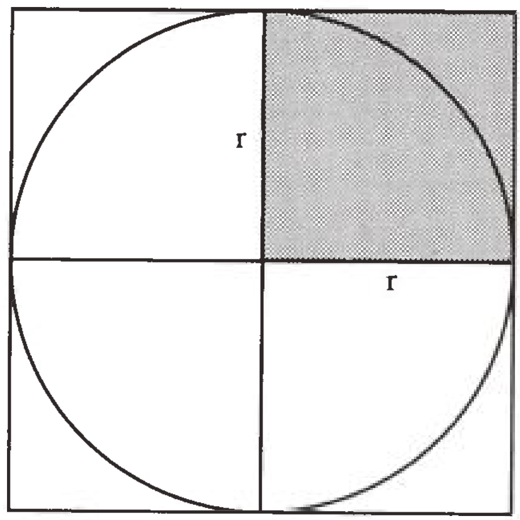
Rectangle	Length	Width	Area
1	6.1	5.9	36.0
2	4.1	0.6	2.5
3	3.5	0.9	3.2
4	3.3	0.9	3.0
5	4.0	0.7	2.8
6	2.6	0.3	0.8
7	0.6	0.5	0.3
8	0.8	0.4	0.3
9	0.6	0.5	0.3
10	0.6	0.6	0.4
11	0.5	0.4	0.2
12	0.5	0.4	0.2

Rectangle	Length	Width	Area
13	0.3	0.2	0.1
14	0.5	0.4	0.2
15	2.1	0.4	0.8
16	0.4	0.4	0.2
17	1.5	0.3	0.5
18	1.6	0.3	0.5
19	0.4	0.3	0.1
20	0.3	0.3	0.1
21	0.5	0.2	0.1
22	0.4	0.3	0.1
23	0.5	0.3	0.2
24	0.4	0.3	0.1

Approx. Area of Circle = 53.0 cm
Radius = 4.2 cm
Radius Squared = 17.6 cm
Approx. of $\pi = 53.0 \div 17.6$
$= 3.01$

Area of Circle: 53.0

Figure 9: Another look at the relationship between area and radius of a circle.



Note: the gray shaded area represents an area of radius squared. Therefore, the circle covers an area somewhat less than 4 x radius squared. Through visual inspection we can determine that each quarter of the circle covers about 3/4 of each square, therefore we might estimate the area of the circle to be about $4(3/4\ r^2)$ or about $3r^2$.