delta-k

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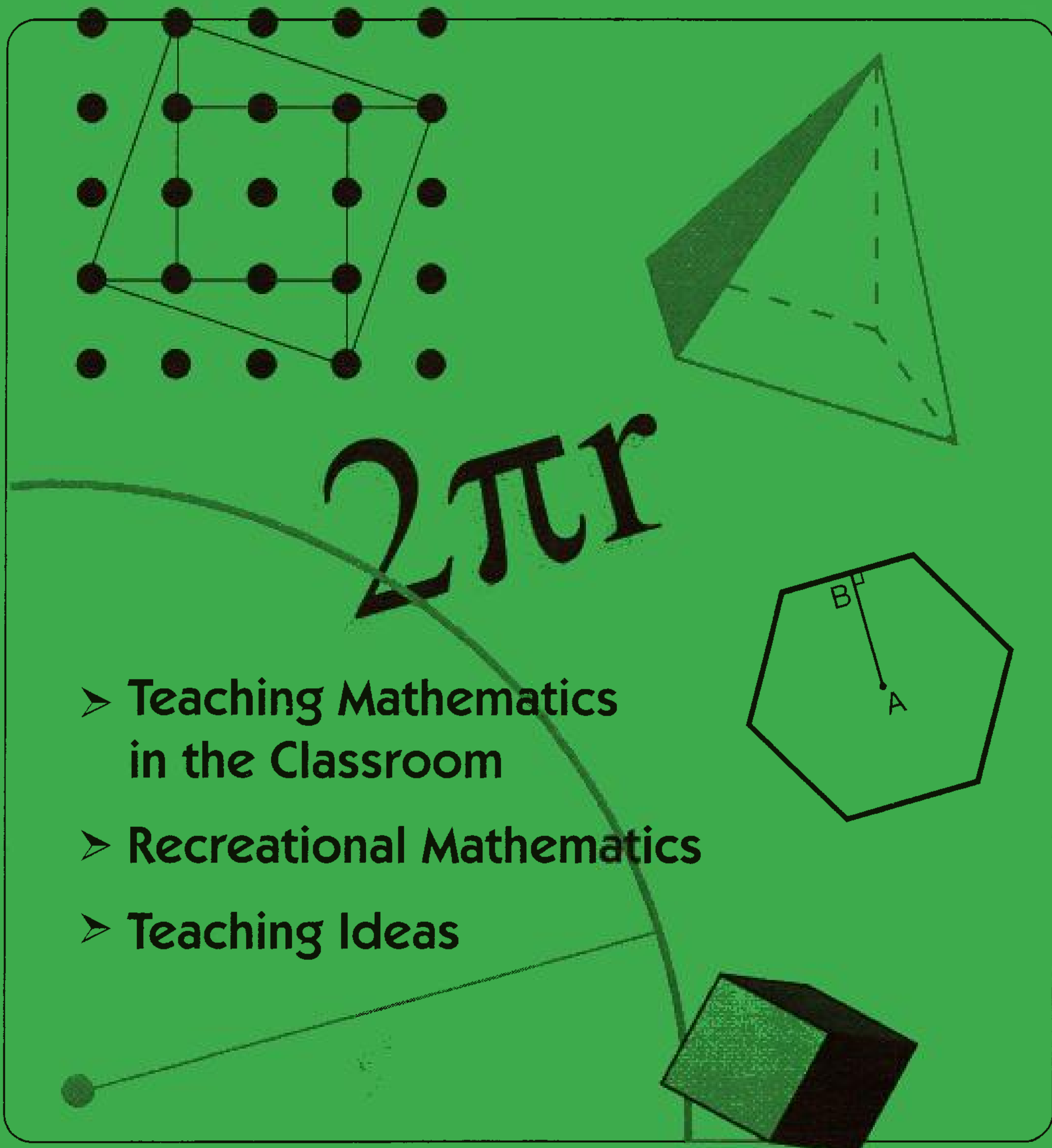

- Teaching Mathematics in the Classroom
- Recreational Mathematics
- Teaching Ideas

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EDITORIAL

The first two articles in this issue address different aspects of teaching mathematics in the classroom. Allen Neufeld focuses on the major goal of enabling students to become strong problem solvers by providing them with various problem-solving experiences. The second article, by Marlow Ediger, addresses the need to consider affective variables in mathematics instruction. We readily accept the need to sequence and plan for achievement in the cognitive domain but all too often forget the importance of the affective domain. Ediger provides some suggestions on how we can focus on the affective domain and thus encourage our students to enjoy their studies.

This issue also contains the traditional sections on recreational mathematics and teaching ideas. Sandra Pulver leads us through an interesting discussion of the subtraction of infinities, while David Duncan and Bonnie Litwiller explore the patterns found through generating geometric sets.

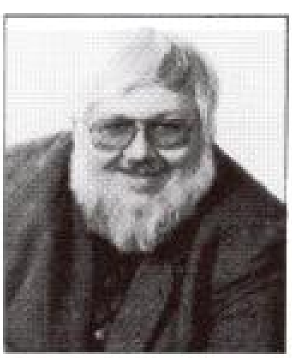
Yvette d'Entremont has adapted a popular television game show and uses it as a highly motivating context for reviewing mathematical ideas in the secondary classroom. I have also included an article that discusses four different calculator explorations for use in the junior high mathematics classroom.

We hope you find the articles interesting, challenging and useful within your own classroom.

We continue to seek articles for upcoming issues of *delta-K*. We are always interested in a variety of articles, but we are particularly interested in innovative teaching ideas to help make the learning of mathematics more interesting, meaningful and enjoyable. Please give some serious thought to what you could contribute to your colleagues across the province. Your ideas are needed and important!

Enjoy!

A. Craig Loewen



This issue of delta-K *is dedicated to the memory of Norm R. Inglis, district representative for Calgary District and longtime PEC liaison for the Mathematics Council, who passed away June 27, 1994. Norm's ready wit, visionary wisdom and valuable advice will be sorely missed.*

Children Solve Mathematical Problems with Multiple Solutions

K. Allen Neufeld

Problem solving has received a major focus in elementary mathematics curricula during the '80s, and emphasis on this major area will grow as curricula are planned for the '90s. More than ever, elementary students need to be equipped with the ability to solve the various problems they encounter in school as well as outside the classroom. "Good problem solvers possess a broad understanding of the concepts involved in a problem" (Irons and Irons 1989).

Attitudes to problem solving are also of considerable importance. "Many students have rather intense effective reactions to mathematical problem solving" (McLeod 1988). Unless teachers establish a positive climate, children will be reluctant to meet the challenge of the unknown. "The problem solver needs enthusiasm to proceed with the solution. Enthusiasm signifies the willingness to accept a challenge or set one's own challenge" (House, Wallace and Johnson 1983).

Teachers can play a significant role promoting problem-solving skills. Many students are the products of earlier curricula when problem solving didn't receive the emphasis it does today. Also, many students had elementary teachers who themselves had negative reactions to the solution of all but the most routine of problems.

> In my work with teacher-interns, I encounter very few who are willing or able to take this approach at the outset. Even though they are college graduates, many of them math majors, they, too, search for the infallible equation and are just as afraid as their students will be of appearing foolish. Teacher educators, then, have to model and encourage general problem solving behavior. (Noddings 1989)

Problems with Multiple Solutions

Many education students encountered standard-textbook problems when they were in elementary school. Often all the problems of a particular set focused on a recently studied computational process featuring the use of a particular equation format. Students were seldom challenged to solve process problems. "A process problem is one that cannot be simply solved by translating the one or more number sentences. Process problems emphasize the thinking processes for obtaining problem solutions" (Charles and Lester 1984).

A positive attitude toward problem solving is important for teachers *and* their pupils. Of equal importance is the willingness to accept a challenge. Pupils who are reluctant to solve problems may be encouraged by the fact that several possible solutions exist, and that finding even one of them is rewarding. Other pupils are challenged to find all the solutions, and they feel rewarded when they use a strategy that will ensure complete success.

> Situations that allow students to experience problems with "messy" solutions or too much or not enough information or that have multiple solutions, each with different consequences, will better prepare them to solve problems they are likely to encounter in their daily lives. (Romberg 1989)

> Problems that have more than one solution tend to foster a problem-posing mindset because they are not as limited as one-answer problems. (Moses, Bjork and Goldenberg 1990)

Children will find mathematics rewarding and challenging only if their teachers are equipped to meet the needs of incorporating excellent problems in their lesson plans. "Because excellent problem-solving questions are seldom created 'on the spot,' teachers will benefit from writing lesson plans that include questions they can use at crucial moments" (Cemen 1989). Of equal importance is the teacher's understanding of the importance of problem solving in the mathematics curriculum.

> If a teacher conceives of mathematics solely in terms of speed, accuracy and one right answer, then it is unlikely that such a teacher will stimulate students to monitor their solution processes,

estimate answers, search for alternate solution methods, pose problems, or engage in similar worthwhile activities. (Grouws and Good 1989)

The factor of multiple solutions is related to the strategy factor. Problems having only one solution may or may not be more difficult than problems having multiple solutions. However, if a student is asked to find all solutions, a natural question is "How can I be sure that I have all solutions?" The effect of multiple solutions on the difficulty of the problem needs to be further researched. (LeBlanc, Proudfit and Putt 1980)

Example Problems and Extensions

In this section, three problems will be presented, one featuring each of the pictorial, symbolic and manipulative modes. Following the examination of the several solutions for each of the problems, ways to revise the problems will be suggested. Solution patterns will also be included for the revised problems.

1. Find all the squares, each of a different area, that can be made on a 4 by 4 (25-dot region) area.

Diagram 1 shows the eight possible solutions if one is restricted to a whole-number solution for each area. The dotted lines within some of the squares are included as an aid to verify the area.

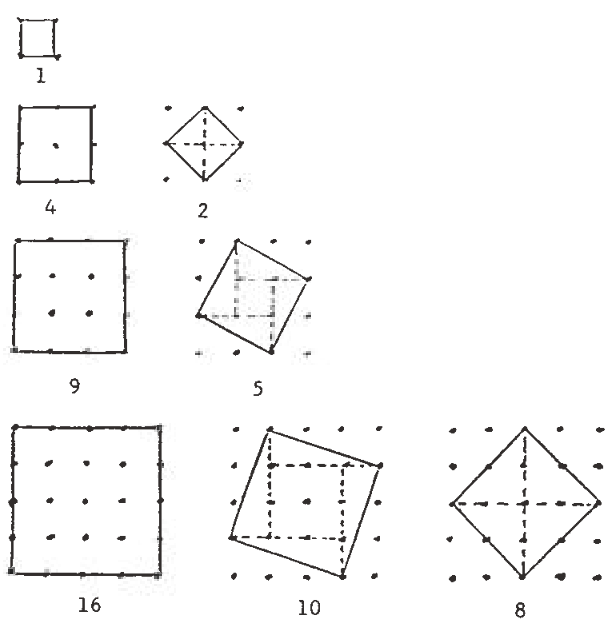


Diagram 1

If the initial problem was revised to feature a 3 by 3 array, the solutions would be indicated by the top five squares in Diagram 1. Similarly, a 2 by 2 array would yield the top three squares in Diagram 1. Diagram 2 shows the seven additional squares that can be made on a 6 by 6 array. A 5 by 5 array would generate all the squares in Diagram 1 as well as the top three squares in Diagram 2.

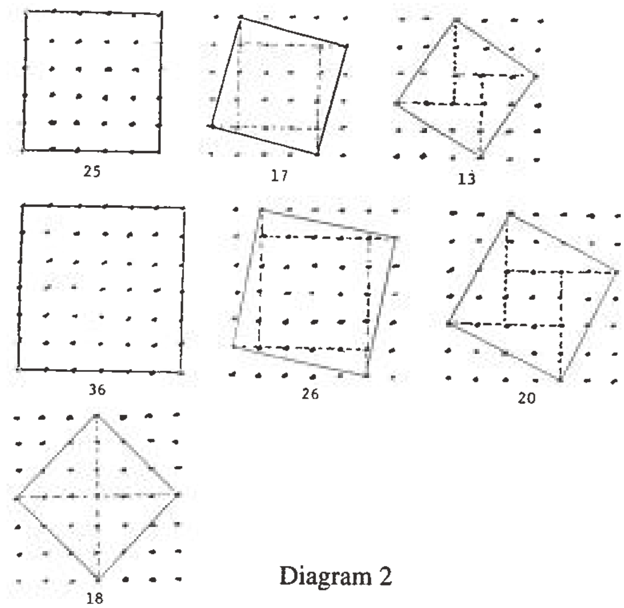


Diagram 2

The following table provides a numerical summary of all the squares that can be made on arrays of ever-increasing dimensions. Students can be encouraged to look for patterns and to predict the number and types of squares for arrays of larger dimensions.

Size of array	**Number of squares**	**Total number of squares**
1 x 1	1	1
2 x 2	1 + 2	3
3 x 3	3 + 2	5
4 x 4	5 + 3	8
5 x 5	8 + 3	11
6 x 6	11 + 4	15
7 x 7	15 + 4	19
8 x 8	19 + 5	24
9 x 9	24 + 5	29
10 x 10	29 + 6	35

2. Find all the ways that bicycles, tricycles and wagons can be made from 20 wheels if there is at least one of each kind and no wheels are left over.

The following chart shows the four possible solutions. The suggested strategy is to begin with one wagon (the vehicle with the largest number of wheels) and to consider the required number of each of the other vehicles to reach the final total of 20 wheels.

Bicycles	Tricycles	Wagons
5	2	1
2	4	1
3	2	2
1	2	3

A simple way to revise this problem would be to change the total number of wheels. The following table shows the possible solutions for numbers of wheels from 16 to 23. The problem featuring 16 wheels has only two solutions, while the problem with 23 wheels has 8, the largest number of solutions.

Wheels	Bicycles	Tricycles	Wagons	Vehicles
16	3	2	1	6
16	1	2	2	5
17	5	1	1	7
17	2	3	1	6
17	3	1	2	6
18	4	2	1	7
18	1	4	1	6
18	2	2	2	6
19	6	1	1	8
19	3	1	1	7
19	4	1	2	7
19	1	3	2	6
19	2	1	3	6
20	5	2	1	8
20	2	4	1	7
20	3	2	2	7
20	1	2	3	6
21	7	1	1	9
21	4	3	1	8
21	1	5	1	7
21	5	1	2	8
21	3	1	3	7
22	6	2	1	9
22	3	4	1	8
22	4	2	2	8
22	2	2	3	7
23	8	1	1	10
23	5	3	1	9
23	2	5	1	8
23	6	1	2	9
23	3	3	2	8
23	4	1	3	8
23	1	3	3	7
23	2	1	4	7

Another variation could be made in the kinds of vehicles chosen. What might be the results if each vehicle had an odd number of wheels (for example, unicycles, tricycles and five-wheeled pentikes)? Students should also be encouraged to look for patterns in the numerical charts.

3. Make a square with the two largest tangram pieces. Find how many ways a square of the same size can be made with other arrangements of tangram pieces. Sketch each way by drawing lines around the chosen sets of pieces. Each way must use a different set of pieces.

Students attempting this problem would benefit by having a sheet of paper containing a number of congruent squares, the size of each to accommodate the square made by the two largest tangram pieces. Diagram 3 shows the five possible congruent squares.

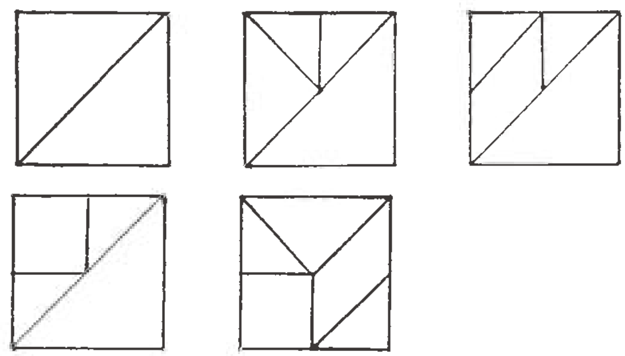


Diagram 3

A variation of this problem would be to ask students to use tangram pieces to make all the different right-angled isosceles triangles. Diagram 4 shows the 12 possible triangles. Interestingly, five different sizes of these triangles exist. Children would probably benefit by having a sheet of paper containing outlines of the 12 triangles.

An extension of either version of the tangram problem featuring the symbolic mode could be accomplished by having students draw each of the five squares and the twelve right-angled isosceles triangles on squared paper. Suppose that the measure of the edges of each square is four units and the measures of the triangles are as follows:

A	hypotenuse	4 units
B—D	equal edges	4 units
E—I	hypotenuse	8 units
J—K	equal edges	6 units
L	equal edges	8 units

Students can then calculate the areas of the squares and triangles as well as the areas of the individual tangram pieces in each illustration. These measures could then be compared with the measures of the actual plastic tangram pieces and combinations.

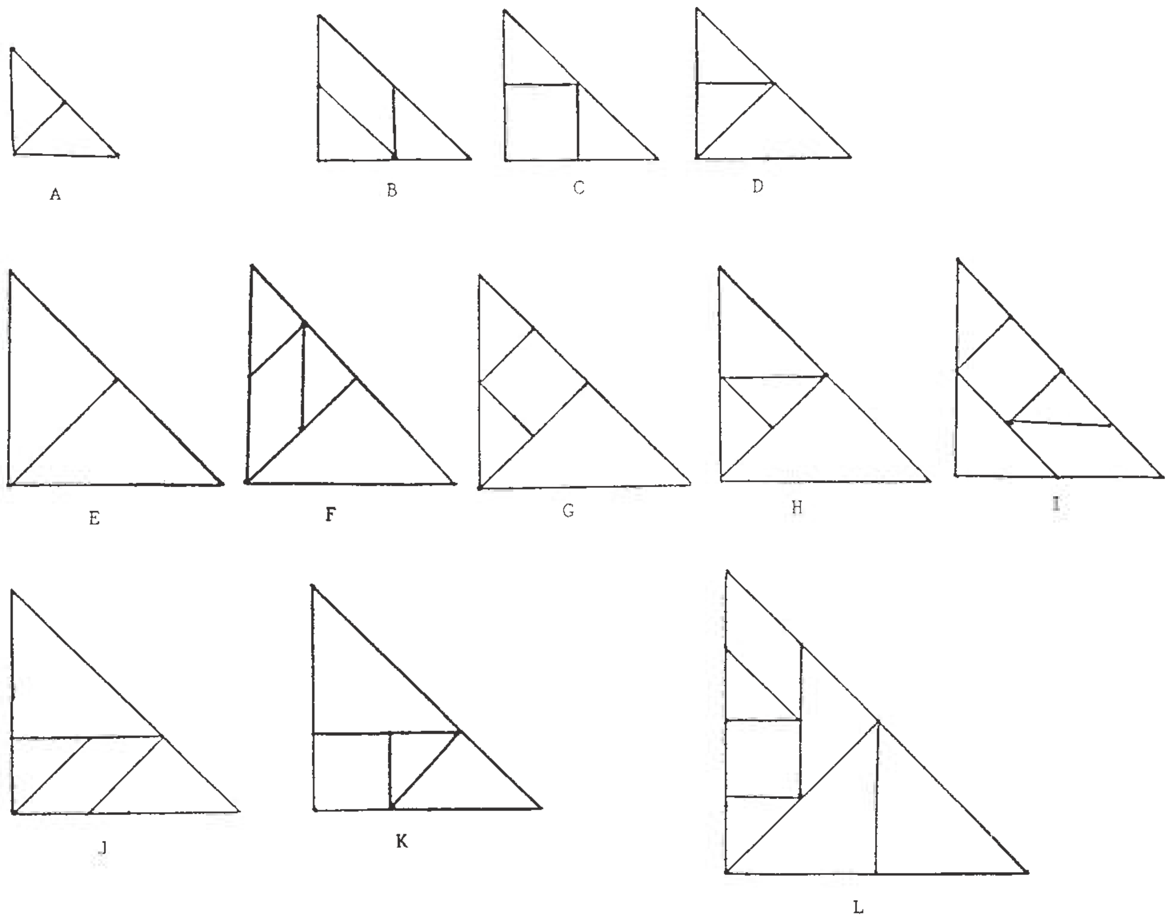


Diagram 4

References

Cemen, P. "Developing a Problem-Solving Lesson." *The Arithmetic Teacher* 37, no. 2 (October 1989).

Charles, R., and F. Lester. "An Evaluation of a Process-Oriented Instructional Program in Mathematical Problem Solving in Grades 5 and 7." *Journal for Research in Mathematics Education* 15, no. 1 (January 1984).

Grouws, D., and T. Good. "Issues in Problem Solving Instruction." *The Arithmetic Teacher* 36, no. 8 (April 1989).

House, P.A., M.L. Wallace and M. A. Johnson. "Problem Solving as a Focus: How? When? Whose Responsibility?" In *The Agenda in Action*, 1983 Yearbook of the National Council of Teachers of Mathematics, edited by G. Shufelt and J.R. Smart. Reston, Va.: NCTM, 1983.

Irons, R., and C. Irons. "Language Experiences: A Base for Problem Solving." In *New Directions for Elementary School Mathematics*, 1989 Yearbook of the National Council of Teachers of Mathematics, edited by P. Trafton and A. Shulte. Reston, Va.: NCTM, 1989.

LeBlanc, J., L. Proudfit and I. Putt. "Teaching Problem Solving in the Elementary School." In *Problem Solving in School Mathematics*, 1980 Yearbook of the National Council of Teachers of Mathematics, edited by S. Krulik and R. Reys. Reston, Va.: NCTM, 1980.

McLeod, D. "Affective Issues in Mathematical Problem Solving: Some Theoretical Considerations." *Journal for Research in Mathematics Education* 19, no. 2 (March 1988).

Moses, B., E. Bjork and E. Goldenberg. "Beyond Problem Solving: Problem Posing." In *Teaching and Learning Mathematics in the 1990's*, 1990 Yearbook of the National Council of Teachers of Mathematics, edited by T. Cooney and C. Hirsch. Reston, Va.: NCTM, 1990.

Noddings, N. "Preparing Teachers to Teach Mathematical Problem Solving." In *The Teaching and Assessing of Mathematical Problem Solving*, Research Agenda for Mathematics Education Volume 3, edited by R. Charles and E. Silver. Reston, Va.: NCTM, 1989.

Romberg, T. *Curriculum and Evaluation Standards for School Mathematics.* Reston, Va.: NCTM, 1989.

Mathematics and the Affective Domain

Marlow Ediger

Cognitive objectives are important to emphasize in the mathematics curriculum. Thus being able to apply what has been learned is salient. Thinking critically and creatively is necessary when determining answers to problems in mathematics. The cognitive domain tends to receive major attention when students attain objectives in mathematics.

Another dimension of teaching stresses students achieving affective goals. Affective or attitudinal goals complement the cognitive dimension. Students then who have positive attitudes toward mathematics should achieve at a more optimal rate in that curriculum area. Positive attitudes influence the self-concept as well as feelings toward others in ongoing lessons and units in the mathematics curriculum.

How might the mathematics teacher assist students to develop quality attitudes in the affective domain?

Quality Attitudes in Mathematics

Teachers are the major decision makers in selecting objectives, learning opportunities and appraisal procedures. They must be highly cognizant of students' attitudes. Perhaps students should increasingly share with mathematics teachers decisions made in ongoing lessons and units. Students' input is vital to guide each to attain more optimally. The role of mathematics teachers might then shift to becoming guides, facilitators and helpers to assist each learner to learn as much as possible. Students need to experience continual success in the mathematics curriculum. The concept of success can be implemented in numerous ways.

Mathematics Textbooks

Using a reputable series of mathematics textbooks, students may achieve optimally on an individual basis. Sequential content in acquiring basic facts, concepts and problem-solving skills might well come from ordered pages in the basal textbook. Students work at their own optimal rate of speed. Mathematics teachers monitor individual progress of each student. Because students progress individually, teachers need to provide background information for each learner prior to beginning a new process. If learners cannot succeed in a given sequence, teachers must guide and direct them. Success at each step of achievement is vital for students on an individual basis. Positive attitudes might then be in the offing for each student. The teacher may emphasize intrinsic or extrinsic motivational approaches to assist optimal mathematics achievement. Branching out from textbook use, students might experience a multimedia approach. Thus software and computers, lifelike experiences as well as audiovisual materials provide enrichment experiences for students.

Students pace their own progress when moving forward sequentially using the basal mathematics textbook as the core of the curriculum. Teachers must provide readiness as needed for each student to understand new facts, concepts and generalizations. Continuous progress in learning is paramount. Quality attitudes toward mathematics should result.

Programmed Learning

Programmed learning (software or books) in mathematics follows models of teaching emphasized by the late B.F. Skinner (1904–1990). Skinner emphasized students learning in small sequential steps. Thus mathematics subject matter is broken down into small segments of knowledge. Students then read from programmed books or from monitors, a small amount of content, after which the learners respond to a multiple-choice item. After responding, the students check their answers with that provided by the teacher. With quality field-tested programs, students individually should respond correctly approximately 90 percent of the time. Programmed materials generally stress the same sequence with read, respond and check. With a high success rate in responding, students should develop quality attitudes. For each sequential step of learning, reinforcement is evident. Each sequential item provides readiness for the next ordered task in ascending complexity.

With programmed learning, students may achieve at their own optimal learning rate. Students should not compare themselves with others in rate of speed and achievement. Each needs to learn as much as possible with feelings of attainment in evidence. Affective goals are then achieved by students individually.

Computer literacy in mathematics is important for all students. Flake, McClintock and Turner (1990, 29) wrote the following:

> Of all the uses of computers, which ones should all students learn to do comfortably and successfully? These questions have stimulated controversy among educators. Some people equate computer literacy with the ability to select and use commercial software. Their argument is that the average person does not need to know how a computer works, what RAM is, or how to write a computer program. They compare using a computer to driving a car—in order to drive, one does not need to know how a car works, what a manifold is, or how to do a tune-up.
>
> On the other hand, some people equate computer literacy with knowledge of programming. Their argument is that in order for a person to use the computer to solve problems, that person needs to have the flexibility and control that a knowledge of programming provides. In short, knowledge is power.

Integration of School and Society

A third philosophy emphasizing the affective dimension in mathematics is the integration of school and society. Life-like endeavors then need to be evident. Separation of the mathematics curriculum from the societal arena is frowned on. That which has utilitarian values in mathematics is emphasized. The practical rather than the abstract becomes paramount. Useful content is acquired by students and applied in the real world of mathematics.

Topics that provide the practical include buying goods and services, balancing a chequebook, budgeting and spending an allowance, comparing prices of items to be purchased, developing a budget, ordering from a sales catalog, shopping wisely, buying items at discount, keeping and maintaining an inventory as well as using credit cards.

Too frequently, students learn abstractions in mathematics that cannot be applied. There is separation then in the abstract from the concrete or the theoretical from the utilitarian. These separations represent dualisms in the mathematics curriculum. Rather, the dual situations need to become one whereby abstractions in mathematics can be used in the real world. The theoretical also is not separated from the practical. Number theory provides guidance and direction on the level of application for each student. The real world and the school curriculum become one in a mathematics curriculum stressing the affective dimension. When students perceive that what has been learned can be used, improved attitudes might well be in the offing.

Pertaining to problem solving, Grossnickle et al. (1983, 177) wrote the following:

> Problem solving is a process by which the choice of an appropriate strategy enables a pupil to proceed from what is given in a problem to its solution. Often the answer is the least important part of the problem-solving process; few of the answers children obtain in school mathematics will have much value in their lives. The ideas used in the process are much more valuable than the answer. Thus, it is important for teachers to determine whether an incorrect answer is due to an error in process or in computation. Do not, however, infer in this discussion that errors in computation are acceptable; rather, keep in mind that overemphasis on answers may impede the pupil's understanding of the process. A pupil with poor computational ability who understands the process can use a calculator to get the answer. A pupil who can compute rapidly and accurately but does not understand the process is lost.

Mastery System of Learning

In a mastery system of learning, mathematics educators select carefully chosen precise objectives prior to instruction. These objectives might be selected several years prior to their implementation in teaching-learning situations if they are chosen on the state or district levels with instructional management systems (IMS).

Prior to instructing a lesson, the teacher announces clearly and concisely what students are expected to learn as a result of teaching and learning. Students then understand what is to be acquired within a given time, such as the implemented lesson plan. The objectives are stated in measurable terms. Either a student does or does not attain what is contained in the objective announced to students prior to instruction. If an objective is not achieved, a different teaching strategy needs emphasis. If an individual student achieved one or more sequential objectives, he or she can move on to the next objective in sequence or order. Students with teacher guidance might then pace their own individual achievement. Each will be at a different place on a continuum in achieving the predetermined measurably stated objectives. Quality attitudes might well be evident when students individually have the opportunity to learn as much as possible.

Contract System

The contract system may become the total mathematics curriculum or emphasize enrichment opportunities to the learner. With adequate background information pertaining to an ongoing unit of study in mathematics, students with teacher guidance plan what to achieve. Each item in the plan is clear and can involve pages to be completed from a single or multiple series of textbooks, construction work in which model geometric figures are constructed and mathematics library books to read and summarize. In the contract, students have numerous vital tasks to record and complete. The due dates are written in the contract.

The contract system emphasizes student selection of content to learn, student sequence of his or her own activities, student purposes in learning and student interest in the mathematics curriculum.

The mathematics teachers' role in the contract system is to encourage, motivate and stimulate student learning. They are guides, monitors and evaluators. No longer are teachers the sole people in determining objectives, learning opportunities and appraisal procedures. Because learners are heavily involved in determining the curriculum, they should become increasingly positive in the affective dimension.

Conclusion

In providing for individual differences and to guide optimal affective achievement, mathematics teachers need to guide students to achieve an adequate self-concept. Adequate self-concept development comes about when teachers assist students to

1. achieve meaningful knowledge so that understanding of acquired subject matter is evident,
2. develop readiness for learning to have learners experience sequence in ongoing activities,
3. increase interest in the mathematics curriculum to attain attention to achieve worthwhile objectives,
4. perceive purpose in achievement to understand reasons for attaining in ongoing lessons, and
5. enjoy mathematics and thus develop quality attitudes in the affective dimension.

Students need to experience procedures in teaching-learning situations which aid optimal achievement in critical and creative thinking, as well as problem solving. Mathematics teachers need to emphasize methodology of instruction that harmonizes with their own unique style of learning. Quality attitudes assist students to achieve vital knowledge and skills.

Software should encourage positive student attitudes in a modern curriculum. Abelson (1992, ix) wrote the following on student control over computers/software:

> Logo is the name for a philosophy of education and for a continually evolving family of computer languages that aid its realization. Its learning environments articulate the principle that giving people personal control over powerful computational resources can enable them to establish intimate contact with profound ideas from science, from mathematics, and from the art of intellectual model building. Its computer languages are designed to transform computers into flexible tools to aid in learning, in playing, and in exploring.
>
> We try to make it possible for even young children to control the computer in self-directed ways, even at their very first exposure to Logo. At the same, we believe Logo should be a general purpose programming system of considerable power and wealth of expression. . . . More than 10 years of experience at MIT and elsewhere have demonstrated that people across the whole range of "mathematical aptitude" enjoy using Logo to create original and sophisticated programs. Logo has been successfully and productively used by preschool, elementary junior high, senior high, and college students, and by their instructors.

References

Abelson, H. *Logo for the Apple II.* Peterborough, N.H.: BYTE/McGraw Hill, 1992.

Flake, J., C. McClintock and S. Turner. *Fundamentals of Computer Education.* 2d ed. Belmont, Calif.: Wadsworth, 1990.

Grossnickle, F., et al. *Discovering Meanings in Elementary School Mathematics.* 7th ed. Chicago: Holt, Rinehart & Winston, 1983.

Subtracting Infinities

Sandra M. Pulver

Consider the example $\int_{-1}^{2} \frac{dx}{x^3}$, a plain and simple integral on the face of it. But this integral is improper because the integrand is discontinuous on the interior of the interval, that is, at $x = 0$. So, to solve, we must break it up.

$$\begin{aligned}
\int_{-1}^{2} \frac{dx}{x^3} &= \int_{-1}^{0} \frac{dx}{x^3} + \int_{0}^{2} \frac{dx}{x^3} \\
&= \underset{b \to 0^-}{Lim} \int_{-1}^{b} x^{-3}\,dx + \underset{b \to 0^+}{Lim} \int_{b}^{2} x^{-3}\,dx \\
&= \lim_{b \to 0^-} \left. \frac{x^{-2}}{-2} \right]_{-1}^{b} + \lim_{b \to 0^+} \left. \frac{x^{-2}}{-2} \right]_{b}^{2} \\
&= \lim_{b \to 0^-} \left. \frac{-1}{2x^2} \right]_{-1}^{b} + \lim_{b \to 0^+} \left. \frac{-1}{2x^2} \right]_{b}^{2} \\
&= \lim_{b \to 0^-} \left(\frac{-1}{2b^2} + \frac{1}{2(1)} \right) + \lim_{b \to 0^+} \left(\frac{-1}{2(4)} + \frac{1}{2b^2} \right) \\
&= \frac{1}{(-2)(-\mathit{very\ small})^2} + \frac{1}{2} \qquad + \text{ second part} \\
&= \frac{1}{(-2)(-\mathit{very\ very\ small})^2} + \frac{1}{2} \qquad + \text{ second part} \\
&= -\infty + \frac{1}{2} \qquad + \text{ second part}
\end{aligned}$$

and the limit of the first part does not exist. So, this integral diverges and we are finished.

But look what can happen when students use $-\infty$ as a concrete limit and *do* go further.

Working on the second part of the limit, students obtain

$$+\left(-\frac{1}{8}\right) + \lim_{b \to 0^+} \frac{1}{2b^2} =$$

$$+\left(-\frac{1}{8}\right) + \frac{1}{2(+\mathit{very\ very\ small})} =$$

$$-\frac{1}{8} + \infty$$

Students then proceed further, saying that the limit of the whole example is therefore $= -\infty + \frac{1}{2} - \frac{1}{8} + \propto$.

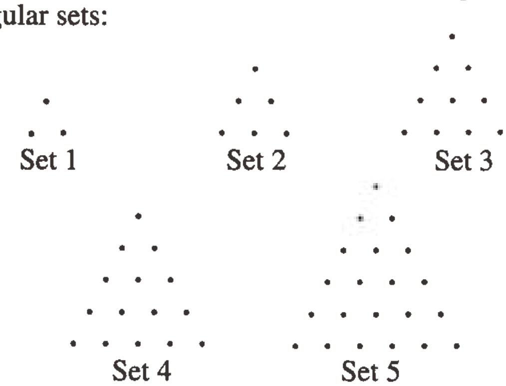
Now the infinities "cancel" each other, leaving $+\frac{1}{2} - \frac{1}{8} = \frac{3}{8}$ as a final result!

Here is an excellent time and place to explain to a student, once again, the concepts of an infinite limit. If a limit is found to be infinite, this means that the limit simply does not exist, and to continue to the second part of the example is foolish. The limit of the whole example cannot exist if the limit of the first part does not exist!

Generating Geometric Sets

David R. Duncan and Bonnie H. Litwiller

Geometric shapes can be used to represent numbers. For example, consider the following five triangular sets:



For each of these triangular sets, three types of points will be counted:

1. The points on the set boundary
2. The points in the set interior
3. The points in the entire set

Table 1 reports the numbers of points in these categories:

Table 1

Triangular Set	**1**	**2**	**3**	**4**	**5**
Points on boundary	3	6	9	12	15
Points in interior	0	0	1	3	6
Total points in triangular set	3	6	10	15	21

Square sets and hexagonal sets can also be pictured and counted in the same ways as for triangular sets.

Square Sets

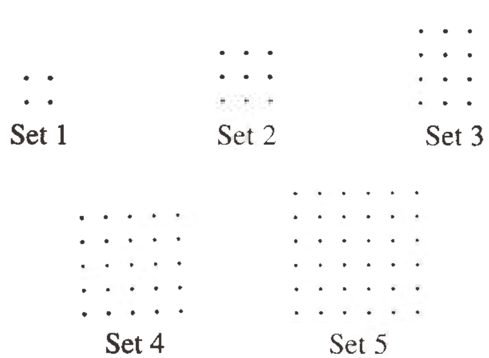


Table 2

Square Set	**1**	**2**	**3**	**4**	**5**
Points on boundary	4	8	12	16	20
Points in interior	0	1	4	9	16
Total points in square set	4	9	16	25	36

Hexagonal Sets

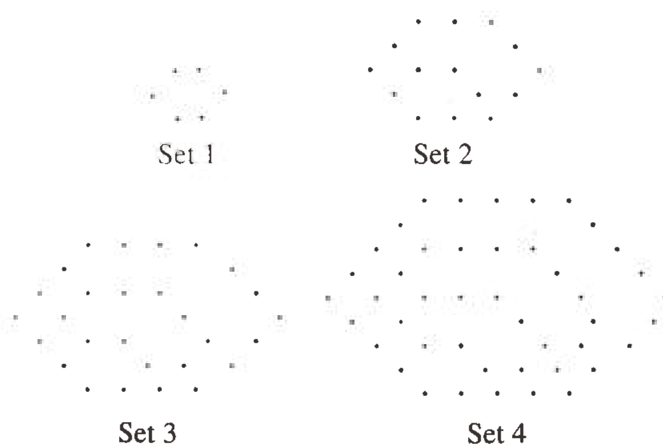


Table 3

Hexagonal Set	**1**	**2**	**3**	**4**
Points on boundary	6	12	18	24
Points in interior	0	3	10	21
Total points in hexagonal set	6	15	28	45

A study of Tables 1, 2 and 3 suggests a variety of patterns. The most obvious of these patterns is that the *boundary* totals for triangular, square and hexagonal sets are consecutive natural number multiples of 3, 4 and 6, respectively. Other patterns are seen on extended Tables 1, 2 and 3; Table 3 has also been augmented with the last row of Table 1:

Table 1 (Extended)

Triangular Set	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Points on boundary	3	6	9	12	15	18	21	24	27
Points in interior	0	0	1	3	6	10	15	21	28
Total points in triangular set	3	6	10	15	21	28	36	45	55

The circled numbers are *three* columns apart.

Table 2 (Extended)

Square Set	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Points on boundary	4	8	12	16	20	24	28	32	36
Points in interior	0	1	4	9	16	25	36	49	64
Total points in square set	4	9	16	25	36	49	64	81	100

The circled numbers are *two* columns apart.

Table 3 (Extended)

Set Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Points on boundary of hexagonal set	6	12	18	24	30	36	42	48	54
Points in interior of hexagonal set	0	3	[illegible]	21	36	55	78	105	136
Total points in hexagonal set	6	15	28	45	66	91	120	153	190
Total points in triangular set	3	6	10	15	21	28	36	45	55

The numbers of interior points (beginning with 3) are *every other* triangle set total (3, 6, 10, 15, 21, 28, 36, 45, 55, . . .).

Remarkably, the three patterns (triangular, square and hexagonal) that relate interior and total points are all different. The first two (triangles and squares) were similar in that they involved only numbers of the type being studied, while the hexagonal pattern involved hexagonal and triangular sets.

The patterns which have been described can be communicated by the use of notation.

Let T_n, S_n and H_n be the total number of points in the nth triangular, square and hexagonal sets; TI_n, SI_n and HI_n be the number of interior points in the nth triangular, square and hexagonal sets; TB_n, SB_n and HB_n be the number of boundary points in the nth triangular, square and hexagonal sets.

The patterns are as follows:

$TB_n = 3n$, $SB_n = 4n$, $HB_n = 6n$
$TI_n = T_{n-3}$ (for $n \geq 4$)
$SI_n = S_{n-2}$ (for $n \geq 3$)
$HI_n = T_{2n-3}$ (for $n \geq 2$)
(for example, for $n = 4$, $HI_4 = 21 = T_{2(4)-3} = T_5$)

Additional patterns can also be deduced, based on the preceding patterns and general observation that, for each figure, the boundary and the interior numbers sum to the total numbers in the sets:

$TB_n + TI_n = T_n$
Thus,
$TB_n = T_n - TI_n$
$3n = T_n - T_{n-3}$ (for $n \geq 4$)

$SB_n + SI_n = S_n$
$SB_n = S_n - SI_n$
$4n = S_n - S_{n-2}$ (for $n \geq 3$)

$HB_n + HI_n = H_n$
$HB_n = H_n - HI_n$
$6n = H_n - T_{2n-3}$ (for $n \geq 2$)

Challenges

1. Construct other polygonal sets (for example, pentagonal, heptagonal and octagonal).
2. Justify the patterns of this article by writing proofs.
3. How could the meanings of T_n, S_n and H_n be extended so that the results are valid if any integer value of n is used?

Geometry Geopardy

Yvette d'Entremont

The fast-paced game shows on television are motivating and fun to watch as we try to match wits with the contestants. "Jeopardy" is one such game show. The unique characteristic of "Jeopardy" is that the contestants are given answers or clues in various categories, and they must respond by posing a question that corresponds to the item described in the clue.

The following activity is based on this popular game show except the questions are all related to geometry. It would be valuable to watch "Jeopardy" a few times to get the feel for the game. Quick thinking is vital in "Jeopardy" because the first person to hit his or her buzzer earns first chance at posing the right question. A correct response adds the dollar value of the item to the player's monetary total, but an incorrect response means the total is reduced by that amount. The more you know of the game "Jeopardy," the more fun Geopardy will be. You will be able to incorporate a Daily Double, Double Geopardy and Final Geopardy into the game which will make Geopardy appealing to the students. All you need to play Geopardy is a basic knowledge of "Jeopardy," an overhead projector, some transparencies and some small Post-it Notes to cover the questions.

Preparing for Geopardy

After watching "Jeopardy" a few times, you must choose your categories, prepare a set of clues and responses for each and decide how to incorporate the Daily Double, Double Geopardy and Final Geopardy into the game. Categories could simply be chapter titles, such as "parallel lines" or "congruent triangles," or more creative themes, such as "words beginning with C." Decide how many clues to have in each category, and try to rank the items from easiest to hardest. Label each clue with its point value, and prepare an answer sheet which will be in the form of questions. Figure 1 is an example of the clue sheet used for this activity.

Playing Geopardy

The clues for the following activity are based on Grades 7, 8 and 9 Journeys in Math series. You will only need transparencies of the game boards containing the clues (Games 1, 2 and 3), an overhead projector, Post-it Notes to cover the clues and a person to keep score. The objective of this activity is to review and reinforce geometrical concepts.

To use this activity in an average-sized classroom, the students should first be organized into four to six teams with each team occupying its own row of desks or seats. It is preferable but not essential that the teams be of equal size. It would be helpful to have one student assisting the teacher by keeping score. Before play begins, review the game rules with the students.

Before play begins, cover all clues on the game boards with small Post-it Notes. To begin play, randomly select one of the front-row students to select an item from the game board. If the student chooses "Definitions for 500," remove the Post-it Note for that item only. Once the clue has been revealed, the front-row students can compete for that item. The first one to raise his or her hand gets the first attempt at the correct question. Depending on the appropriateness of the student's response, the team's score will be increased or decreased by the amount of the item value. Once that item has been completed, the item should be left uncovered. For the second item, all students in the second rows will be the contestants and must choose a covered item and the game continues. Daily Double, Double Geopardy and Final Geopardy may be incorporated into the game in the same fashion as done in the television series.

Conclusion

Games can be fun, challenging and motivating. They also provide a change of pace and can involve all members of the class. Encourage the students to try to silently respond even when they are not active contestants. Most students will try to do this anyway to see if they can beat the others. The game moves fast enough to keep the students from tuning out. You may wish to photocopy the blank game board (Figure 1) and create your own categories and clues. You also may wish to create a game of

Geopardy with categories from algebra or any other concept. It provides an excellent means of review while holding the students' attention.

Possible Questions as Student Responses

Game 1

Lines and Angles

(100) What is the angle measurement of a right angle?
(200) What is the angle measurement of an acute angle?
(300) What are opposite angles?
(400) What are complementary (or adjacent) angles?
(500) What are alternate-interior angles?
(600) What is an exterior (or an obtuse) angle?

Definitions

(100) What is a square?
(200) What is the vertex?
(300) What is an angle bisector?
(400) What is a hexagon?
(500) What is perpendicular?
(600) What is 12?

Diagrams

(100) What is a right triangle?
(200) What is an isosceles triangle?
(300) What is a scalene triangle?
(400) What are congruent triangles?
(500) What is a reflex angle?
(600) What is a contained angle?

Game 2

Lines and Angles

(100) What is the angle measurement of a straight angle?
(200) What is 45 degrees?
(300) What is the angle measurement of an obtuse angle?
(400) What are supplementary angles?
(500) What are corresponding angles?
(600) What are co-interior angles?

Diagrams

(100) What is a cube?
(200) What is a parallelogram?
(300) What are parallel lines?
(400) What is a trapezoid?
(500) What is a rectangular pyramid?
(600) What is a rhombus?

Symbols

(100) What is pi?
(200) What is a segment?
(300) What is a line?
(400) What is perpendicular?
(500) What is congruent?
(600) What is the metric symbol?

Game 3

Diagrams

(100) What is the hypotenuse?
(200) What is 180 degrees?
(300) What is the radius?
(400) What is the median?
(500) What is an inscribed angle?
(600) What is the apothem?

Definitions

(100) What is a degree?
(200) What is a heptagon?
(300) What is a rhombus?
(400) What is a reflex angle?
(500) What is the centroid?
(600) What is the orthocentre?

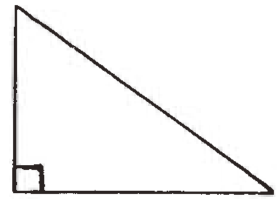
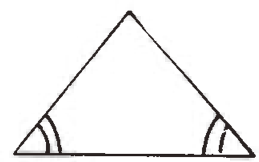
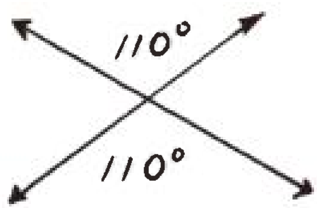
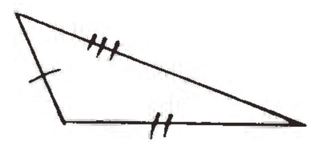
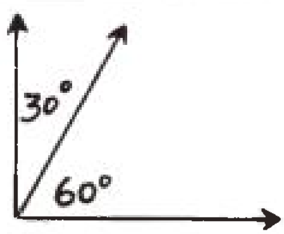
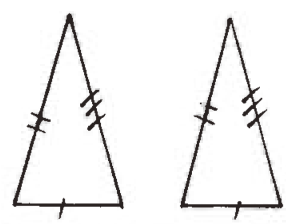
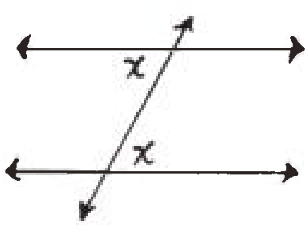
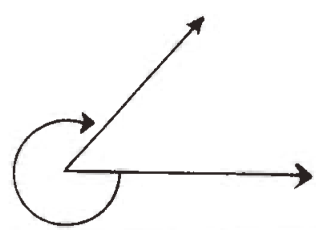
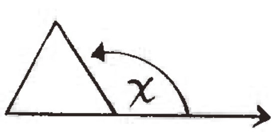
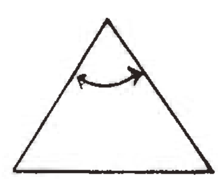
Formulas

(100) What is the formula for the area of a parallelogram?
(200) What is the formula for the circumference of a circle?
(300) What is the formula for the area of a triangle?
(400) What is the formula for the area of a circle?
(500) What is the formula for the area of a trapezoid?
(600) What is the formula for the volume of a cylinder?

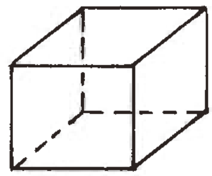
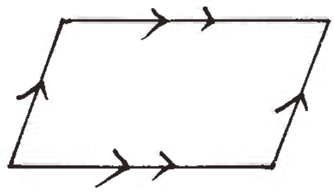
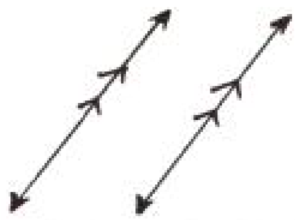
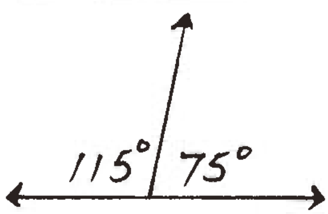
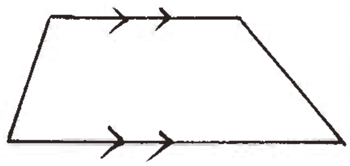
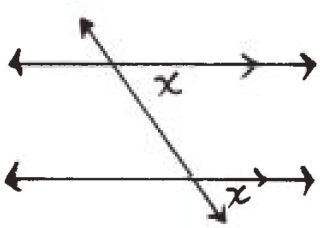
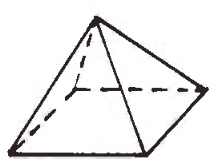
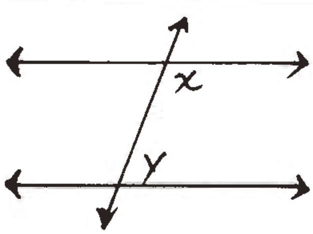
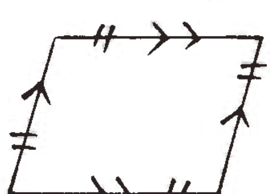
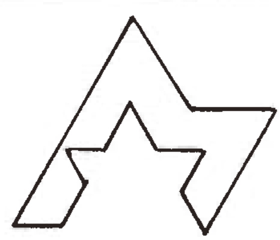
Figure 1

Value			
100			
200			
300			
400			
500			
600			

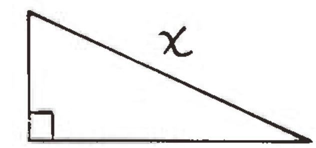
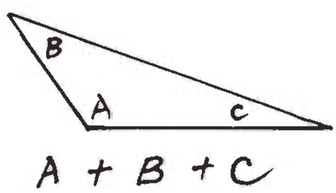
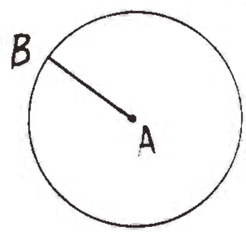
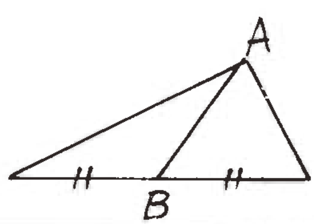
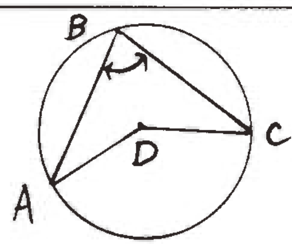
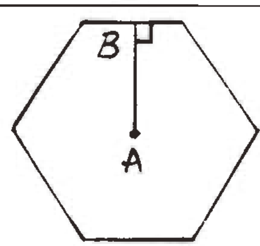
Game 1

Value	Lines and Angles	Definitions	Diagrams
100	90°	a figure with 4 equal sides and 4 right angles	
200	< 90°	the point of intersection of the 2 rays of an angle	
300		cuts an angle into two equal parts	
400		the shape of a honeycomb	
500		the relationship between north/south and east/west	
600		the number of sides of a dodecahedron	

Game 2

Value	Lines and Angles	Diagrams	Symbols
100	$180°$		π
200	degrees in a bisected right angle		
300	$>90° < 180°$		
400			$\perp$
500			$\cong$
600			

Game 3

Value	Diagrams	Definitions	Formulas
100		a unit of angle measure	bh
200		a polygon with 7 sides	$2\pi r$
300		parallelogram with 4 equal sides	$\frac{bh}{2}$
400		an angle between 180° and 360°	πr^2
500		point of intersection of 3 medians of a triangle	$\frac{h(a+b)}{2}$
600		point of intersection of 3 altitudes of a triangle	$\pi r^2 h$

Calculator Explorations in Junior High Mathematics

A. Craig Loewen

The fact that calculators are inappropriate for presenting or discussing all mathematical topics is probably true; like any good instructional tool, they have their limitations. However, we must be careful not to rule out all instructional applications of calculators due to our reluctance to employ them in specific situations.

It is becoming difficult to argue that calculators should not be a part of mathematics instruction: calculators are simply too common in our society to be denied—it almost seems impossible or even unreasonable to expect or ask students *not* to use them. Our task as teachers is to decide when and how calculators are to be used, not if they should be used. Teachers must learn to use the calculator as a tool for facilitating instruction and experimentation. The calculator can be a useful tool for helping us all explore the nature and limitations of formulas and other mathematical relationships addressed in the junior high mathematics curriculum. This article will present four such explorations.

What Is a Calculator Exploration?

In a calculator exploration, the teacher introduces and monitors an experiment whereby students can derive or validate a mathematical idea through many exemplars. A calculator exploration activity involves the following steps: (a) a topic, idea, formula, property or other mathematical relationship is briefly introduced to the students, (b) the brief discussion is followed by a period of investigation where the limitations and plausibility of the idea or relationship are explored, and (c) the exploration concludes with a summary and generalization of the main idea, formula, fact or relationship.

A Familiar Example

Our first example of a calculator exploration involves an activity designed to determine the approximate value of pi (π). In this exploration, the students will use a piece of string to measure the circumference and diameter of a variety of cylindrical objects. The students will construct a list of the measurements for each cylinder and organize the information in a table, comparing circumference, diameter and the quotient of circumference divided by diameter (Figure 1). The students should find that circumference for each cylinder is a little more than three times the diameter of that cylinder. If students combine their results with those of other students, they may be able to compute an approximation of the value of π.

To engage the exploration, the teacher could start by rolling a tin can one rotation along the surface of a table or blackboard—the path of the can may be shown by marking the starting and ending points and connecting the two with a straight line (it is important that students realize the length of the line drawn represents the can's circumference). The distance the tin can has rolled may now be compared with the diameter of the can by tracing the can, measuring the diameter and the length of the path or marking diameter lengths along the path (Figure 2). The teacher may wish to repeat this process with another can of a different size to show that in both cases the circumference is approximately three times the diameter (or conversely that the diameter is 1/3 the circumference). The students are now ready to enter into an exploration of the value of π.

As the students measure the diameter and circumference of each cylinder, they should be sure to record the values in a table. A great deal of information is typically collected, analyzed and summarized in calculator explorations, so one must employ a mechanism for keeping account of this information (the need to generate and/or keep a table is a powerful problem-solving strategy and should not be de-emphasized in the investigative process).

A large number of examples will be needed to attain a reasonable approximation of the value of π. The examples may be collected in various ways such as having each student in the class measure one object and report the results to the entire class, or by having students work in small groups where each group is responsible for measuring five to ten different objects. Once all the data have been collected, students can compute the ratio of circumference to diameter for each object as well as compute an arithmetic mean of these quotients. This mean should be relatively close to the actual value of π (the teacher may wish or need to engage a discussion of experimental error). Students may enjoy calculating a whole class mean, comparing this mean with the results of individual groups (who was closest?) and to the actual value of π (3.14159. . .). Conducting calculator explorations using group structures is not difficult—calculator explorations and cooperative learning activities are extremely compatible.

After the calculations are finished, students will quickly realize that few or no wildly divergent ratios between the circumference and diameter exist—if the activity is completed carefully, each ratio is approximately three to one. Students should be encouraged to summarize the relationship in an equation such as

Circumference ≈ 3 × diameter or Circumference = π × diameter

The large number of consistent examples encourages a sense of confidence and believability in an otherwise extremely abstract relationship. The sense of believability enhanced by this physical experience will encourage retention. In essence, the variety of examples combined with the concrete exploration has encouraged a form of informal inductive proof.

Explorations, such as the one described above, are possible only through the use of the calculator. Students would be unwilling to complete the requisite number of computations (and probably incapable of accurately completing them) without the aid of the calculator. In calculator explorations, students are working with real data which are collected firsthand. Real data are rarely whole numbers (students often describe these as "messy numbers"). However, students need to learn to work with "messy" numbers drawn from real objects, and such experiences may serve to provide the concept with a sense of relevance.

In calculator explorations, the calculator serves as a tool to free the student's mind to focus on a relationship without the frustration of repetitive computations. Generally, in calculator explorations, the relationship or idea of interest takes precedence over the computations. In any classroom activity where this generalization is true, the calculator may be effectively employed as a learning/teaching tool. Three other calculator explorations are provided for the reader's consideration.

Exploration 1: Summing the Interior Angles of a Triangle

Objective

Determines that the sum of the interior angles in a triangle is 180°.

Materials

Protractor, Figure 3 (duplicated as student handout), pencil, ruler, calculator.

Process

To participate in this activity, students must be able to use a protractor to find the measure of a given angle. Once this skill has been established, the teacher can distribute the handout (Figure 3) and explain how the chart at the bottom of the page is to be completed. The students should be encouraged to begin with the given triangles but then branch out to include other triangles they have constructed.

Generalization

In this activity, students will quickly learn that regardless of the shape of the triangle (or the type of angles used to build the triangle, that is, acute, obtuse or right), the three angles always sum to a constant value (180°). The teacher may wish to adapt and extend the activity to address the sum of the interior angles of any quadrilateral. In the extended activity, students should realize that any quadrilateral can be divided into two triangles by connecting diagonally opposite corners (Figure 4), and that this realization can be used to explain why the interior angles of any quadrilateral sum to 360°. The activity can be further extended to five- (and more) sided figures.

Exploration 2: The Pythagorean Theorem

Objective

Recognizes the relationship between the lengths of the sides of any right-angled triangle, that is, that the length of the hypotenuse squared is equal to the sums of the squares of the lengths of the remaining two sides.

Materials

Protractor, Figure 5 (duplicated as student handout), pencil, ruler, calculator.

Process

To begin this activity, the teacher will need to emphasize that we are working with right-angled triangles, and that one side of the right-angled triangle has a special name: hypotenuse. The exploration is conducted by asking students to measure the lengths of each of the sides of the given right triangles and to record these lengths in the chart shown at the bottom of Figure 5. After measuring the given triangles, students are again encouraged to construct one or more of their own right-angled triangles and include the measurements of these triangles in their charts.

Generalization

Through this activity, students should generalize that $a^2 + b^2 = c^2$ where c is the length of the hypotenuse (the side opposite the right angle) and a and b represent the lengths of the remaining two sides for *any* triangle containing a right angle.

Exploration 3: Area of a Circle

Objective

Determines the value of pi (π) through exploring the relationship between the radius and area of a given circle.

Materials

Figures 6 and 7 (duplicated as handouts), pencil, millimetre ruler, calculator, protractor (or photocopied sheets of various-sized circles).

Process

Before undertaking this activity, the student must first review the process of determining the area of a rectangle. The student must also be able to identify and measure the radius of a circle.

To begin the activity, students draw the largest rectangular shape they can inside the circle (Figure 7). The area of this rectangle is calculated after measuring its length and width. This value should be entered into the chart in Figure 6. The process of constructing and measuring rectangles should continue as long as the rectangles are sufficiently large for reasonably accurate measurement (a completed example is shown in Figure 8). After the last rectangle has been drawn and measured, the areas of all rectangles are summed with the aid of the calculator. The final computation involves dividing the experimental area by the square of the radius of the circle. The students should be encouraged to repeat the process and computations beginning with a different circle. Each time the process is completed, the students should find that the ratio of area to radius squared is a value a little larger than three.

This relationship between area and radius can also be seen through considering Figure 9. The shaded area within the square may be described as having an area equal to the radius of the circle squared (r^2). The entire area of the circle then could be described as something less than $4r^2$ (and through visual inspection) probably approximately $3r^2$.

Some questions to ask:

- How does this process approximate the area of the circle?
- Assuming we have stayed within the boundaries of the circles with all our rectangles, how should our estimate compare with the actual area of the circle?

Generalization

While the value π is the constant relating circumference of a circle to its radius, π similarly relates area of a circle to its radius squared. The area of a circle is approximately equal to a little more than three times the radius squared.

A final note: The teacher may wish to allow students to develop their own experimental process, that is, to devise their own activity to test a given relationship or formula. Further, the teacher may wish to encourage this process when students suggest or even adopt inappropriate relationships. Though it is not true that these explorations constitute valid proofs of mathematical ideas, the process of revealing situations where a relationship does not hold may be useful in breaking students' error patterns.

Figure 1: Calculating the value of pi (π) through investigation of circumference of circles (cylindrical objects)

OBJECT	CIRCUMFERENCE (C)	DIAMETER (d)	CIRCUM ÷ DIAM ($C \div d$)
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

INSTRUCTIONS:

[a] Measure the circumference and diameter of 10 cylindrical objects and record the values in the table above.

[b] For each object calculate the value of the circumference divided by the diameter.

[c] Answer the questions below.

QUESTIONS:

[a] What do you notice about the values in the right most column of your table? What does it mean?

[b] Calculate the average of all of the values in the right most column. These values should average to approximately 3.14. How close is your average? Why is your average not exact?

[c] Compare your average with that of a classmate. Who was closest? Calculate the average for the entire class. How close is this average to 3.14?

Figure 2: Approximating the value of pi (π) through investigation of circumference of circles.

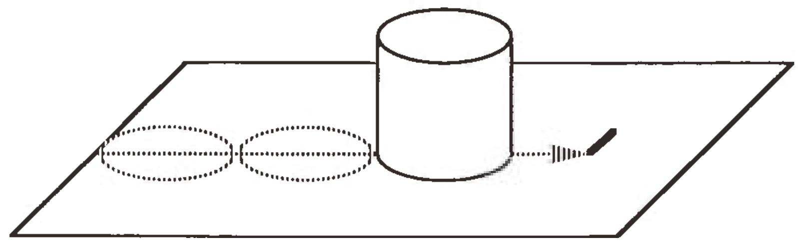
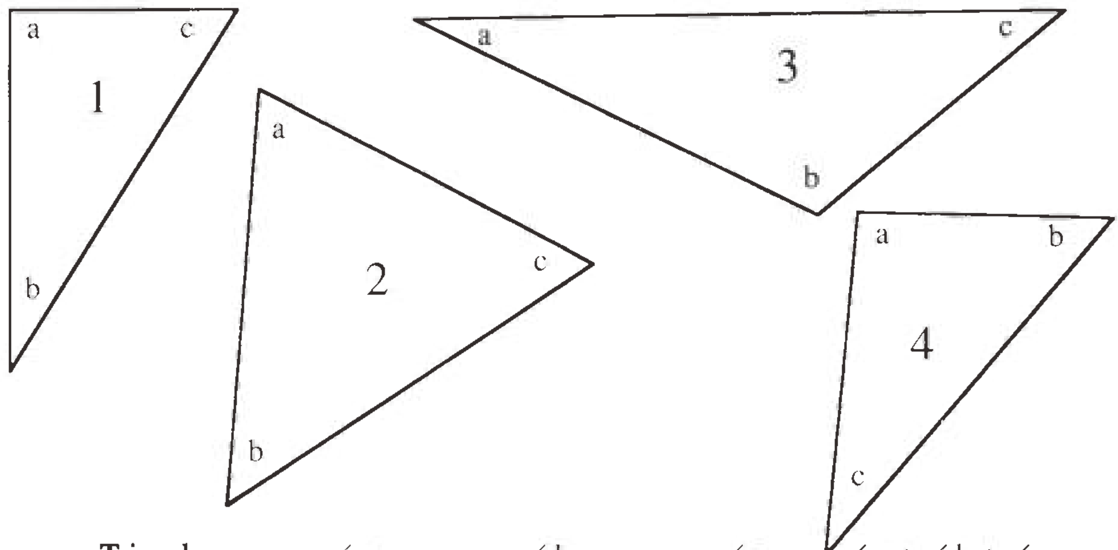


Figure 3: Calculating the sum of the interior angles of a triangle.



Triangle	∠a	∠b	∠c	∠a + ∠b + ∠c
1				
2				
3				
4				
5				

Instructions:

[a] Using a protractor, measure each of the angles in each triangle above. Record the measure of each angle in the table above.
[b] Sum the measures of the angles to complete the fifth column of the chart.
[c] Draw one more triangle on the back of this page to complete the fifth row.
[d] Answer the questions below.

Questions:

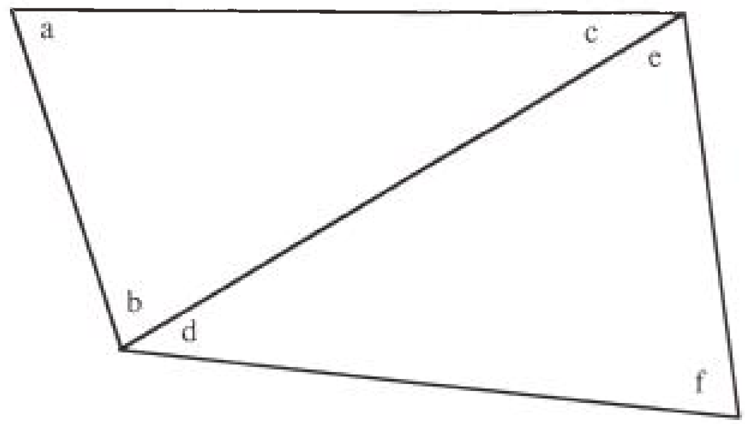
[a] What do you notice about the values in the last column of your table? Generalize to a rule about all triangles.

[b] Construct a triangle with one very large angle. Measure. Does the rule still hold?

[c] Hypothesize: what would the interior angles of any four sided figure sum to? Construct an experiment to test your hypothesis.

[d] Does it matter which angle of these triangles is labeled a, b, or c? Why?

Figure 4: Demonstrating that the measure of the interior angles of a quadrilateral sum to 360°.

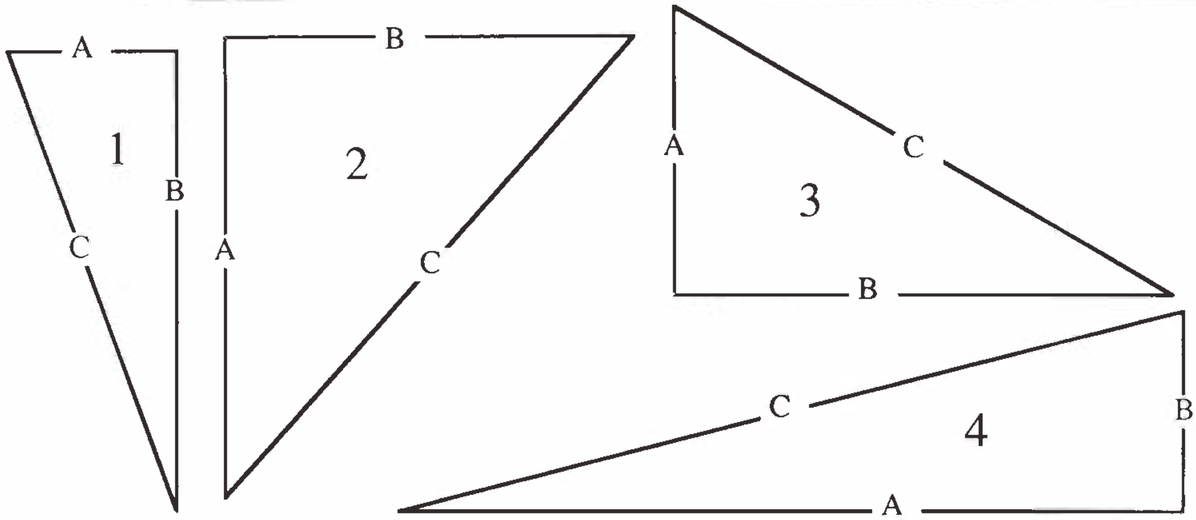


$\angle a + \angle b + \angle c = 180°$

$\angle d + \angle e + \angle f = 180°$

$\angle a + \angle b + \angle c + \angle d + \angle e + \angle f = 360°$

Figure 5: Exploring the Pythagorean relationship.



Triangle	Length of A	Length of B	Length of C	A^2	B^2	$A^2 + B^2$	C^2
1							
2							
3							
4							
5							

Instructions:

[a] Using a ruler, measure the length of each side of each triangle above. Record the length of each side in the table above.

[b] Using these measurements, complete the remaining columns in the chart.

[c] Draw one more triangle with a right angle on the back of this page to complete the fifth row.

[d] Answer the questions below.

Questions:

[a] What do you notice about the two last columns in the chart above? Summarize this result to a rule about right angled triangles.

[b] Does it matter which of the sides of triangle are labeled A, B or C? Why?

[c] Does this relationship hold true for triangles that do not have one right angle? Draw two or more such triangles and test your hypothesis.

Figure 6: Exploring the relationship between area and radius of a circle.

Radius of Circle:

Rectangle	Length	Width	Area	Rectangle	Length	Width	Area
1				13			
2				14			
3				15			
4				16			
5				17			
6				18			
7				19			
8				20			
9				21			
10				22			
11				23			
12				24			

Area of Circle:

Instructions:

[a] Measure and record the radius of the circle.
[b] Construct a rectangle inside the circle.
[c] Measure the length and width of rectangle and calculate its area.
[d] Continue constructing and measuring rectangles until the circle is virtually filled.
[e] Sum all of the areas of all of the rectangles to approximate the area of the circle.
[f] Repeat with the second circle. Try again with a large circle you draw.

Questions:

[a] Calculate the approximate area of the circle given.

[b] Divide the area of the circle by the square of the radius of that circle.

[c] Do you think dividing the area by the square of the radius would give a similar value for all circles? Why or why not?

Figure 7: Exploring the relationship between area and radius of a circle.

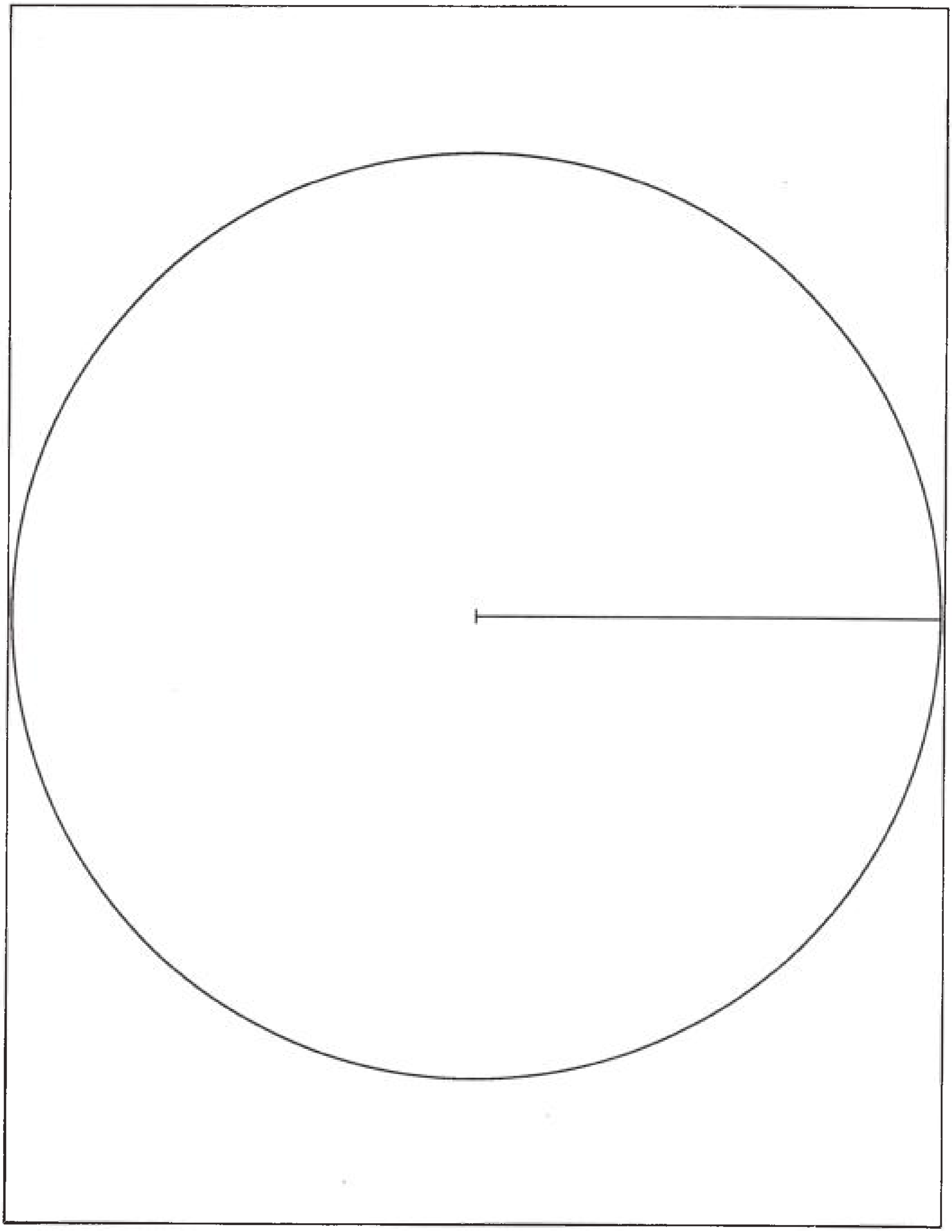
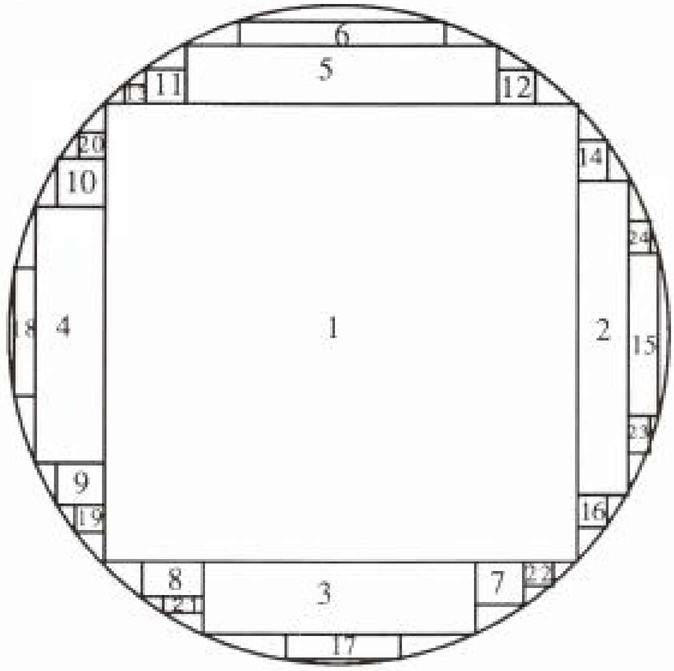


Figure 8: Completed example of Exploration 3.



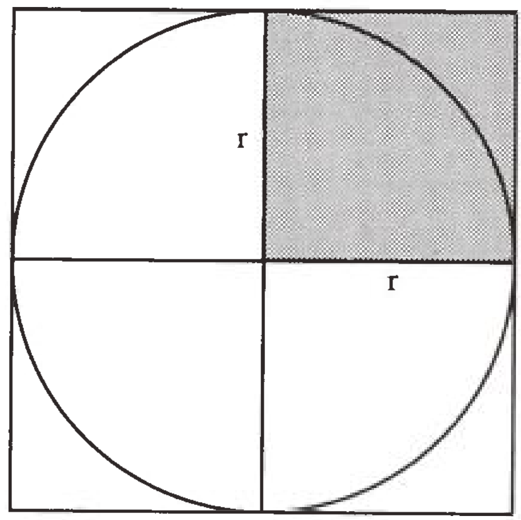
Rectangle	Length	Width	Area
1	6.1	5.9	36.0
2	4.1	0.6	2.5
3	3.5	0.9	3.2
4	3.3	0.9	3.0
5	4.0	0.7	2.8
6	2.6	0.3	0.8
7	0.6	0.5	0.3
8	0.8	0.4	0.3
9	0.6	0.5	0.3
10	0.6	0.6	0.4
11	0.5	0.4	0.2
12	0.5	0.4	0.2

Rectangle	Length	Width	Area
13	0.3	0.2	0.1
14	0.5	0.4	0.2
15	2.1	0.4	0.8
16	0.4	0.4	0.2
17	1.5	0.3	0.5
18	1.6	0.3	0.5
19	0.4	0.3	0.1
20	0.3	0.3	0.1
21	0.5	0.2	0.1
22	0.4	0.3	0.1
23	0.5	0.3	0.2
24	0.4	0.3	0.1

Approx. Area of Circle = 53.0 cm
Radius = 4.2 cm
Radius Squared = 17.6 cm
Approx. of π = 53.0 ÷ 17.6
= 3.01

Area of Circle: 53.0

Figure 9: Another look at the relationship between area and radius of a circle.



Note: the gray shaded area represents an area of radius squared. Therefore, the circle covers an area somewhat less than 4 x radius squared. Through visual inspection we can determine that each quarter of the circle covers about 3/4 of each square, therefore we might estimate the area of the circle to be about $4(3/4\ r^2)$ or about $3r^2$.

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