Mathematically Speaking: Communication in the Classroom

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One key standard proposed by the National Council of Teachers of Mathematics in *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989) is "Mathematics as Communication." *Professional Standards for Teaching Mathematics* (NCTM 1991) offers direction for teachers to address this standard through their teaching practices.

Focusing on communication requires neither a defensive position nor a fabrication of connections between language and mathematics learning. Indeed, the burden of proof should be put on those who wish to dichotomize them. Whether implicitly believed or explicitly stated, connections between language and mathematics are pervasive. Significantly, however, these connections are open to wide interpretation; that is, they depend on how one perceives language, mathematics and learning. This view may be illustrated by three scenarios taken from personal experience.

In the first instance, I was in a store and overheard a conversation between a salesperson and a customer. I came along just as the salesperson was telling the customer that he was a former high school mathematics teacher and what his philosophy of teaching was. "I see mathematics as a language," he said, "and I taught it that way. I taught the grammar of mathematics." The salesperson's perceptions of mathematics and language were evident.

The second scenario involves a student who was taking a course in teaching English as a second language at the same time she was in my curriculum and instruction course in junior high mathematics. On several occasions, she mentioned many similarities between the two courses and that my views of learning were much like those of the other professor. In particular, she noted our mutual emphasis on the need for context and the role of personal experience. Interestingly, when I had a conversation about this with the other professor, he expressed surprise that there were connections between learning a language and learning mathematics. He saw mathematics as comprising rules and facts best learned in a rote fashion. This reminded me that I have encountered teachers who have embraced the principles of whole language learning, yet who still think of mathematics as basically a series of algorithmic procedures such as long division.

The third example arose from an exam question that addressed the importance of using manipulative materials in mathematics. One student gave the following response:

> Most of our language is taught at first with symbolic ideas. "Horse" is given meaning in the presence of the animal or a picture of one. No one tries to teach children what a horse is without some sort of representation. If we make the language of mathematics more tangible, the concepts therein will be more easily learned.

This was a future teacher who was surprised and delighted when he found that even he could benefit from the use of concrete materials.

These incidents show that cognizance and acceptance of the connections between language and mathematics may require re-examination of fundamental beliefs about learning. The role of language in learning mathematics may be clarified by illustrations of how teachers may implement guidelines offered by the NCTM for "Mathematics as Communication."

Use Problems and Materials to Promote Communication

If you want to cultivate communication among your students, you must provide problems or tasks that stimulate discussion. Some problems are more likely to stimulate discussion. This is not due to the mathematical understanding required to solve the problems but rather to other features inherent in them. Ask students to clarify and justify their ideas orally and in writing by relating them to models such as manipulative materials, pictures and diagrams. Where appropriate, the use of concrete materials fosters language and mathematical vocabulary development because children can talk about what they are doing with the materials and see how the concrete action is related to written symbols. While inventive teachers may enjoy making up appropriate assignments, many textbook problems may be adapted to include the particular features discussed in this article.

Problems that require the solvers to supply data necessitate collaboration. Requiring each student in a group to contribute to a problem before it can be solved allows less verbal students to enter into discussion naturally and fosters continued participation. Some typical "involvement" problems are listed below:

- If you each take enough candles for your next birthday cake, how many candles will you need in all for your group? (Model: real birthday candles)
- If all the students in our class bring their sisters and brothers to our class picnic, will the ratio of boys to girls at the picnic be the same as the ratio of boys to girls in our class? (Model: two colors of chips or blocks to represent girls and boys)
- Estimate how many minutes it takes each group member to get to school. Organize and display your data. Find the mean, median and mode. How could this information be useful? (Model: stem and leaf plot)
- Each person selects any two natural numbers whose difference is 2. Find the product of the two numbers. Find the square of the average of the two numbers. Compare the answers of all members in the group. Is there a pattern? Can you show if the pattern would be true for any such pair of numbers? (Model: chart)

Problems that include extraneous conditions and/or data promote communication because students must agree on what information is required to solve the problem.

- Sam had 8 hockey cards and 13 baseball cards. Jenny gave him 5 more hockey cards. How many hockey cards did Sam have then? (Model: hockey cards)
- There are 8 members in a chess club. There are 5 girls and 3 boys. How many different ways can the members pair up to play? (Model: name cards)
- A square garden was enclosed by a fence attached to 2-m high posts 5 m apart. If 20 posts were used to make the fence and 6 posts were left over, what is the area of the garden? (Model: diagram)
- Bill paid $6.10 for his lunch of 2 hamburgers and a soda using only dimes and quarters. He used 9 more quarters than dimes and had 5 dimes left in his pocket. How many quarters and dimes did he use to pay for his lunch? (Model: chart)

Problems that have more than one answer stimulate discussion. When two or more students arrive at different answers to the same problem, a common assumption is that only one of them is correct. "Show us how you got your answer" is a natural instruction that requires each student to justify his or her answer to the other students in the group. Some multiple-answer questions are listed below:

- Tom said, "I have 6 coins worth 30¢ altogether. What coins do I have?" (Model: real coins)
- What is the next number in this pattern?
  [2] [3] [5] [8] [ ]
  (Model: chips to represent each number in the pattern)
- A bag of cookies can be shared fairly among 2, 3, 4, 5 or 6 friends. How many cookies are in the bag? (Model: chips)
- Find seven numbers that have a median of 10, a mode of 5 and a median of 8. (Model: chips and stem and leaf plot)

Problems that may have more than one interpretation provide good catalysts for communication. In mathematics teaching, we tend to avoid ambiguity. If a teacher or student finds that a problem does not seem to be stated clearly, we may disregard it on that basis. The intention is good: we want to be very clear so as not to confuse our students. One reason for this has been the focus on one correct answer. In math, however, as in other aspects of everyday life, when everything is clear and agreeable, little remains to be said. Arguments and opposing opinions tend to generate more talk and thus more language. Examples of this problem type are shown below:

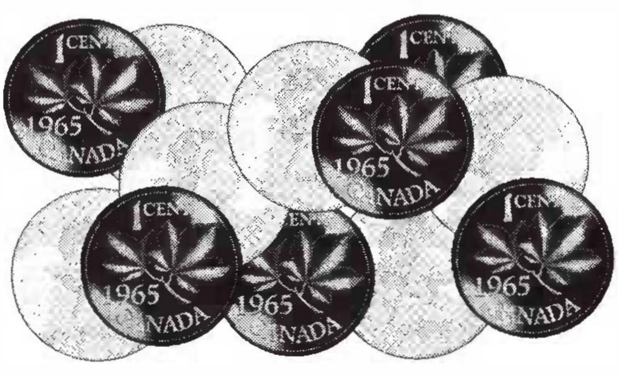
- Jaime ate 3 chocolate raisins and 1 jelly bean. Dale ate 2 gum drops. Who ate the most candy? (Does *most* mean how many candies in a discrete sense, or is the size of the candies important?) (Model: real candies)
- You each have been given a carrot. Who has the biggest carrot? (Students must come to a consensus on how to determine *biggest*. They may use length, mass or diameter as the attribute for comparison.) (Model: real carrots)
- How many different ways can you make double-decker ice cream cones with 8 flavors of ice cream? (Would chocolate on top of vanilla be *different* from vanilla on top of chocolate?) (Model: colored chips)
- There are 99 lockers in a school hallway. Every other locker has a poster on it. Every third locker is unlocked. How many lockers have a poster and are locked? (Does every other locker mean that you start with the first or the second locker?) (Model: diagram)

Problems that require active sharing of decision making or materials for the solution process cannot be solved unless each student in the group understands the strategy being used, such as in the following examples:

- If each person chooses an Attribute block, can you line up them so that each block is different from the block in front of it in exactly one way? (Model: Attribute blocks)
- I will draw four cards, one at a time, from a set of numeral cards from 0 to 9. Work as a team to make the largest 4-digit number possible. You must agree on the place value of each numeral as it is drawn. (Model: sets of numeral cards)
- Each group takes a geoboard. The first person makes the smallest possible square on the geoboard. The area of this square is one. In turn, each person in the group must make a square with a different area and explain how to find the area of that square. (Model: geoboard)
- One person in the group chooses any 2-digit number and any 3-digit number. Using the constant multiplier on the calculator, the others take turns estimating what number to multiply the 2-digit number by to get as close as possible to the 3-digit number. This can be carried out for as many decimal places as possible. (Model: calculator)

Problems that have no correct answer evoke the response, "I can't do it!" The value of such problems lies in the students' justifications for why it can't be done. Four examples of these unsolvable questions are given below:

- Use 14 toothpicks to make a square. Place them end-to-end to make the sides. (Model: toothpicks)
- You have 35¢ in pennies and nickels. If there are 6 coins, what are they? (Model: coins)
- Seven students wrote a math test. The average score was 74 percent. If the marks of six of the students were 72, 61, 88, 93, 54 and 46, what was the mark of the seventh student? (Model: chart)
- A man is three times as old as his son. The sum of their ages is an odd number. How old are they? (Model: chart)



Nurture the Development of Mathematical Terminology

Help students relate their everyday language to mathematical language and symbols. Listening to students when they use their own words to communicate mathematical ideas is as important as talking to them. Encourage them to articulate, through their own language, what they understand implicitly. If we begin with their language, we can help them clarify their understandings and introduce formal mathematical terms and symbols when appropriate. This approach, as illustrated by the following examples, is applicable at all grade levels.

My young son was playing with a pail of plastic magnetic letters, which he dumped on the kitchen floor. I said, "Thomas, please pick them up because we are having supper now." He answered, "But I'm rhyming them." I didn't really have to look to see what he was doing because his language made it fairly obvious. While children are natural sorters, they do not naturally use formal terminology. That is, he did not say, "I am classifying the alphabet."

Picture a Grade 5 student sorting the figures in a set of cardboard polygons. "These shapes are the same," he says, "because they all have four sides and square corners." The teacher responds, "Yes, Steve, you are correct. We call these polygons rectangles."

A junior high school student is asked if she can give answers to the following: 2×7, 4×6, 10×5, $3 \times n$. She responds, "I can answer the first three: $2 \times 7 = 14$, $4 \times 6 = 24$, $10 \times 5 = 50$; but I cannot do $3 \times n$ because I don't know what n is. It can be different numbers." The teacher replies, "That's a good explanation, Cindy. Another way to say that things can be different is to say that they vary, so we call n a variable."

A Math 20 class was using graphing calculators to explore the effects of a on the graph of $f = ax^2$. Describing the changes, one student said, "The graph gets narrower or wider." His teacher agrees, "That's how it looks. In math, we say it compresses or expands."

Difficulties may arise when words have meanings in everyday life different from their meanings in math. The English language has many examples, such as *difference*, *mean*, *root* and *square*. Some words have similar meanings in math and everyday life. *Divide*, *intersect* and *cone* have specific mathematical meanings, but they are related to everyday interpretations. On the other hand, some words such as *decimal*, *polygon* and *integer* are mathematical terms.

Make Discussion a Natural Part of Learning Mathematics

Children should come to realize that discussion is a natural and important part of learning and using mathematics. Discomfort with talking about mathematics leads to mathematics anxiety, a very real problem in our society. Not only is this anxiety prevalent but also it is considered okay to feel that way. In fact, some people even brag about their inability to do mathematics. Mathematics often is portrayed as a rigid, formal system of concepts and skills that are difficult to understand if you are *not mathematically minded.* Once learned, they must be drilled [to be remembered] and are applied through precise algorithmic processes.

Communication in the mathematics class has tended to be a one-way process, from teacher to students. Communication from students to teacher usually has been repeating what the teacher said. This acts as a verification that students understood what the teacher said. Few adults recall being asked for an *opinion* about a mathematical concept or being given a *choice* about which procedure they preferred to use.

Viewing mathematics learning as an individual pursuit means that looking at your neighbor's work is cheating—after all, there is only one correct method and only one correct answer. Working in a small group is less threatening than speaking up in front of the class. Students may be surprised by the different procedures that can be used to solve a problem, and they can clarify their own thinking when justifying their methods to others. Interactive communication in mathematics class does not just happen. Teachers must plan opportunities that foster discussion, model appropriate mathematical language themselves and evaluate their students' progress for the purpose of enhancing communication skills.

References

National Council of Teachers of Mathematics (NCTM)—Commission on Teaching Standards for School Mathematics. *Curriculum and Evaluation Standards for School Mathematics.* Reston, Va.: NCTM, 1989.

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