Maple: A Computer Algebra System

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In the September 1993 issue of *delta-K*, Dale Burnett gave some examples using the computer algebra system Mathematica. This article describes an equivalent system. Since about 1980, mathematicians at the University of Waterloo have developed a computer algebra system called Maple V Release 2 (Maple). Those involved have founded a company that maintains the software package to which mathematicians from around the world contribute. There are 400,000 users worldwide. This company now employs more than 50 people and is associated with three university research groups. More than 20 people are directly involved in design reviews and extensions. Similar computer algebra systems are available, such as REDUCE and MACSYMA. At the moment, Maple is one of the more successful user-friendly packages. Maple performs well on a Macintosh with 4 MB of RAM and a hard drive, making it a resource-efficient software package. The same hardware requirement applies to the IBM-PC and its compatibles. The Ontario Ministry of Education has obtained a provincewide license for its high schools. This was preceded by a study that gave the system a favorable recommendation.

The first computer algebra systems were developed in the '60s, but the programming languages were not portable, and their platforms were mainframe computers. In the '90s, with technological improvements in the capabilities of personal computers, matters have changed. In the '70s, the slide rule vanished from the classroom and was replaced by the handheld calculator. In the '90s, a similar event is taking place in mathematics proper with profound implications.

This time, the change is a little farther removed from everyday life because many applications are geared toward the activities of mathematicians, scientists, engineers and teachers of mathematics and science. It is especially useful in large calculations. Maple is friendly enough that a high school student with proper guidance can learn the system's basic workings. It will solve equations of a single variable up to degree four in terms of radicals or systems of linear equations. It graphs algebraic functions and conic sections and performs statistical plots and so on. Of course, it does the ordinary operations such as factoring, simplifying and multiplying algebraic expressions, which are some regular high school topics. Maple also has a built-in Pascal-like programming language specifically directed toward programming mathematical functions. It will interact with programming languages such as Fortran and C.

Educators will have to start thinking about these systems because some handheld calculators perform similar functions, although they are not quite as powerful. For many students, the computer will be just another fixture in their future working life. In the immediate future, classrooms will not likely have a computer on every desk, but a program such as Maple allows the teacher to demonstrate certain aspects of mathematical routines pertaining to the curriculum and will allow students to experiment with the system.

That the computer can perform complex calculations changes mathematics itself in ways that will affect the curriculum. In the not-so-distant past, logarithmic tables instigated by Napier (1550–1617) and others were a major aid in scientific calculations. Over time, the tables were improved, and certain conventions for easier use were adopted. The same will happen to computer algebra. The introduction of computer algebra at the high school level will eventually force curriculum changes. We have some experience with the use of calculators that perform arithmetical operations, and mental arithmetic is still a necessity of life. Algebraic operations such as factoring and multiplication and many other routines are still necessary, even if they can be done by computer. To illustrate, let me give two anecdotes. I was trying to solve problem 497 from *College Mathematics Journal* (Aboufadel 1994). The problem asked for a limit. Maple produced the answer *1* in 17 seconds on my Macintosh LCIII. It took me several hours to fill in the gaps and show that the answer was correct. However, such searches are not always successful. This means students who use the system should also be required to provide careful documentation of solutions they present. The second example comes from the June 1993 issue of *Crux Mathematicorum*. Editor Bill Sands asks in exasperation for easier arguments after presenting the solution to problem 1680: "The equalities . . . have been verified by helpful colleague Len Bos using MACSYMA. But the editor hasn't the foggiest idea how they were obtained!" (p. 271).

The computer is going to play a permanent role in mathematics. Maple also offers great opportunities for self-study and remedial tasks. Undoubtedly, basic skills in mathematics will always be required. However, we will have to redefine what the specific bounds are. Powerful systems cannot be used without being able to judge the results produced. Students need to be able to document their answers carefully. It is also necessary to be able to discover incorrect results. In the choice of software, you want to be assured that the package is going to be properly maintained, be well documented and that assistance is readily available, which was the case with Maple.

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Appendix 1

```
> P:=proc(n)
local  a,b;
a:=1;b:=1;
while  a<n  do
if gcd(a,b) = 1    then
(a^2  -  b^2,2*a*b,a^2  +  b^2) fi;
b:=b-2;
if b<= 0 then b:=a ;a:=a+1 fi;
print(a^2  -  b^2,2*a*b,a^2  +  b^2) ;
od;
end;

P  :=  proc(n)
      local a,b;
          a := 1;
          b := 1;
          while a < n do
                if gcd(a,b) = 1 then a^2-b^2,2*a*b,a^2+b^2 fi;
              b := b-2;
               if b <= 0 then b := a; a := a+1 fi;
                print(a^2-b^2,2*a*b,a^2+b^2)
          od
      end

> P(5);
```

$$3, 4, 5$$

$$5, 12, 13$$

$$7, 24, 25$$

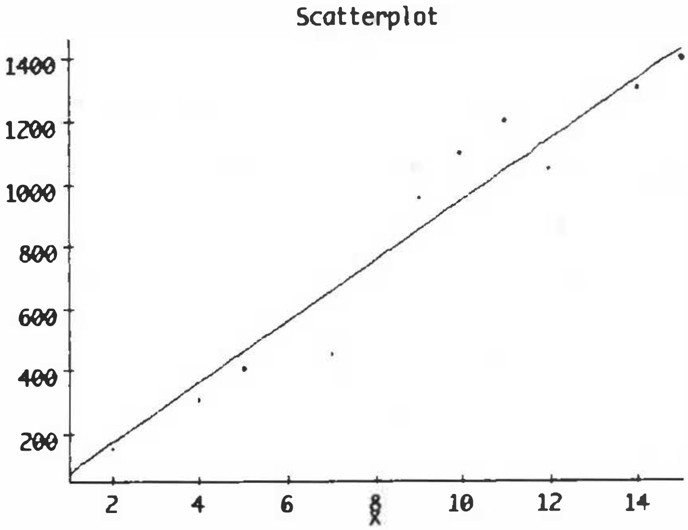
$$15, 8, 17$$

$$9, 40, 41$$

```
> This program prints the first 5 Pythagorean triples.
```

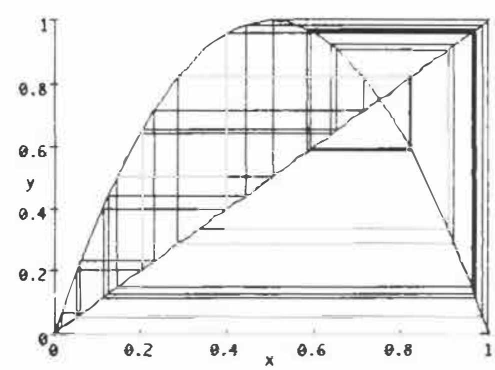
If the program is correct, Maple prints a duplicate version; otherwise, it stops at the error.

Appendix 2



Appendix 3

```
> f:=x->4*x*(1-x):

> plotpoints:=[seq((f@@      (trunc((i+2)/4)))(0.2),i=1..120)]:

> plot2:=plot(x,x=0..1,y=0..1):

> plot3:=plot(4*x*(1-x),x=0..1,y=0..1):

> plot1:=plot(plotpoints,x=0..1,y=0..1,style=LINE,style=PATCH):

> plots[display]({plot1,plot2,plot3});

>
```



This kind of iteration goes back to Fatou and Julia, two French mathematicians from the first half of this century. This was picked up again by Mandelbrot and others.

Compare this with the article "Introducing the Derivative Through Iteration of Linear Functions" in *Mathematics Teacher* (May 1994). To vary the picture, vary the function and the number of iterations.

Appendix 4

```
> with(stats):
```

```
>
dat:=array([[y,x],[200,1],[400,5],[450,7],[150,2],[1100,10],[1300,14],[950,9],[300,4],
[1400,15],[1200,11],[550,6],[1050,12]]):
```

```
> regression(dat,y=A+B*x);
```

$$\{B = 96.95652174, A = -21.48550725\}$$

```
> statplot(dat,y  =subs(",  A  +  B*x),style=point);
```

```
> dat1:=(5,9,4,5,6,7):
```

```
> average(dat1);
```

$$6$$

```
> median(dat1);
```

$$5$$

```
> sdev(dat1);
```

$$\frac{4}{5}\sqrt{5}$$

```
>
```