Building a Professional Memory: Articulating Knowledge About Teaching Mathematics

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In many ways, teaching is a profession without a memory. Unlike architecture and engineering, few detailed records are kept of what teachers do and how they do it. Architects and engineers leave behind drawings, specifications, models, contracts and buildings. Such artifacts provide a record of the problems faced, solutions tried and products produced. Teachers, however, leave few descriptions that record their experiences introducing a new topic or their struggles with particularly challenging curriculum. Many records that are left do little to capture the complexity of teachers' pedagogy.

An integral part of the education of doctors, lawyers and business people is the study of "cases" that record their profession's history. At present, there is no comparable body of literature to which beginning and practising teachers can turn to discover the wisdom of their predecessors. The development of a case literature of mathematics teaching should be a professional priority.

A useful focus for articulating the complexities of subject matter pedagogy is Shulman's (1987) view that effective teachers transform knowledge of subject matter into forms accessible to their pupils, rather than transmitting knowledge or pouring ideas into someone else's head. Shulman calls this amalgam of subject matter and pedagogy *pedagogical content knowledge*. It arises from deliberations about how to teach particular content to particular pupils in particular contexts and consists (among other things) of misconceptions pupils typically bring to instruction, alternative ways of representing subject matter and effective teaching strategies for changing misconceptions. To a significant degree, the acquisition of relevant pedagogical content knowledge—a way of thinking that helps the teacher understand the learner's difficulty and subsequently transform the content so the learner can understand—distinguishes effective mathematics teachers from those who are less effective.

Figure 1
Transforming Subject Matter Knowledge

Subject Matter Knowledge → **Teachable Subject Matter Knowledge**

Pedagogical Content Knowledge

- Student misconceptions
- Strategies for altering misconceptions
- Alternative representations

When learning primarily involves acquiring information, instruction can proceed in a transmission mode. This typically involves motivating pupils, delivering content, providing opportunities for practice and evaluating learning. In these situations, pupils employ familiar ways of thinking to assimilate new information presented by teachers. Teachers have little need to use pedagogical content knowledge because they can transmit, relatively intact, their knowledge of the subject matter to their pupils. A good deal of mathematics, however, incorporates ways of thinking that are not intuitively obvious. Effective mathematics teaching demands that subject matter be transformed to allow it to be learned meaningfully by novices. Consequently, mathematics teachers find themselves in need of extensive repertoires of pedagogical content knowledge. This knowledge needs to be captured in case studies.

Having discussed mathematics teaching with many prospective teachers over the last few years, we have found that few are able to provide suitable representations for many rudimentary mathematical abstractions. This lack of understanding often stems from their own experiences of school mathematics and the resulting perception of mathematics as a collection of isolated rules to be memorized. We will use an elementary concept, division of fractions, to illustrate the importance of appropriate representations in mathematics and the necessity for teachers to acquire pedagogical content knowledge regardless of the grade they teach.

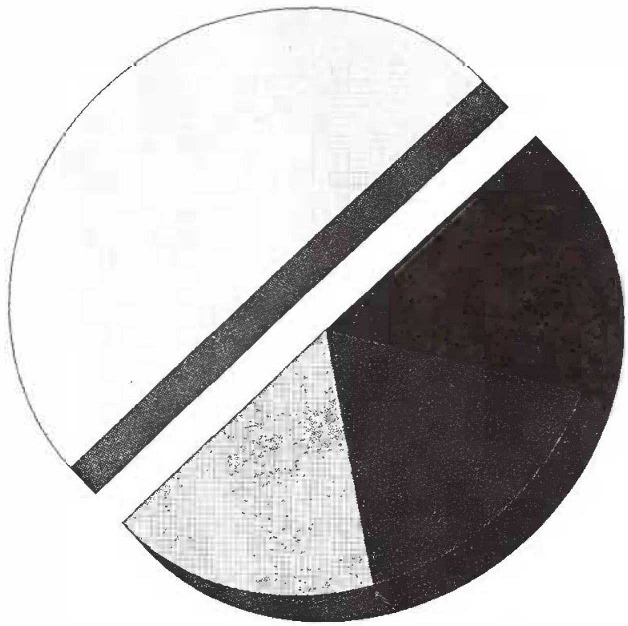
Transforming Knowledge: Division of Fractions

A question that often provides some indication of how a person has learned mathematics is 10 divided by ½. When answering this question, a few prospective teachers contribute the answer 5 (the answer being sensible for the representation they are using—10 shared in half), but most give the correct answer 20. However, when asked to explain why the answer is 20, a majority are unable to provide a representation or model. They resort to rules memorized during their own schooling. As one student teacher wryly explained, "Ours was not to reason why, just invert and multiply."

An understanding of pupils' misconceptions is important pedagogical content knowledge. Without this knowledge, teachers are not in a position to help pupils clarify their understanding. For example, in the situation described above, teachers need to know that many pupils will be comfortable with the smaller answer 5 when they divide 10 by ½ due to their previous experiences with natural numbers. Many pupils have developed the idea that "multiplication makes bigger" and "division makes smaller," which is true for natural numbers. Pupils will also find comfort in the *partition* or sharing model for division and happily share 10 in half, again obtaining 5 for the answer. Unless teachers are aware that such errors are extremely common among 12 and 13 year olds, they are unlikely to spend the necessary time introducing the concept.

Being aware of possible misconceptions, however, is insufficient to enable teachers to advance their pupils' more complete understanding of division. To assist their pupils, teachers need a second model for division, sometimes referred to as the *quotition model*, and have available suitable representations for it. One representation of the quotition model occurs when one asks, "How many people can attend a party if there are 10 pizzas and everyone receives half a pizza?". The correct answer, 20, is usually quickly contributed, but often the connection between the verbal question and its equivalent mathematical expression, 10 divided by ½, is not made. When answering a question of the form 10 divided by ½, pupils have to ask themselves, "How many halves are there in 10?" if they are to make sense of the symbolism. This notion becomes especially important when dividing a fraction by another fraction, for example, ½ divided by ⅙. Many pupils want their answers to be a fraction and, what is more, a rather messy fraction. They are often surprised when they obtain the whole number 3. However, when pupils are provided with a pictorial representation (Figure 2) and are asked, "How many sixths in one half?", most Grade 5 or 6 pupils understand why the answer is 3. Pupils are comfortable that there are six sixths in a whole pie, and therefore there should be three sixths in half the pie.

Figure 2
Pictorial Representation to Illustrate ½ ÷ ⅙



Unfortunately, children, and sometimes teachers, are concerned only with the final correct answer to mathematical questions, especially arithmetical questions. This preoccupation often leads to understanding becoming of secondary importance. It is even thought of as inconsequential by many pupils. Justifying the rationale underlying a procedure helps demonstrate its importance. Only when pupils travel comfortably between real entities and symbolic representations can the term *mathematically literate* be appropriately applied to them.

Conclusions

Having suitable representations for oneself and having the knowledge to help pupils develop their own models, stories and analogies for mathematical symbolism comes not only with teaching experience but also with the philosophical outlook that such ideas are important. If mathematics is seen simply as a set of rules and procedures to be reproduced on a test, pedagogical content knowledge is unnecessary. However, if the teacher's role is to assist pupils to understand why their answer is correct or incorrect, the teacher must have more than a knowledge of general pedagogy and mathematics.

To transform subject matter content knowledge into a form accessible to pupils, teachers need to know particulars about the content relevant to its teachability, particulars that probably would not have been revealed until the task of teaching had been assumed. This pedagogical content knowledge is in some sense a result of the interaction of content and pedagogy. It is knowledge about the content derived from consideration of how best to teach it.

Certainly, teachers who are aware of a misconception, are cognizant of its origin and who possess multiple representations to correct it have the pedagogical content knowledge to make the subject matter accessible to pupils. Seldom, however, is this expertise shared systematically among colleagues, and the wisdom of practice is lost to the teaching profession (Shulman 1986). Without records of experience, pedagogical content knowledge has to be continually reinvented by each new generation of teachers—consuming time and energy that should be used constructing new understanding.

Teaching is one of the few professions where most people expect the novice to perform in a similar fashion to the veteran. If teaching is to progress, we have to recognize the intricacies associated with it and understand that teachers are also learners. Teacher educators are gradually recognizing the importance and relevance of using case studies that capture the complexities of teaching. Consequently, there is an increased awareness that the teaching profession needs cases written by practising teachers (Cochran-Smith and Lytle 1990; Shulman and Mesa-Bains 1990). Through such cases, teachers hear their peers' voices. When we have built a shared professional memory that carefully articulates knowledge about teaching mathematics, present and future teachers will have at their command examples of exceptional teaching with which to face teaching's challenges and complexities. Only then will teachers really be able to benefit from peers' wisdom and practice.

References

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