

Writing Mathematics

E.G. Bernard

Young people retain stories such as Cinderella accurately after hearing them surprisingly few times. And they remember such stories for their whole lives. Young children, then, possess the ability to learn a system of symbolic knowledge. Can techniques from language experiences assist in the learning and assessment of mathematics?

The Test

To attempt to answer this question, I visited several mathematics classes and told them the following:

I am going to give you an unusual mathematics assignment today. First, I'll give you a non-mathematical example of the assignment and let you try it. We'll discuss and take up aspects of this assignment. Second, I'll give you the mathematical assignment. Third, I'll return in a few days, take up the assignment, explain to you what I'm trying to accomplish and get your views.

I'm going to put two nonmathematical expressions on the board. When you see the two expressions, I want you to tell me the content (the story) that you associate with these two expressions.

Here are the two expressions: *glass slipper* and *pumpkin*. Now tell me the content (the story) that you associate with these two expressions.

Most students said the expressions referred to the story of Cinderella. When asked to commence telling the story, most students began "Once upon a time" and were able to tell the story. I pointed out that, although they had likely not heard this story for a long time, they had an amazing amount of correct detail in their stories and the content was well sequenced. I continued:

Now for the mathematics assignment. This time, I will put a title on the board, and you write the story "The Differences Between Adding and Multiplying Fractions."

Remember what you learned in The Writing Process in your English classes? List the appropriate content, that is, what you know about the title; sequence the content; imagine a story into which to put your content; and write out a full account of the story. Give a lot of detail. Use only words.

Results

When I read the stories, I found that many students had employed introductory and concluding sentences. For instance, "There are a few differences between the addition and multiplication of fractions" and "These are the differences between the addition and multiplication of fractions."

In a few cases, students incorporated their content into a traditional type of story. For instance,

Once upon a time, there was a boy named Bob who didn't know how to do fractions. His friend helped him after school one day. . . . Bob finally understood how to do fractions and passed his fraction test.

In another version, the conclusion stated: "And the boy not only passed his next fraction test, years later he became a math teacher." Another student's story went like this:

Adding and Multiplying were talking to each other one day. Adding asked Multiplying to tell how he is different than Adding regarding fractions. . . . "Well, although we both deal with numbers, we are different in a few ways."

I had prepared a marking scheme and awarded marks for the title being present, for an introductory sentence (with a bonus for a creative introduction), for a conclusion and for spelling, grammar and writing style. In marking the content of the addition of fractions, I was looking for some discussion of finding a common denominator, of adding the numerators and placing the sum of the numerators over the common denominator. In marking the multiplication of fractions, I was looking for a description of multiplying the numerators and placing this product over the product of the denominators. I also awarded marks for a reference to reducing answers to their lowest terms, of dividing in the multiplication of fractions to reduce the size of the numbers, a reference to the order of operations and a discussion of dealing with mixed numbers.

As many as 40 percent of the students' marks increased on this assignment over their marks on traditional fraction tests—the classes I visited had recently completed their study of fractions, and I had the students' scores on their fraction tests. In many

cases, the increases were substantial, for example, 47 to 65, 23 to 45, 63 to 80 and 72 to 100. Some students received marks similar to their marks on the traditional fraction tests. As many as 40 percent of the students' marks decreased, for example, 71 to 60, 60 to 35, 53 to 10 and 63 to 0. But most of the students whose marks decreased were new Canadians who were unable to describe or write about mathematical operations, albeit some of them scored the highest on the traditional types of tests.

Reflections

First, I believe that exercises of this type allow students to exhibit knowledge not requested in traditional tests. It is important, in all subject areas, that we employ many types of questions and various evaluation instruments that suit the varied talents of our students—to allow students to be successful in some area of each course. For instance, in visual arts courses, students are not evaluated in only one medium; various media are explored, such as sketching, watercolors, oil, charcoal and clay. As well, they are evaluated on art history. The greater the variety of tests we use, the more likely we will match an aspect of a course with a talent of a student. We should employ traditional types of testing and assignments of the kind I am suggesting here.

Second, the business world and postsecondary schools want graduates who can communicate clearly and creatively. They want young people who can describe as well as apply what they know. If this is true, a student who can apply theory and articulate it clearly should score higher than someone who can only perform the calculations properly. If writing mathematics is an important aspect in understanding and reporting mathematical concepts, I hope this article points to a way of commencing this activity in our classrooms.

A third factor was emphasized when I returned to the classes to explain what I was doing. When I commented on the fact that a young child can repeat a story such as Cinderella after hearing it surprisingly few times, some students said

But there was no pressure or expectation to learn Cinderella at our parents' knees. And, at the time, Cinderella was interesting to us. Fractions (or mathematics in general) are not interesting to many of us.

These students pointed out the importance of using this kind of exercise (writing mathematics) at a much earlier stage in the educational process.

To implement "learning and assessing Writing Mathematics," we need to apply principles that pertain to the learning of language and stories:

- In learning language and stories, we give countless examples of how words are used. In mathematics, we need to give countless examples of how numbers are used. Further, we need to give examples of applications in, for instance, science, technology, computer studies, sports, geography, business and so on.
- Mathematics topics need to be presented in a large context rather than as isolated pieces. Language and stories are learned holistically. We did not learn Cinderella by hearing bits of the story now and then, and out of sequence. By learning (or seeing) the whole story, association and sequence are present. Association and sequence are two aspects of what is meant by the material having "meaning," not being "nonsense." There is a lot of research on the amount of time and effort required to learn nonsense syllables as opposed to learning meaningful material (research suggests four times the effort is required).

Does mathematics come across as nonsense syllables to some children? We need to present the "whole fraction story": what fractions are, examples of what fractions look like, how fractions are used, pictures of pieces of pie, the parts of a fraction named and labeled, the four operations as they apply to fractions (noting the similarities and differences among the operations), examples of the four operations and so on. As well as association and sequence being present as a result of this approach, students would have the whole story of fractions in their notes in one place for study purposes.

- Part of the testing of mathematics knowledge, then, should include the natural questioning used in evaluating language and stories as well as the traditional means of testing.
- Assignments and assessments in mathematics need to involve great variety, not just textbook-like questions. For instance, in introductory geometry, assignments and assessments need to include problem solving, research, writing, conceptual knowledge, esthetics and so on. These aspects can be addressed while children are learning the shapes; learning the parts of each shape; labeling; learning perimeter, area and volume; applying the formulas for perimeter, area and volume; creating mobiles of shapes; researching how the shapes appear in nature; writing the story of, say, Area; using computers in generating shapes; estimating widths of rivers and heights of trees; and so on.
- The common elements of "problem solving" need to be discussed, noting that there is often more than one solution to a problem. The learning style

of each student may mean that each student prefers one way over another.

- One important method of learning in the humanities involves students asking questions prior to commencing a topic and then having their questions answered during their readings. We remember the answers to our questions better than we remember the answers to someone else's (the teacher's) questions. As students accumulate experience asking their own questions, they will get better at asking questions prior to commencing a topic—and will get better at remembering significant details about a topic.

In the case of fractions, some questions students will come up with include: What are fractions? What does the word *fraction* mean? Can you add, subtract, multiply and divide fractions? What are the rules for these operations with fractions? What are some applications of fractions? How are fractions used? What new vocabulary arises from the study of fractions? Is there a relationship between fractions and decimals, and fractions and percent, and so on?

Reprinted with permission of The Canadian School Executive, Volume 14, Number 2, June 1994, 11–13.