Linda's Trisection

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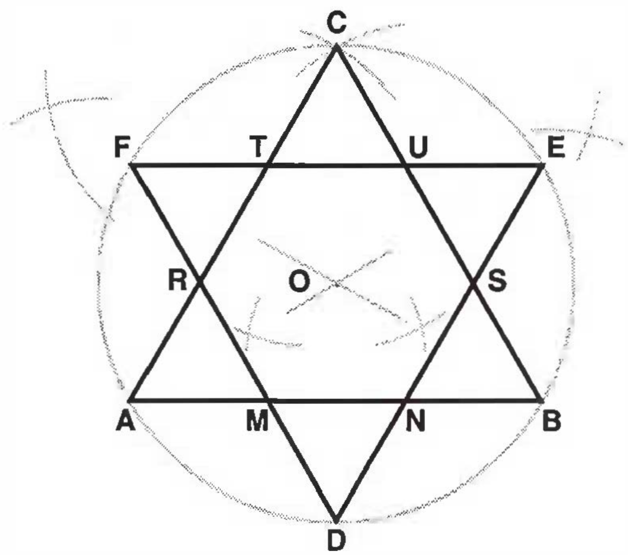
Editor's note: Linda is a student at St. Mary's High School in Calgary. On her own initiative, she came up with the following procedures for trisecting a segment and trisecting an angle. Student initiatives should be encouraged. I am prepared to seriously consider student work for inclusion in *delta-K*.

Trisecting a Segment



Trisect $\overline{AB}$

1. Construct equilateral triangle ABC.
2. Find centre O of ΔABC (bisect $\overline{AC}$ and $\overline{BC}$).
3. With radius $\overline{OA}$, draw circle.
4. With radius OA and centre at A, intersect circle at D and F. With centre at B, intersect circle at E.
5. Draw $\overline{FD}$ and $\overline{ED}$. Label intersection of $\overline{AB}$, as M and N.
6. $\overline{AM} = \overline{MN} = \overline{NB}$.

   $\therefore \overline{AB}$ has been trisected.

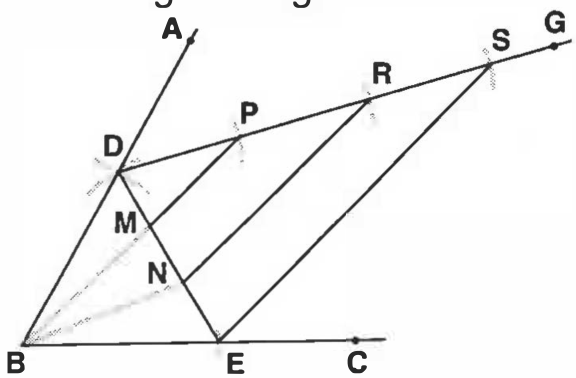
Note: This construction is similar to the construction of a regular hexagon.

Proof

Prove that $\overline{AM} = \overline{MN} = \overline{NB}$ given the above construction.

$\angle CAB = \angle CBA = 60°$	ΔABC equilateralΔ
ADBECF regular hexagon	constructed
$\angle DBE = 120°$	regular hexagon
$\angle EDB = \angle BED = 30°$	isosceles ΔDBE
ΔADB is isosceles	constructed
$\angle ADB = 120°$	regular hexagon
$\angle DBN = \angle DAB = 30°$	isosceles ΔADB
$\angle DNB = 120°$	sum $\angle$'s in Δ
$\angle SNB = 60°$	supplementary $\angle$'s
$\angle BSN = 60°$	sum $\angle$'s in Δ
$\therefore \Delta SNB$ is equilateral	$\angle$'s all 60°
similarly ΔRAM is equilateral	
and $\Delta SNB \cong \Delta RAM$	since $\Delta DBE \cong \Delta ADF$
$\angle DNB = \angle MNS = 120°$	vertical $\angle$'s
similarly	
$\angle RMN = \angle TRM = \angle UTR =$	
$\angle SUT = \angle NSU = 120°$	
$\therefore$polygon TUSNMR	
regular hexagon	
$\overline{RM} = \overline{MN} = \overline{NS}$	regular hexagon
$\overline{RM} = \overline{AM}$ and $\overline{NS} = \overline{NB}$	equilateral Δs
$\therefore \overline{AM} = \overline{MN} = \overline{NB}$	substitution

Trisecting an Angle



Trisect $\angle ABC$

1. Construct isosceles ΔBDE.
2. Draw $\overline{DG}$.
3. On $\overline{DG}$, mark off three congruent segments.
4. Construct SE.
5. Copy $\angle DSE$ at R and P; extend lines to intersect $\overline{DE}$ at M and N.
6. Construct $\overline{BM}$ and $\overline{BN}$.

   $\angle ABC$ is trisected.