Enhancing Mathematics Teaching in the Context of the Curriculum and Professional Standards of the National Council of Teachers of Mathematics

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This article briefly reviews the curriculum and professional standards of the National Council of Teachers of Mathematics (NCTM). These standards have driven and directed curriculum development, teaching, evaluation and professional development in mathematics since their publication in 1989. This article provides readers with an overview of the curriculum and professional standards for mathematics.

Inherent in the standards is a consensus that all students need to learn more, and often different, mathematics and that instruction in mathematics must be significantly revised. The need for standards for school mathematics is clearly evident in that they ensure quality, indicate goals and promote change. The NCTM considers all three reasons equally important. Schools, and in particular school mathematics, must reflect the important consequences of the current reform movement in mathematics if students are to be adequately prepared to live in the 21st century. Today's society expects schools to ensure all students have an opportunity to become mathematically literate, are capable of extending their learning, have an equal opportunity to learn, and become informed citizens capable of understanding issues in a technological society.

Educational goals for students must reflect the importance of mathematical literacy. Toward this end, the K–12 standards articulate five general goals for *all* students:

- That they learn to value mathematics
- That they become confident in their ability to do mathematics
- That they become mathematical problem solvers
- That they learn to communicate mathematically
- That they learn to reason mathematically

These goals imply that students should be exposed to numerous and varied related experiences that encourage them to value the mathematical enterprise, to develop mathematical habits of mind and to understand and appreciate the role of mathematics in human affairs; that they should be encouraged to explore, to guess and even to make and correct errors so that they gain confidence in their ability to solve complex problems; that they should read, write and discuss mathematics; and that they should conjecture, test and build arguments about a conjecture's validity.

The mathematics classroom must be permeated with these goals and experiences so that they become commonplace in students' lives. Exposing students to the experiences outlined in the standards will ensure that students gain mathematical power. This term denotes an individual's abilities to explore, conjecture and reason logically, as well as his or her ability to use various mathematical methods effectively to solve nonroutine problems. This notion is based on the recognition of mathematics as more than a collection of concepts and skills to be mastered; it includes methods of investigating and reasoning, means of communication and notions of context. In addition, for each individual, mathematical power involves development of self-confidence.

The NCTM (1989, 1991) has presented 78 standards divided among eight categories: Grades K–4 curriculum, Grades 5–8 curriculum, Grades 9–12 curriculum, evaluation of students, teaching mathematics, evaluation of the teaching of mathematics, professional development of teachers, and support and development of mathematics teachers and teaching.

Curriculum Standards

Curriculum standards for school mathematics are value judgements based on a broad, coherent vision of schooling derived from several factors: societal goals, student goals, research on teaching and learning, and professional experience. Each standard starts with a statement of what mathematics the curriculum should include, followed by a description of the student activities associated with that mathematics and

a discussion that includes instructional examples. Three features of mathematics are embedded in the standards. First, "knowing" mathematics is "doing" mathematics. Doing mathematics is different from mastering concepts and procedures. That is not to say that informational knowledge has no value, only that its value lies in the extent to which it is useful in the course of some purposeful activity. Students clearly must know the fundamental concepts and procedures from some branches of mathematics; established concepts and procedures must be relied on as fixed variables in a setting in which other variables may be unknown. However, instruction should persistently emphasize "doing" rather than "knowing that."

Second, some aspects of doing mathematics have changed in the last decade. The computer's ability to process large sets of information has made quantification and the logical analysis of information possible in such areas as business, economics, linguistics, biology, medicine and sociology. Because mathematics is a foundation discipline for other disciplines and grows in direct proportion to its utility, the mathematics community believes that the curriculum for all students must provide opportunities to develop an understanding of mathematical models, structures and simulations applicable to many disciplines.

Third, changes in technology and the broadening of the areas in which mathematics is applied have resulted in growth and changes in the discipline of mathematics. More than half of all mathematics has been invented since World War II. The new technology not only has made calculations and graphing easier but also has changed the very nature of the problems important to mathematics and the methods mathematicians use to investigate them.

Because technology is changing mathematics and its uses, the NCTM believes

- appropriate calculators should be available to all students at all times,
- a computer should be available in every classroom for demonstration purposes,
- every student should have access to a computer for individual and group work and
- students should learn to use the computer as a tool for processing information and performing calculations to investigate and solve problems.

Access to this technology is no guarantee that any student will become mathematically literate. Calculators and computers for users of mathematics are tools that simplify, but do not accomplish, the work at hand. Similarly, the availability of calculators does not eliminate the need for students to learn algorithms. Students should be aware of the choices of methods when calculating an answer to a problem. When an approximate answer is adequate, students should estimate. If a precise answer is required, students must be capable of choosing an appropriate procedure. Many problems should require students to conduct mental calculations or use paper and pencil. For more complex calculations (for example, long division or column addition), students should be able to use calculators. Finally, if many iterative calculations are needed, a computer program should be written or used to find answers (for example, finding a sum of squares).

With respect to mathematical content, the standards represent the minimum that all students will need to be productive citizens. The standards do not specify alternative instructional patterns prior to Grade 9. For Grades 9–12, the standards have been prepared in light of a core program for all students, with explicit differentiation in terms of depth and breadth of treatment and the nature of applications for college-bound students. There is an implied expectation that all students have an opportunity to encounter typical problem situations related to important mathematical topics.

Student activities are the second aspect of each standard. Two general principles have guided the description of these activities: first, activities should grow out of problem situations; and second, learning occurs through active as well as passive involvement with mathematics. Traditional teaching emphases on practice in manipulating expressions and practising algorithms as a precursor to solving problems ignore the fact that knowledge often emerges from the problems. Thus present strategies for teaching may need to be reversed: knowledge often should emerge from experience with problems. Furthermore, students need to experience genuine problems regularly. A genuine problem is a situation in which, for the individual or group concerned, one or more appropriate solutions have yet to be developed. However, instruction should vary and include opportunities for

- appropriate project work,
- group and individual assignments,
- discussion between teacher and students and among students,
- practice on mathematical methods and
- exposition by the teacher.

Another premise of the standards is that problem situations must keep pace with the mathematical and cultural maturity and experience of the students. For example, the primary grades should emphasize the empirical language of the mathematics and whole numbers, common fractions and descriptive geometry. In the middle grades, empirical mathematics should be extended to other numbers and the

emphasis should shift to building the abstract language needed for algebra and other aspects of mathematics. High school mathematics should emphasize functions, their representations and uses, modeling and deductive proofs.

The standards specify that instruction should be developed from problem situations. Situations should be sufficiently simple to be manageable but sufficiently complex as to provide for diversity in approach. They should be amenable to individual, small-group or large-group instruction; involve a variety of mathematical domains; and be open and flexible as to the methods to be used.

The first three standards for each grade level are problem solving, communication and reasoning, although these vary between the levels on what is expected of students and of instruction. The fourth curriculum standard at each level is mathematical connections. This label emphasizes the belief that, although it is often necessary to teach specific concepts and procedures, mathematics must be approached as a whole. Concept, procedures and intellectual processes are connected. Thus the curriculum should include deliberate attempts, through specific instructional activities, to connect ideas and procedures among different mathematical topics and with other content areas. Following the connections standard, nine or ten specific content standards are stated and discussed. Some have similar titles, which reflects that a content area needs emphasis across the curriculum; however, others emphasize specific content that needs to be developed at that level.

Student Evaluation Standards

These standards are viewed and presented in three categories. The first set consists of three evaluation standards and discusses general assessment strategies related to the curriculum standards. These standards present principles for judging assessment instruments, including alignment, multiple sources of information, and appropriate assessment methods and uses.

The second set contains seven standards under student assessment and focuses on providing information to teachers for instructional purposes. They closely parallel the curriculum standards of problem solving, communication, reasoning, mathematical concepts and mathematical procedures, in addition to two separate standards on mathematical disposition and mathematical power. These seven standards are to be used by teachers to judge students and their mathematical progress.

The final set of four standards falls under program evaluation and addresses the gathering of evidence with respect to the quality of the mathematical program. The standards are indicators of program evaluation, curriculum and instructional resources, instruction and evaluation team. These standards are to be used by teachers, administrators and policymakers to judge the quality of the mathematics program and the effectiveness of instruction.

Standards for Teaching Mathematics

Central to the curriculum and evaluation standards is the development of mathematical power for all students. Mathematical power includes the ability to explore, conjecture, reason logically, solve nonroutine problems, communicate about and through mathematics, and connect ideas within mathematics and between mathematics and other intellectual activities. Mathematical power also involves the development of personal self-confidence and a disposition to seek, evaluate and use quantitative and spatial information in solving problems and in making decisions. Students' flexibility, perseverance, interest, curiosity and inventiveness also affect the realization of mathematical power.

To reach the goal of developing mathematical power for all students requires the creation of a curriculum and an environment, in which teaching and learning are to occur, that are very different from much of our current practice. The image of mathematics teaching needed includes elementary and secondary teachers who are more proficient in

- selecting mathematical tasks to engage students' interest and intellect (that is, worthwhile mathematical tasks);
- providing opportunities to deepen their understanding of the mathematics being studied and its applications;
- orchestrating classroom discourse in ways that promote the investigation and growth of mathematical ideas;
- using, and helping students use, technology and other tools to pursue mathematical investigations;
- seeking, and helping students seek, connections to previous and developing knowledge; and
- guiding individual, small-group and whole-class work.

The professional standards for teaching mathematics are a major shift away from the current practice to mathematics teaching for student empowerment. Current practice is characterized by a predictable sequence of activities. First, answers are given for the previous day's assignment. Difficult problems are

worked on by the teacher or students at the chalkboard. This is followed by the introduction of a new concept and the assignment of homework. Students work on the homework for the remainder of the class period, with the teacher moving around the room to answer questions.

The professional standards for teaching mathematics call for a change in the environment of mathematics classrooms. We need to shift toward

- classrooms as mathematical communities and away from classrooms as simply a collection of individuals;
- logic and mathematical evidence as verification and away from the teacher as the sole authority for right answers;
- mathematical reasoning and away from merely memorizing procedures;
- conjecturing, inventing and problem solving and away from an emphasis on mechanistic answer finding; and
- connecting mathematics, its ideas and its applications and away from teaching mathematics as a body of isolated concepts and procedures.

The phrase "all students" is used throughout the standards. This means that schools and communities must accept the goal of mathematical education for every child from Kindergarten to Grade 12. This does not mean that every child will have the same interests or capabilities in mathematics. It does mean that we will have to examine our fundamental expectations about what children can learn and can do and that we will have to strive to create learning environments in which raised expectations for children can be met.

Standards for the Evaluations of the Teaching of Mathematics

This section presents eight standards for evaluating the teaching of mathematics organized under two categories. The first category describes the process of evaluation and includes standards dealing with the evaluation cycle, teachers as participants in evaluation and sources of information.

The evaluation process should reflect that the overall intent is to improve instruction, that it should be a dynamic and continual process, that teachers should be an integral part of that process, and that because of the complexity of teaching, it should involve a variety of sources of information gathered in various ways. The standards emphasize that teachers should be encouraged and supported to engage in self-analysis and to work with colleagues in improving their teaching. When evaluation involves supervisors or administrators, their relationship with teachers should be collegial with the intent to improve instruction.

The second category of standards in this section describes the foci of evaluation and includes five standards dealing with mathematical concepts, procedures and connections; mathematics as problem solving, reasoning and communication, mathematical disposition; assessing students' mathematical understanding; and learning environment.

These standards, particularly the standard dealing with mathematical disposition, emphasize the importance of significant mathematics when reevaluating mathematics teaching. Through encounters with significant mathematics, students develop mathematical power. But attaining mathematical power requires more. It requires a disposition to do mathematics and an environment in which the processes of doing mathematics are continually emphasized.

This can occur only when teachers present stimulating tasks and create an environment in which problem solving, reasoning and communication are valued and promoted. Further, the message teachers send students should not be limited to instruction alone; it must also include what and how mathematical learning is assessed. Through assessment, we communicate to our students what mathematical outcomes are valued.

A consistent message throughout the standards for the evaluation of teaching is the importance of teachers reflecting on their teaching and working with colleagues and supervisors to improve their teaching. While the standards provide a focus for improvement, such improvements will occur only when teachers consciously decide to engage in ongoing professional development. This, in turn, requires support and encouragement at all levels.

Standards for the Professional Development of Teachers of Mathematics

Five standards are presented in this section: experiencing good mathematics teaching, knowing students as learners of mathematics, knowing mathematical pedagogy, developing as a teacher of mathematics and the teacher's role in professional development.

Mathematics teachers must have good role models during their preservice and continuing inservice training. Teachers often teach the way they have been taught. Therefore, preservice instructors need to address the major components of teaching: tasks, discourse, environment and analysis of teaching.

The education of mathematics teachers should develop their knowledge of the content and discourse of mathematics. Teachers' comfort with, and

confidence in, their own knowledge of mathematics affects what they teach and how they teach it. Their conceptions of mathematics shape their choice of worthwhile mathematical tasks, the learning environments they create and the discourse in their classrooms. Knowing mathematics includes understanding specific concepts and procedures as well as the process of doing mathematics. Mathematics involves the study of concepts and properties of numbers, geometric objects, functions and their uses—identifying, counting, measuring, comparing, locating, describing, constructing, transforming and modeling. The relationships and recurring patterns among these objects and the operations on these objects lead to building such mathematical structures as number systems, groups or vector spaces and to studying the similarities and differences among these structures.

Such knowledge ought not to be developed in isolation. The ability to identify, define and discuss concepts and procedures; to develop an understanding of the connections among them; and to appreciate the relationship of mathematics to other fields is critically important.

The standards clearly state that, to sufficiently understand the mathematical topics specified for each level, teachers teaching mathematics should have not less than nine semester hours of coursework in content mathematics at the K–4 level, fifteen semester hours of coursework at the 5–8 level, and the equivalent of a major in mathematics at the 9–12 level.

The preservice and continuing inservice training of teachers of mathematics should provide multiple perspectives on students as learners of mathematics by developing teachers' knowledge of research on how students learn mathematics; the effects of students' age, abilities, interests and experience on learning mathematics; the influence of students' linguistic, ethnic, racial and socioeconomic backgrounds and gender on learning mathematics; and ways to affirm and support full participation and continual study of mathematics by all students.

Knowing mathematical pedagogy is an important standard that can be achieved through preservice and continuing inservice training of teachers of mathematics by developing

- teachers' knowledge of and ability to use and evaluate instructional materials and resources, including technology;
- ways to represent mathematics concepts and procedures;
- instructional strategies and classroom organizational models;
- ways to promote discourse and foster a sense of mathematical community; and
- means for assessing student understanding of mathematics.

Developing as a teacher of mathematics is really at the heart of teaching. Teachers of mathematics must have ongoing opportunities to examine and revise their assumptions about the nature of mathematics, how it should be taught, and how students learn mathematics; observe and analyze a range of approaches to mathematics teaching and learning, focusing on the tasks, discourse, environment and assessment; work with a diverse range of students individually, in small groups and in large-class settings with guidance from and in collaboration with mathematics education professionals; analyze and evaluate the appropriateness and effectiveness of their teaching; and develop dispositions toward teaching mathematics.

Essentially, being a teacher of mathematics means developing a sense of self as such a teacher. Such an identity grows over time and is built from many different experiences with teaching and learning. Further, it is reinforced by feedback from students that indicates they are learning mathematics, from colleagues who demonstrate professional respect and acceptance, and from a variety of external sources that demonstrate recognition of teaching as a valued profession.

Teachers develop as professionals on an ongoing basis. The standard regarding the teacher's role in professional development suggests that teachers of mathematics should take an active role in their own professional development by accepting responsibility for experimenting thoughtfully with alternative approaches and strategies in the classroom; reflecting on learning and teaching individually and with colleagues; participating in workshops, courses and other educational opportunities specific to mathematics; participating actively in the professional community of mathematics educators; reading and discussing ideas presented in professional publications; discussing with colleagues issues in mathematics and mathematics teaching and learning; participating in proposing, designing and evaluating programs for professional development specific to mathematics; and participating in school, community and political efforts to effect positive change in mathematics education. Schools and school systems must support and encourage teachers in accepting these responsibilities. What is essential is that teachers of mathematics view themselves as agents of change, responsible for improving mathematics education at many levels: the classroom, the school, the district, the region and even the nation.

Standards for Supporting and Developing Mathematics Teachers and Teaching

Professional Standards for Teaching Mathematics (NCTM 1991) presents a vision of teaching that calls for a teacher who is educated, supported and evaluated in ways quite different from current practice. To create a teaching environment as described in the standards, teachers must have access to educational opportunities over their entire professional lives that focus on developing a deep knowledge of subject matter, pedagogy and students.

Teachers can, and do, implement successful mathematics programs with little help or encouragement. However, sustaining them or expecting that they flourish without adequate support is not reasonable. The changes called for by the curriculum and evaluation and the professional teaching standards need the support of policymakers in government, business and industry; school administrators, school board members and parents; college and university faculty and administrators; and leaders of professional organizations.

Policymakers in government, business and industry should take an active role in supporting mathematics education by accepting responsibility for

- participating in partnerships at the national, provincial and local levels to improve the teaching and learning of mathematics;
- supporting decisions made by the mathematics education professional community that set directions for mathematics curriculum, instruction, evaluation and school practices;
- providing resources and funding for, and assistance in, developing and implementing high quality school mathematics programs that reach all students, as envisioned in *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989) and *Professional Standards for Teaching Mathematics* (NCTM 1991).

School administrators and school board members should take an active role in supporting teachers of mathematics by accepting responsibility for

- understanding the goals for the mathematics education of all students set forth in the NCTM standards and the needs of teachers of mathematics in realizing these goals in their classrooms;
- recruiting qualified teachers of mathematics, with particular focus on the need for the teaching staff to be diverse;
- providing a support system for beginning and experienced teachers of mathematics to ensure that they grow professionally and are encouraged to remain in teaching;
- making teaching assignments based on the qualifications of teachers;
- involving teachers centrally in designing and evaluating programs for professional development specific to mathematics;
- supporting teachers in self-evaluation and in analyzing, evaluating and improving their teaching with colleagues and supervisors;
- providing adequate resources, equipment, time and funding to support the teaching and learning of mathematics as envisioned in the standards;
- establishing outreach activities with parents, guardians, leaders in business and industry, and others in the community to build support for quality mathematics programs; and
- promoting excellence in teaching mathematics by establishing an adequate reward system, including salary, promotion and conditions of work.

College and university administrators need to actively support mathematics and mathematics education faculty by accepting responsibility for establishing an adequate reward system, including salary, promotion and tenure, and conditions of work, so that faculty can and are encouraged to

- spend time in schools working with teachers and students;
- collaborate with schools and teachers in the design of preservice and continuing education programs;
- offer appropriate graduate courses and programs for experienced mathematics teachers;
- provide leadership in conducting and interpreting mathematics education research, particularly school-based research;
- cooperate with pre-college educators to articulate the K–16 mathematics program; and
- make concerted efforts to recruit and retain teacher candidates of quality and diversity.

The leaders of professional organizations need to take an active role in supporting teachers of mathematics by accepting responsibility for

- promoting and providing professional growth opportunities for those involved in mathematics education,
- focusing attention of the membership and the broader community on contemporary issues dealing with the teaching and learning of mathematics,
- promoting activities that recognize the achievements and contributions of exemplary mathematics teachers and programs, and
- initiating political efforts that effect positive change in mathematics education.

Summary

In *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989) and *Professional Standards for Teaching Mathematics* (NCTM 1991), the argument is made that what is needed is a design change strategy. This means that new ways of doing things within the system—new roles for teachers and students, new goals, new structures—must be explored to find solutions to persistent problems that result in students failing to become mathematically powerful.

While the standards have directed us to some degree to the issues that need to be addressed, they are not a prescription for what must done at each grade level. However, if we make a long-term commitment to the standards set forth, if we endeavor to persevere and if we continue to modify our course as new knowledge comes to the fore, we will make progress toward the goal of developing mathematical power for all students. Such massive change as is proposed in the standards will take time and much work and dedication from teachers and many others.

First, we must challenge all those charged with responsibility to teach mathematics in our schools to work collaboratively in using the curriculum and evaluation and professional standards as the basis for change so that the teaching and learning of mathematics in our schools is improved.

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When patterns are broken, new worlds can emerge.

—Tuli Kupferberg

When you reread a classic, you do not see more in the book than you did before, you see more in you than there was before.

—Clifton Faidman