Exploring Products Through Counting Digits, Part 2: Factors with an Unequal Number of Digits

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Number sense is a kind of intuitive idea about how numbers behave. Among other things, children with good number sense "recognize the relative magnitudes of numbers" and they "know the relative effect of operating on numbers" (NCTM 1989, 38). These "intuitions" enable students to make realistic estimates of computational results.

In Part 1 (Cathcart 1995), activities were suggested that would help children make generalizations about the minimum and maximum number of digits in a product of two numbers that had an equal number of digits. The number of digits in the product will be either $2n - 1$ or $2n$, where n is the number of digits in each product. Applying this understanding may help children avoid some unreasonable computational results.

But what happens if the factors do not have the same number of digits? That is the focus of this article.

One-Digit (Non-Zero) Multipliers

Have the children make a table similar to Table 1 below or provide them with the headings and nine rows (a 10 by 5 grid). Ask them to fill in the minimum product column by *mentally* doing the computation and the maximum product column by using a *calculator*. Then assign the investigation questions below for group discussion. A whole class discussion afterward would also be useful to synthesize observations and generalizations.

For Investigation

- What can you say about the minimum product? Describe the pattern that you see.
- What can you say about the maximum product? Describe the pattern that you see.
- What can you say about the number of digits in the minimum product?
- What can you say about the number of digits in the maximum product?
- What can you say about the number of digits in the product of any number and a one-digit number?
- In the case of two numbers with an equal number of digits, the number of digits in the product will be $2n$ or $2n - 1$, where n is the number of digits in each product (Cathcart 1995). How can this generalization be modified so that it is true for the case of a two-digit or greater number multiplied by a one-digit number?

Table 1
Products of One-Digit Multipliers

Factors	Minimum Product	Number of Digits in Minimum Product	Maximum Product	Number of Digits in Maximum Product
2-digit × 1-digit	10	2	891	3
3-digit × 1-digit				
⋮				
9-digit × 1-digit	100,000,000	9		

Two-Digit Multipliers

Have the children make another table similar to Table 1 with the title, Products of a Two-Digit Multiplier. In the factors column, have the children list 3-digit × 2-digit, 4-digit × 2-digit, 5-digit × 2-digit and so on to 9-digit × 2-digit.

Ask the children to

- fill in the minimum product column using mental arithmetic and
- fill in the first four rows (three-digit through six-digit) using a calculator.

For Investigation

- What can you say about the minimum product? Describe the pattern that you see.
- Examine the column with the maximum products.
  - Is there a pattern? Describe it.
  - Use the pattern to complete the rest of the maximum product column.
- Compare the maximum product column in Table 1 for one-digit multipliers and the one just completed for two-digit multipliers.
  - How are they the same?
  - How are they different?
- What can you say about the number of digits in the product of any number and a two-digit number?
  - Does the new generalization you made for one-digit multipliers in Table 1 hold for two-digit multipliers? If not, modify it so that it works for both cases.

Three-Digit Multipliers

Have students make another table like the previous ones but for three-digit multipliers. This time they will need only six rows: 4-digit × 3-digit, 5-digit × 3-digit, and so on to 9-digit × 3-digit.

Ask the children to

- fill in the minimum product column using mental arithmetic and
- fill in only the first two rows of the maximum product column using a calculator. This is all they will be able to do directly (in one step) on an eight-digit display calculator. (One possible strategy to do computations with larger numbers is outlined in Cathcart 1995).

For Investigation

- What can you say about the minimum product? Describe the pattern that you see.
- Examine the two products that you completed in the maximum product column.
  - How are these two numbers the same as the first two maximum products for two-digit multipliers?
  - How are they different?
  - Use this information to write in the rest of the products in the maximum product column for three-digit multipliers.
    - Discuss the pattern that you used with a friend. Did he or she discover the same pattern?
- Examine the pattern for minimum products in the three tables you have completed. How is the pattern the same? How is it different?
- Examine the pattern for maximum products in the three tables you have completed. How is the pattern the same? How is it different?
- Does the previous rule for how many digits there will be in a product hold for three-digit multipliers?

Larger Multipliers

Use what you have learned to

- predict the least number of digits in the product of a five-digit number and a four-digit number;
- predict the greatest number of digits in the product of a five-digit number and a four-digit number;
- write the smallest product you could get when you multiply a five-digit number by a four-digit number; and
- write the largest product you could get when you multiply a five-digit number by a four-digit number.

Conclusion

Children should be more successful at mathematics if they have good number and operation sense. Being able to see patterns is one component of number and operation sense. Explorations of the relationships and patterns suggested in this article should help children develop number and operation sense and should provide one more way for them to check the reasonableness of a product—does my answer have the number of digits that it should have?

References

Cathcart, W. G. "Exploring Products Through Counting Digits: Equal Factors." *delta-K* 32, no. 3 (August 1995): 7–9.

National Council of Teachers of Mathematics (NCTM). *Curriculum and Evaluation Standards for School Mathematics.* Reston, Va.: NCTM, 1989.