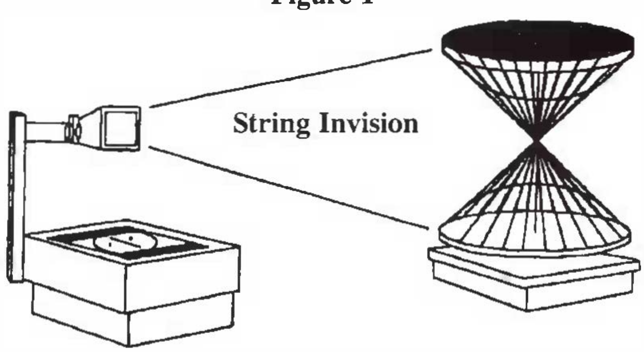
String Invision: Conics Teaching Model

Don Hillacre

The String Invision conics teaching model was designed as a classroom manipulative for Math 30 students, to enhance their ability to understand the physical properties of the conic sections with respect to the intersection of a plane and a cone. Use of the model in the classroom allows students to experiment with various conic scenarios and to demonstrate their understanding of the conic properties. The three-dimensional and interactive qualities of the model provide opportunity for hands-on learning and concept reinforcement that the two-dimensional and static models fail to do.

String Invision (Figure 1) is a tabletop model consisting of two wooden disks held apart on a centre pole (the axis of symmetry), mounted on a square wooden base. The top disk is fixed to the pole and the bottom disk is free to rotate up and down on the centre pole. The rotatable disk is suspended from the top disk by string segments, which form a cylinder with the top and bottom disks as ends. As the bottom disk is rotated, the strings form a double nappe cone with the vertex on the axis pole. An overhead projector is used to shine a light-plane onto the strings of the model. The light-plane is created by using a mask on the overhead projector that blocks out all light except for a fine line. The mask is designed to allow versatility in the placement of the light plane.

Figure 1

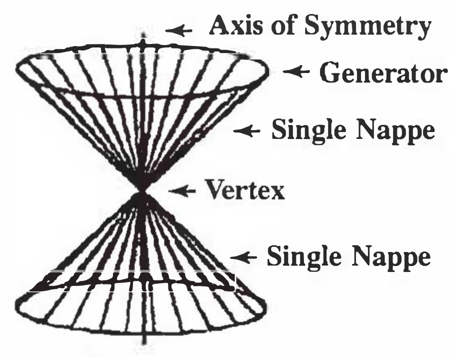


Conics Teaching Model

Because of the model's ability to demonstrate the double nappe cone and the cylinder, students can view all nine conic cross-sections and both arcs of the hyperbola. Gradually changing the angle and position of the light-plane allows discovery of how these different cross-sections are formed and how they can be changed in size and shape. The model also shows the formation of the degenerate curves. Because of the hands-on nature of the model, students are encouraged to formulate their own theories and definitions as to how the curves change with modification to the position and angle of the light-plane. Teacher-led discussion and testing promotes further refinement of these mathematical conjectures orally or in writing.

For a typical classroom lesson, the students gather around the model, taking care not to block the projection of the light-plane. The demonstration is best conducted in a semi-darkened room, using a high intensity projector or, if available, a laser light pen equipped to create a single line of light. (The Math Factor video series to be released by ACCESS Network will include a demonstration of the model, using a laser light source.) The first step of the lesson would be to familiarize the students with the parts of the model (Figure 2): generator line, axis of symmetry, vertex and both nappes of the double nappe cone. When the mask is removed from the overhead projector, the light from the projector casts a two-dimensional shadow of the model on the wall. This can be used to assist students in drawing the three-dimensional model.

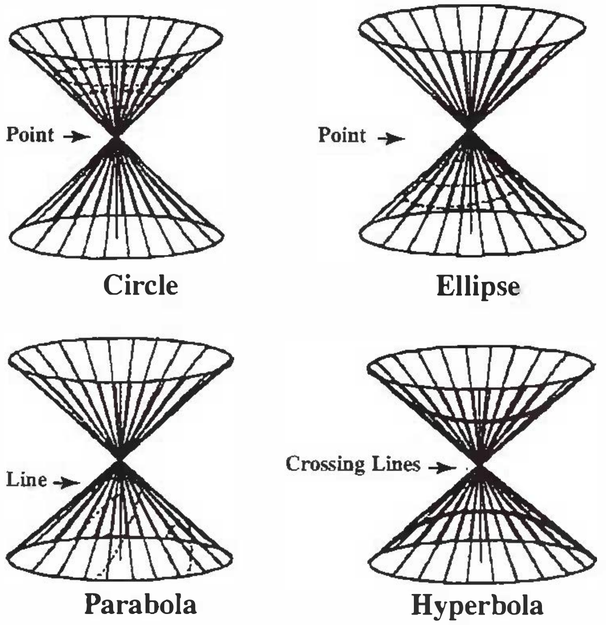
Figure 2



Double Nappe Cone

The second step in the lesson would be to project the light-plane onto the strings at various angles and to observe the four typical (basic) conic cross-section curves: the ellipse, the parabola, the hyperbola and the circle (Figure 3). The circle can be seen as the limiting position of the ellipse. At this point, the students can be encouraged to demonstrate their understanding of how the curves are formed by positioning the plane on a two-dimensional diagram and by participating in oral discussion. Definitions of the four typical conic curves can then be formally written down.

Figure 3

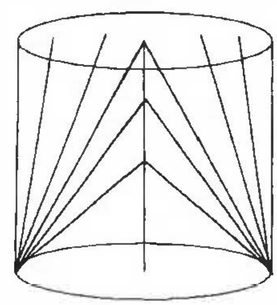


The third step will be to test these definitions using the model and then to refine them to disallow the degenerate cases. For example, an initial definition of a parabola might be stated as "the curve formed by the intersection of a plane parallel to the generator." The definition is correct to a point but needs to be made more restrictive to disallow the degenerate straight line. Another example is the hyperbola and its degenerate, the two intersecting lines. This naturally leads into discussion about degenerates and how they are formed.

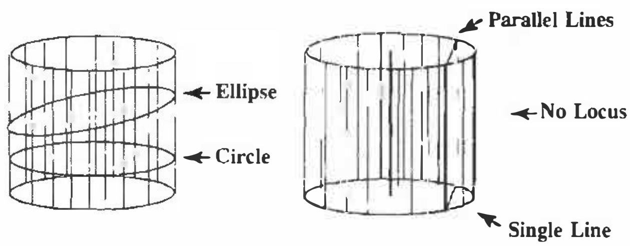
The fourth step will be to observe the effects of moving each of the four typical conic curves toward the vertex and thus to their degenerates. This will show the circle and ellipse getting smaller and eventually becoming a degenerate point. The circle and the ellipse degenerate into a point since the circle is really the limiting case of an ellipse. The parabola narrows and becomes a single line. The hyperbola narrows with its vertices approaching each other and becomes two crossing lines, two V-shaped curves meeting at the vertex.

For the fifth step, begin with a recap of the four basic curves and their three degenerates, then move on to discover the remaining two curves: two parallel lines and no locus. Where are the other two curves? They cannot be observed on the double nappe cone, so the cone must be degenerated into a cylinder. This can be demonstrated by grasping the vertex in one hand and releasing the clip that holds the double nappe cone in place with the other hand. Then move the hand holding the vertex along the centre pole, allowing the string to slide through the hand. As the vertex moves up, the sides of the cone can be seen to move toward the vertical and thus at infinity would become a cylinder. The increasing slope of the walls of the lower nappe can be best observed by noting the model's shadow on the wall. At this point, the vertex can be released, and the cylinder will be created on the wall (Figure 4).

Figure 4



Cone to Cylinder



Replace the mask on the projector and recreate as many of the seven curves mentioned as are possible on the cylinder. Students will soon discover they are unable to create the parabola, the hyperbola, the two crossing lines, the single line through the vertex and the point. But they can now create the two parallel lines, the single line and no locus by making the light-plane parallel to the centre pole and increasing its distance from the pole. When the plane's distance is less than the radius of the cylinder, two parallel lines can be produced. At a distance equal to the radius of the cylinder, a single line is formed. Distances greater than the radius will not intersect the cylinder and thus no locus is formed. Summarize which curves can be made on the double nappe cone, which can be made on the cylinder and which can be made on both.

To reinforce and conclude the lesson, the students may choose a variety of activities, such as creating a bulletin board display, doing small-group presentations on what they learned in the lesson, answering the questions in the teacher's resource manual and so on. Students often wish to participate in further exploration of concepts such as the following:

- What happens to the original curves when the double nappe cone is converted to the cylinder with each curve on the model?
- What observations can be made in the shadow that is cast on the wall when the double nappe cone is converted to the cylinder?
- What observations can be made in the shapes of the curves if, instead of a single straight line of light, other shapes are projected on the strings such as parallel lines of varying thickness, curved lines such as circles and sine waves, or regular figures such as triangles and squares?
- Make a mobile art show as the model spins and the projector head is oscillated up and down.

The String Invision model and its attendant discovery approach to learning have been well received by teachers and students. One of the model's most noticeable effects on the students is that students can actually see in three dimensions how the conic sections are formed, as opposed to trying to grasp the concepts from seeing two-dimensional diagrams or static single nappe cones. Course-end student surveys always include positive comments and encouragement for continued use of the model in future classes. Students enjoy and benefit from the active participation and interaction that happens around the model.

The surest way to be late is to have plenty of time.

—Leo Kennedy

Yes, I'm growing older. But the important word is not "older." It is "growing."

—Joan Sutton