delta-k

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MATHEMATICS COUNCIL
OF THE ALBERTA
TEACHERS' ASSOCIATION

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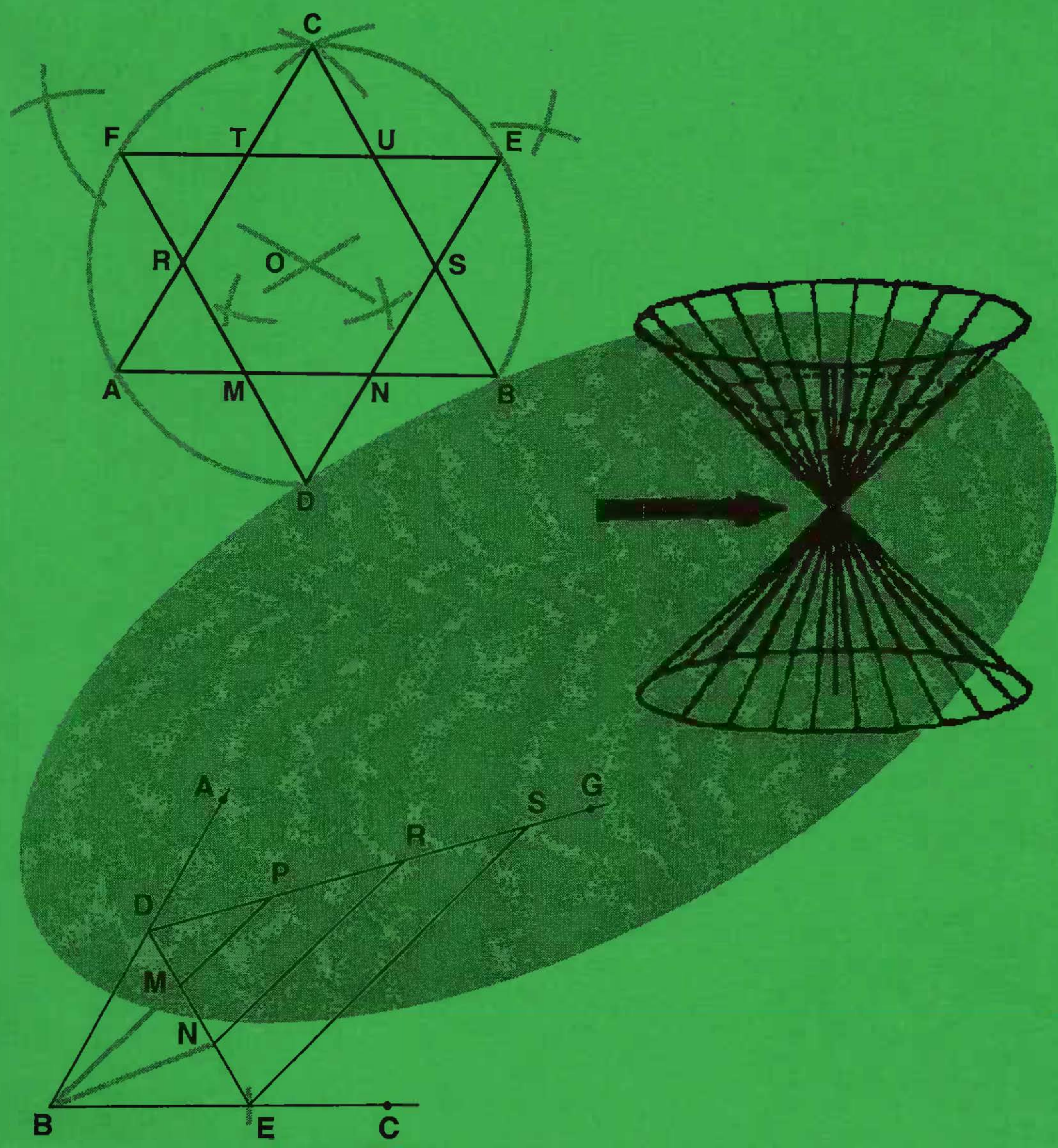
December 1995



This issue of *delta-K* is dedicated to Norm Inglis.

delta-k

Volume 33, Number 1

December 1995

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 delta-K is published by the ATA for the Mathematics Council (MCATA). EDITOR: Arthur Jorgensen, 4411 5 Avenue, Edson, Alberta T7E 1B7. EDITORIAL AND PRODUCTION SERVICES: Central Word Services staff, ATA. Opinions expressed herein are not necessarily those of MCATA or of the ATA. Address correspondence regarding this publication to the editor. *delta-K* is indexed in the Canadian Education Index. ISSN 0319-8367

COMMENTS ON CONTRIBUTORS

W. George Cathcart is a professor in the Faculty of Education at the University of Alberta.

Linda Chiem is a student at St. Mary's High School in Calgary.

Yvette d'Entremont is an associate professor in elementary and secondary math methods at the University of Alberta.

Marcia Groves teaches at Windsor Park Elementary School in Edmonton.

Don Hillacre teaches at Spruce Grove Composite High School in Spruce Grove. In March 1995, Don received a $2,500 fellowship from the Roy C. Hill Charitable Foundation for his use of the string invision model in his classroom.

Louise M. Lataille is a professor at Springfield College in Springfield, Massachusetts.

Werner W. Liedtke is a professor at the University of Victoria.

A. Craig Loewen is a professor at the University of Lethbridge.

Zenia Nemish teaches at Windsor Park Elementary School in Edmonton.

Jack Price is president of NCTM.

Klaus Puhlmann is the superintendent of the Grande Yellowhead Regional Division No. 35 in Edson.

Connie K. Varnhagen is an instructor at the Centre for Research in Child Development, Department of Psychology, University of Alberta.

Fran Yeske teaches at Windsor Park Elementary School in Edmonton.

delta-K is the official journal of the Mathematics Council (MCATA) of The Alberta Teachers' Association (ATA).

The journal's objective is to assist MCATA to achieve its goal of improving teaching practices in mathematics by publishing articles that increase the professional knowledge and understanding of teachers, administrators and other educators involved in teaching students mathematics. The journal seeks to stimulate thinking, to explore new ideas and to offer various viewpoints. It serves to promote MCATA's convictions about mathematics education.

EDITORIAL

By the time you receive this issue of *delta-K*, you will be almost halfway through another school year, with all its challenges. The annual conference is already history; you will likely have distributed report cards for the first time; and you will be getting ready for the Christmas break and looking forward to 1996. It is hard to believe there are only four years left in this century.

The MCATA publications schedule is different this year. Instead of publishing a newsletter and a journal this fall, we are publishing only this issue of *delta-K*, which is a combination journal and newsletter. The proposed timeline for future issues of *delta-K* is February, April and July. Deadlines for receiving materials will be February 10 for the April issue and April 25 for the July issue.

Other publications available to members this school year are the historical document, *Thirty-Four Years and Counting: A History of the Mathematics Council of The Alberta Teachers' Association* that provides an overview of MCATA from 1961 to 1995, and a monograph on statistics and probability being prepared by Florence Glanfield which should be available by late spring. Also, Andy Lui of the University of Alberta is the appointed editor for *Math for Gifted Kids II*, a joint publication with the Gifted and Talented Education Council that is due out in spring 1996.

MCATA is looking for ways to best serve its members and is prepared to help with organizing miniconferences and regionals. It would be great to see some regionals established in this province. MCATA also supports mathematics contests.

Remember the 1996 annual conference will be held in Red Deer in early November. Plan to take an active part.

All the best for the festive season and 1996.

Special Thanks

Special thanks to Joan Worth for her many hours of effort in assembling materials for the historical publication *Thirty-Four Years and Counting*. Without her dedication and hard work, it would never have happened.

Thanks also to all who responded to our request for articles outlining experiences and memories related to the Mathematics Council.

The publications staff at Barnett House deserve a special bouquet for their commitment to the project. Despite all their other assignments, they made every effort to ensure this historical document was one we could be proud of. Thanks, Kate Ballash, Penny Harter, Heather Parker, Lisa Pashniak, Yuet Chan (designer) and Lisa Maltby (ATA archivist).

I might add that I have always found these folks most helpful with all MCATA publications. Without their help, my assignment would be impossible.

Arthur Jorgensen

I wouldn't have seen it if I hadn't believed it.

—Marshall McLuhan

From the President's Pen

Each year promises its own surprises, anticipations and initiatives. As is the case each fall, we are happy to welcome the new executive members and look forward to their contributions toward making MCATA a continued success. We bid a fond farewell to those members who are leaving the executive—we know they will continue to serve mathematics education in the province in other arenas.

We all enter the coming year with some intrepidness and cautious optimism about what lies ahead for education, particularly mathematics education. One MCATA mandate continues to be that of communication, information sharing and service to our members.

Our first major function of the school year was the September 29–30 conference in Lethbridge. The conference was a worthwhile professional development opportunity for teachers. The executive intends to reach out to as many members as possible in a number of ways. Procedures will be identified and put in place as the year unfolds.

I thank Past President Wendy Richards for her leadership over the last two years. A great number of positive developments took place during those years, and we appreciate Wendy's part in many of them.

Special thanks and congratulations are due to Art Jorgensen, publications editor; Joan Worth; and staff at Barnett House for the job well done in producing the archival and historical publication, *Thirty-Four Years and Counting: A History of the Mathematics Council of The Alberta Teachers' Association.*

In closing, I extend a genuine invitation for anyone with questions, comments, ideas or suggestions to share them with any executive member. We look forward to hearing your opinions and receiving your input. Best wishes for an exceptional year.

George Ditto

From the Membership Director

In future, I would like to use this space to report on activities that members are performing to promote math education in their classrooms, schools or communities. Do you or any of your colleagues have innovative ideas to share? Are math inservice sessions or miniconferences being held in your area that you would like others to know about? Will your school be celebrating Math Education Week, and what activities will the school be involved in? Let me know what is happening with math education at your school: Daryl Chichak, 1826 51 Street NW, Edmonton T6L 1K1; fax 469-0414.

Daryl Chichak

George Ditto, Bob Hart and Dick Kopan made the presentation to Bill Bell (r).

Norm Inglis Memorial

In memory of Norm Inglis, a TV and VCR were presented to Bill Bell, principal of A. E. Bowers Elementary School in Airdrie. The TV and VCR will be used in the Norm Inglis Technology Centre at the school.

Norm served on the MCATA executive as the PEC liaison from 1990 until his death in June 1994.

Bob Hart

Art Peddicord stands with Cindy Meagher.

Farewell to Art Peddicord

Art Peddicord is retiring after long and dedicated service to education in Alberta. For many years, he served as the Department of Education representative on the MCATA executive.

Art was primarily responsible for starting the mathematics symposia that have successed in bringing mathematics teachers from across the province together for an inspiring day of presentations and discussions. Although retiring, Art indicated a desire to assume a major share of the responsibility for organizing future symposia.

On behalf of MCATA, Cindy Meagher presented Art with a retirement gift at the symposium in Calgary on May 5, 1995.

[Editor's note: I understand that Art has not been particularly well. All MCATA members wish him a speedy recovery.]

1995 Outstanding Mathematics Educator Award: Florence Glanfield

Florence Glanfield receives the 1995 Outstanding Mathematics Educator Award from Cindy Meagher.

The 1995 Outstanding Mathematics Educator Award was presented to Florence Glanfield at the annual conference in Lethbridge at the end of September.

Florence has been active in various roles in the field of mathematics education since 1982. Some of her roles include the following: senior high mathematics teacher; research assistant; faculty consultant for student teachers of senior high mathematics; university instructor, curriculum designer for Mathematics 14 and 24; examination manager for Mathematics 30 diploma exams; and author and reviewer for junior high mathematics resources. Florence is an *active* member of many mathematics organizations, including the National Council of Supervisors of Mathematics (NCSM), the National Council of Teachers of Mathematics (NCTM), the Association of State Supervisors of Mathematics (ASSM), the Mathematics Council of The Alberta Teachers' Associaton (MCATA), and the Ontario Associaton of Mathematics Educators (OAME).

Florence's professional activities include acting as a referee for the National Awards of the Prime Minister's Award for Teaching Excellence in Science, Technology and Mathematics; serving as

- a member of the Program Committee for the National Forum on Mathematics Education,
- Western Canada representative, Regional Services Committee, NCTM,
- conference chair, NCTM 1994 Canadian Regional conference and
- Alberta Education representative to MCATA;

and participating in the annual meetings of NCTM, NCSM and ASSM.

Florence has given presentations and provided inservice sessions for organizations such as teachers' conventions, the Langley School District, the Manitoba Association of Mathematics Teachers, the Yukon Teachers' Association, international NCTM meetings, the Saskatchewan Mathematics Teachers' Society, the MCATA, the Ontario Association of Mathematics Teachers, the National Council of Supervisors of Mathematics, the ATA Computer Council and the County of Parkland. The range of locations for these presentations covers Yellowknife, Lethbridge, Medicine Hat, Edmonton, Calgary, Winnipeg, Seattle, Toronto, Rocky Mountain House, Leduc, St. Albert, Slave Lake, Red Deer, Hay River, Sangudo, Bonnyville and many more.

Phil Campbell (assistant director, Student Evaluation, Alberta Education) commented that Florence's efforts have enabled teachers to understand what students should know and be able to do in mathematics and have established standards for student achievement in Alberta. Frank Horvath (director, Student Evaluation) stated that Florence has the unique inclination to act on ideas and to make opportunities for communicating the importance of mathematics education and the role of math in people's lives.

Florence has demonstrated throughout her career her commitment to improving student learning. She has contributed extensively to the professional development of mathematics teachers. She has shown the ability to be creative and innovative in her involvement with mathematics education. Her untiring determination to enhance mathematics teaching and learning has earned her this award.

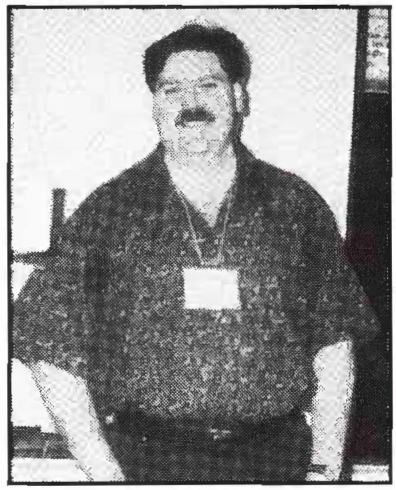
Cindy Meagher

1995 Conference and Symposium Photos

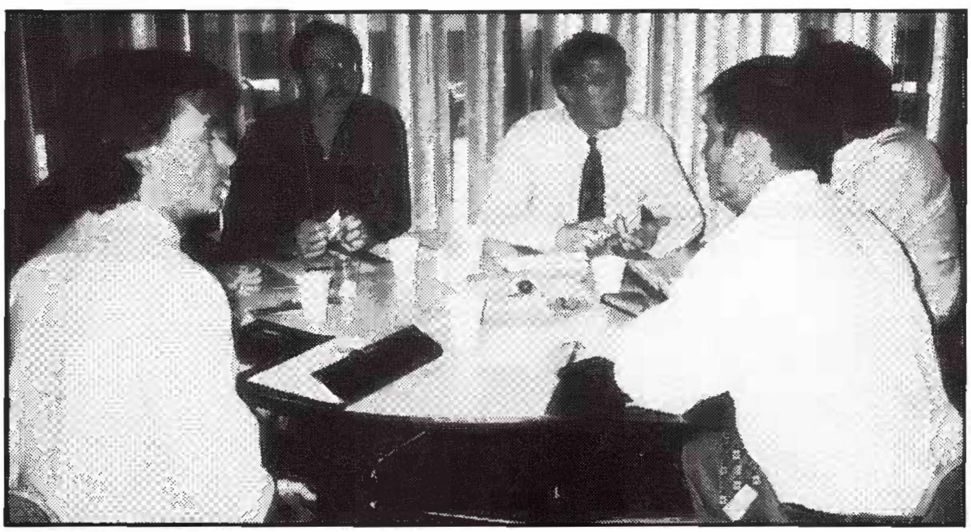
These photos were taken at the Math symposium and the annual conference in Lethbridge September 28–30, 1995.

Art Jorgensen

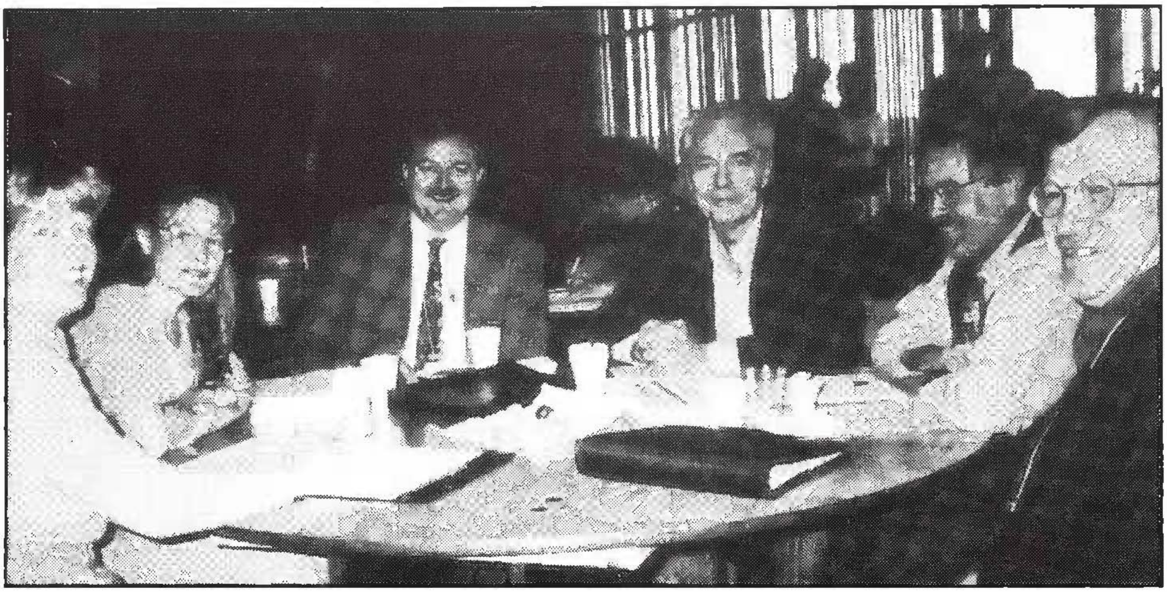
Marie Hauk



A. Craig Loewen



Let's assess the donuts.



Roundtable discussion on assessment

Katherine Willson

George Ditto

Conference participants

Nouveautés en mathématiques

Élémentaire

Un nouveau programme de mathématiques à l'élémentaire vient de paraître. Il a été publié en deux formats: le premier, en vertical, renferme toutes les attentes du programme de mathématiques à l'élémentaire; le deuxième, en horizontal, contient des tableaux séquentiels montrant la progression des thèmes de la maternelle à la 6e année.

Un lexique de mathématiques provisoire a été envoyé à toutes les écoles. La version finale de ce lexique paraîtra à l'automne et inclura le vocabulaire du *Cadre commun des programmes d'études de mathématiques M–12.*

À paraître dans les mois suivants: liste de ressources annotées.

Maternelle à la 12e année

Le *Cadre commun des programmes d'études de mathématiques M–12* est le résultat du Protocole de collaboration sur l'éducation de base dans l'ouest canadien. Les ministères de l'éducation des provinces de l'ouest et des territoires ont accepté ce cadre comme étant le programme officiel et ont promis de l'implanter dès qu'ils le jugeront opportun.

L'Alberta a choisi d'implanter le programme aux niveaux de la 7e et de la 9e année à compter du mois de septembre 1996, et les autres niveaux, c'èst-à-dire de la maternelle à la 6e année ainsi que la 8e année, à compter de septembre 1997.

Secondaire

Le nouveau programme de mathématiques 31 a été distribué cet été et doit être implanté à compter de septembre 1995.

Un guide pédagogique et un document d'information suivront sous peu.

Nouvelles ressources

Pour les élèves et les enseignants

- Jean-François Vincent, *Lexique mathématique,* 1994, Guérin, 4501, rue Drolet, Montréal H2T 2G2; ISBN 2-7601-3755-4.
- Ronald Côté et al., *LEXIMATH, lexique mathématique de base,* Éditions Beauchemin, 3281, avenue Jean Béraud, Chomedey, Laval H7T 2L2; ISBN 2-7616-04460-6.

Pour les enseignants

- Denis de Champlain et al., *LEXIQUE MATHÉMATIQUE, enseignement secondaire,* Éditions du Triangle d'Or, Modulo Éditeur, 233, avenue Dunbar, Bureau 300, Mont Royal, Montréal H3P 2H4; ISBN 2-89422-007-3.

Getting people to like you is just the other side of liking them.

—Norman Vincent Peal

The Right Angle

1996 Mathematics 30 Diploma Examinations

Each Mathematics 30 diploma examination is designed to reflect the core content outlined in the course of studies and is limited to those expectations that can be measured by a paper-and-pencil test. Two and one-half hours of writing time are allotted for the exam, but students may take an additional half hour to complete it.

Content for the 1996 Mathematics 30 diploma examinations in the machine-scored section is emphasized as follows:

Multiple-Choice and Numerical Response	Percent Emphasis
Polynomial Functions	11
Trigonometric and Circular Functions	12
Statistics	5
Quadratic Relations	10
Exponential and Logarithmic Functions	11
Permutations and Combinations	10
Sequences and Series	11
	70
Written Response	30

As published in the 1994–95 Mathematics 30 diploma examination information bulletin, written-response questions will assess whether students can draw on their mathematical experiences to solve problems and to explain mathematical concepts. Therefore, the written-response questions will not necessarily fall into a particular unit of study but may cross more than one unit or may require students to make connections between mathematical concepts.

The three mathematical understandings of procedures, concepts and problem solving are addressed throughout. Each understanding has the following emphasis:

Multiple-Choice and Numerical Response	Percent Emphasis
Procedures	21
Concepts	24.5
Problem Solving	24.5
Written Response	**Percent Emphasis**
Procedures, Concepts, Problem Solving	30

Check the 1995-96 bulletin, sent to all Albertan schools for more information regarding standards, assessment of communication and problem solving.

Mathematics 33

The Mathematics 33 bulletin for 1995–96 has been sent to all Albertan schools.

Achievement Assessment

Results of the Grades 3, 6 and 9 June 1995 achievement assessments were sent to schools in October. School and jurisdiction reports followed in November.

Information bulletins regarding the June 1996 assessments went to schools in October.

Kay Melville

The Space Detective Agency: A Summer Program of Mathematics and Science

In August 1995, children aged 10–14 participated in a week-long mathematics and science program sponsored by the Centre for Mathematics, Science and Technology Education (CMASTE) at the University of Alberta. A space detective agency theme provided a focus for an interesting assortment of hands-on investigations. Students spent the week as detectives, collecting and analyzing clues in the form of science and mathematics problems to solve intergalactic mysteries. Activities included finding information through the Internet on the environmental features of cold planets and the physical features of animals living in the Arctic; experiments with liquid nitrogen and dry ice; decoding messages sent from outer space; fingerprint and locomotion analyses; building rockets and CO_2 cars; plus many more challenging and interesting activities in which observation, classification, extrapolation, deduction and a healthy imagination were required.

The culminating event of the week had students analyzing an alien space probe that included objects and information about the aliens' location in space, their physical being, and atmospheric and environmental aspects of their planet. Each group then presented a live news broadcast of its findings to a receptive audience of parents, peers and faculty members.

The program was organized and conducted by mathematics and science education doctoral students, in conjunction with faculty members from the Department of Secondary Education at the University of Alberta. A research component of the project focused on students' understanding of mathematics and science in high activity, interactive and variable-entry investigations.

The Space Detective Agency pilot summer program proved enjoyable for everyone involved. Its success has inspired talk of running the program again next summer. For more information, contact Elaine Simmt, Lynn Gordon Calvert or Leo MacDonald in the Department of Secondary Education, University of Alberta, at 492-3674.

A Challenge

Evaluate the following problems and send your solutions to C. Georges Mullings, 2-4518 54 Avenue, Barrhead T7N 1K7.

1. How do you use a grid or graph to teach division in algebra?

2. What is the algorithm to find the cube root of any number?

3. How do you evaluate the following equation?

$$\int_0^{\pi/2} \sqrt{\sin\theta}\ d\theta =$$

Canadian Forum for Education in Mathematics

Last May, I represented MCATA in Quebec City at the Canadian Forum for Education in Mathematics. Sponsored by the Canadian Mathematical Society, the forum brought together about 150 mathematics educators at all levels—from classroom teachers to professors of mathematics—from across Canada. As well, representatives from ministries of education, Industry Canada, educational publishing, and technology and telecommunications companies attended.

The forum began with a presentation on SchoolNet, a cooperative initiative of governments, educators and industry, that aims to link all Canadian schools to the electronic highway. (See "SchoolNet" sidebar.)

We then broke into discussion groups to examine one of four topics: curricula, expectations, lifelong learning or math at work. Each group agreed on key issues and goals, and then worked to identify strategies and procedures to accomplish them. We also predicted how mathematics education would look if our aims were achieved. This provided for some lively debate and some interesting conclusions.

The second day, we reported on our previous deliberations and then met in provincial/territorial action groups. This was especially valuable, as we were able to share information about the Western Canada Protocol for a Common Math Curriculum with college and university colleagues, most of whom had never heard of it.

I left Quebec City with a more global perspective of mathematics education in our country. It was gratifying to realize that efforts are being made across the country to improve the quality of our mathematics teaching. I was encouraged by the apparent realization in postsecondary institutions that different types of teacher education courses in mathematics are needed. Concerns were voiced from every province about the need for continuing professional development and inservice sessions in new curricula and the lack of funds for implementation. (I hope the ministry representatives took note. . . .)

Everyone came away believing that we had begun a dialogue that was important and should be pursued, and that it would be possible to continue to stay in contact with one another as math educators via the Internet. I urge each of you to explore that same possibility.

SchoolNet

SchoolNet is a cooperative initiative of Canada's provincial, territorial and federal governments, educators, universities and colleges and industry. It aims to link Canada's 16,000 plus schools to the electronic highway as quickly as possible.

SchoolNet's objective is to electronically network Canadian schools to

- enhance educational opportunities and achievements in elementary and secondary schools across Canada by making national and international education resources available to Canadian teachers and students no matter where they are located;
- foster significant improvements in learning performance by facilitating the development of electronic delivery of the most advanced and proven educational techniques through new software applications and access to electronically based resources;
- stimulate learning and produce a school graduate population with a strong command of information and telecommunications technologies, which will be key employability skills in a knowledge-based economy;
- identify and develop new educationally relevant services from government, industry, universities and colleges, and facilitate their electronic provision to schools, teachers and students;
- build shared learning experiences among teachers and students in schools across Canada through electronically based educational projects; and
- stimulate Canadian information technology, software and multimedia businesses by providing new market opportunities.

Our forum presentation was given by Nadya Cournoyer, Industry Canada, 235 Queen Street, Ottawa, ON K1A 0H5; phone (613) 991-6057, fax (613) 941-1296, e-mail cournoyernadya@ic.jc.ca

Wendy Richards

Professional Dates, Resources and News

American High School Mathematics Examination (AHSME)

The AHSME will be held Thursday, February 15, 1996. Registration fees are as follows:

- US$15 before January 15
- US$20 January 16–February 1
- US$25 after February 1

Exams, including student answer forms, are sold in bundles of 10 at US$7.50 per bundle. Registration forms and brochures are available from Dr. Walter E. Mientka, Executive Director, American Mathematics Competitions, Department of Mathematics and Statistics, 1740 Vine Street, University of Nebraska-Lincoln, Lincoln, NE 68588-0658.

NCTM

- Annual Conference, San Diego, April 25–28, 1996
- Canadian Regional Conference, Vancouver, August 22–24, 1996

Mathematics Education Month

April 1996 is Mathematics Education Month. Start planning now to make the most of it in your classroom and your school.

MCATA Conference

The Council's next annual conference, "Math: Making Connections," will be held November 1–3, 1996, in Red Deer. The program committee is putting together an interesting program—professionally and socially. A Call for Presenters form is enclosed with this issue of *delta-K*; if you or someone you know has a topic to put forward, please submit the form as soon as possible.

Of course, conference organizers are hoping for a large turn out of teachers to hear these wonderful presentations. Plan now to attend.

National Council of Supervisors of Mathematics (NCSM)

The NCSM is an organization for leaders in mathematics, from early childhood to adult. NCSM is unique in its purpose: supporting mathematics leadership at the school, district, college/university, state/province and national levels. Its membership constitutes an international force, collaborating to initiate and implement reform in mathematics education.

Contact Ralph Connelly, Faculty of Education, Brock University, St. Catharines, ON L2S 3A1; phone (905) 688-5550, fax (905) 688-0544, e-mail rconnell@dewey.ed.brocku.ca or David Glatzer, West Orange High School, 51 Conforti Avenue, West Orange, NJ 07052; phone (201) 669-5301, fax (201) 669-1260.

Mathpacks, A Home-School Connection

The Mathpacks project evolved from a concern about how to make connections between home and school regarding children's mathematics learning. The project provided teachers and parents with a wonderful bridge between home and school. For more information, contact

- Liz Krezanoski, St. Gabriel Catholic Elementary School, 466-0220;
- Elaine Trepanier, Mount Carmel Catholic Elementary/Junior High School, 433-1062;
- Pat Hauck, ECS consultant, Edmonton Catholic Schools, 441-6144; or
- Betty Morris, math consultant, Edmonton Catholic Schools, 441-6104.

Assessment in Practice

The Mathematical Sciences Education Board (MSEB) and the National Research Council in Washington, D.C., periodically publish a newsletter about assessment activities in mathematics. To date, there is no subscription cost. To get on the mailing list, contact Assessment in Practice, MSEB, HA 476, National Academy of Sciences, 21101 Constitution Avenue NW, Washington, DC 20418; phone (202) 334-1472, fax (202) 334-1453, e-mail mseb@nas.edu

CMASTE Monographs

- *Chaos, Fractals, and Infinity: Introductory Ideas for the Secondary Classroom* edited by J. M. Barnes and T. E. Kieren. $15.
  Includes seven papers produced by a group of mathematics educators who set out to explore the topics of chaos theory, fractal geometry and infinity as curriculum ideas.
- *Applying the NCTM Standards in Alberta* edited by A. Olson and D. A. Reid. $10.
  Applies the recommendations of the National Council of Teachers of Mathematics *Curriculum and Evaluation Standards for School Mathematics* and *Professional Standards for Teaching Mathematics* in the context of the realities of mathematics classrooms.
- *Graphing Calculators in High School Mathematics* edited by L. G. Calvert and E. Simmt. $15.
  A collection of activities and worksheets created by teachers for use in high school mathematics, with a strong emphasis on guided discovery as a means of facilitating student understanding of algebra and analytical geometry.

Make cheques payable to CMASTE. Mail your order and payment to CMASTE Book Orders, 341 Education South, University of Alberta, Edmonton T6G 2G5.

New Journal Launched

The Council for Canadian Learning Resources has launched a new national journal to encourage the selection and use of Canadian books, videos, computer software and other media for Canadian schools and libraries. *Resource*Links: Connecting Classrooms, Libraries and Canadian Learning Resources* began publication for teachers, librarians and other educators in September.

*Resource*Links* will review print resources (fiction and nonfiction books, pictures books, magazines, government publications, curriculum guides and textbooks), audiovisual resources (films, multimedia kits and videos) and electronic resources (CD-ROMs, listservs, multimedia resources and resources on the Internet). *Resource*Links* will recommend exceptional resources for specific curriculum themes and units and resources reflecting Canada's multicultural and regional diversity. The journal will suggest classroom tie-ins, resources to connect young people, literacy, curriculum and culture, effective units of instruction and approaches to literature-based reading programs.

Subscriptions are $49 for six issues per year and are available from the Council for Canadian Learning Resources, 604-810 West Broadway, Vancouver, BC V5Z 4C9; phone (604) 925-0266, fax (604) 925-0566.

Potpourri

- *Statement of Principles on Technology in the Reform of Mathematics and Science Education* has been published by the U.S. Department of Education and the National Science Foundation. For a free copy, contact the National Library of Education, 555 New Jersey Avenue NW, Room 1101, Washington, DC 20208; phone 1-800-424-1616.
- *Lessons in Mathematics for the Classroom and Inservice Sessions with Alternative Assessment Procedures* is a collection of 25 lessons dealing with mathematics topics appropriate for secondary school classrooms. Cost is US$12. Make cheques payable to LDEC Project, and mail your order to May Friedrich, Project Manager, University of Northern Iowa, Department of Mathematics, Cedar Falls, IA 50614-0506; phone (319) 273-6952.
- *Explorations in Science Culture 1995* is a directory of projects funded under the Science Culture Canada program. Mathematics projects are included. Contact Science Culture Canada, Science Promotion and Academic Affairs Branch, Industry and Science Policy Sector, Industry Canada, 8th Floor, West Tower, 235 Queen Street, Ottawa, ON K1A 0H5; phone (613) 953-1077.
- *Understanding Racial-Ethnic Differences in Secondary School Science and Mathematics Achievement* is a study from the National Center for Education Statistics (NCES). Free while supplies last. Contact Ruth Harris, NCES, Room 408, 555 New Jersey Avenue NW, Washington, DC 20208; phone (202) 219-1831.

News from NCTM

Canadian Regional Leadership Conference

The Canadian Regional Leadership Conference was held June 30–July 3, 1995, in Regina and was attended by representatives from nine provinces and the two territories. Conference highlights include the following:

- NCTM representatives spoke on new initiatives, delegate assembly resolutions, grants and joint membership
- Assessment standards
- Presentation of new technology
- Sharing of activities between provinces

One of the conference's most exciting outcomes was having the opportunity to dialogue with members from other provincial math councils. A lot of ideas, concerns and perspectives on mathematics across Canada were shared.

Details of the conference and information regarding the discussions will appear in future Mathematics Council publications.

Resources

The following resources are available from the NCTM, 1906 Association Drive, Reston, VA 22091-1593. To phone in an order, call 1-800-235-7566 (orders only).

Reach Out to Preservice Teachers

What is one of the best ways to spread the news about NCTM to preservice teachers? The NCTM offers specifically designed kits of NCTM materials to help teacher educators introduce the benefits of membership to their students. The NCTM also offers minisubscriptions for NCTM journals to individual preservice teachers.

The kit consists of materials including the NCTM news bulletin, an educational materials catalog, informational brochures and a catalog of special products. The kit also contains three recent copies of an NCTM journal—*Teaching Children Mathematics, Mathematics Teaching in the Middle School* or *Mathematics Teacher*—depending on the appropriate teaching level. The kit costs US$9.

The minisubscription is a four-month subscription to *Teaching Children Mathematics* or *Mathematics Teacher*. No other materials accompany the journal. A minisubscription costs US$9.

More Issues of *Mathematics Teaching in the Middle School* Available

Mathematics Teaching in the Middle School, NCTM's newest journal which addresses educational concerns of teachers and students in Grades 5–8, is increasing its publication to five issues per year (an increase of one over the 1994–95 academic year). NCTM members may receive *Mathematics Teaching in the Middle School* by making it an add-on subscription to their current NCTM membership for US$15. Institution memberships are US$50.

Assessment Standards Now Available

Assessment Standards for School Mathematics is now available for US$15, a discount of 40 percent from the previously listed cost. NCTM individual members in good standing as of October 1995 will receive a complimentary copy of the 102-page book.

Package and quantity discounts are available.

Troubled by Math Anxiety?

Information on math anxiety is available from NCTM: a math anxiety information sheet; a two-page bibliography of recent articles, books and studies on the subject; and a one-page annotated bibliography.

Pointers for Parents

Help Your Child Learn Mathematics—Even When You Don't Know the Answers

Your child may feel more comfortable with mathematics—and be better able to do well in school and in tomorrow's business world—if you show enthusiasm for his or her work. Here are five ways to help your child deal successfully with mathematics:

- **Connect mathematics to the real world.** Young people often say "When am I ever going to use this?" If parents talk about ways that mathematics relates to what's going on around them, it is likely that young people will become much more interested. Talk with your child about mathematics that comes up on the evening news or even in sporting events.
- **Encourage independence.** Let children figure out the answers on their own, rather than telling them how to do it. Don't be as concerned with your child getting the right answers as with learning how to arrive at them.
- **Show that you believe they can succeed.** Be ready to listen carefully to your child's explanation of a problem to understand how you can help. When checking over homework, make positive comments.
- **Set the stage.** Provide a special place for study geared to your child's learning style, whether it is at a desk or sprawled out on a bed. To stay informed of your child's progress, discuss homework assignments together. Let your child show you how to do several problems.
- **Don't be afraid to ask.** If you don't understand a mathematics problem, search elsewhere for help. Locate a friend, neighbor or relative in a mathematics or science field who might be familiar with the subject.

For a free copy of the NCTM brochure "Help Your Child Learn Math," send a self-addressed, stamped, business-size envelope to NCTM, 1906 Association Drive, Dept. M-NAPS, Reston, VA 22091-1593.

Jack Price

Looks are so deceptive that people should be done up like food packages with the ingredients clearly labeled.

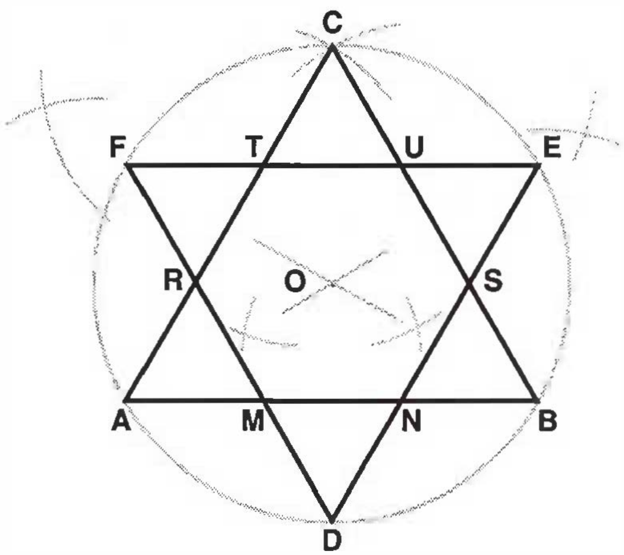
—Helen Hudson

Linda's Trisection

Linda Chiem

Editor's note: Linda is a student at St. Mary's High School in Calgary. On her own initiative, she came up with the following procedures for trisecting a segment and trisecting an angle. Student initiatives should be encouraged. I am prepared to seriously consider student work for inclusion in *delta-K*.

Trisecting a Segment



Trisect $\overline{AB}$

1. Construct equilateral triangle ABC.
2. Find centre O of ΔABC (bisect $\overline{AC}$ and $\overline{BC}$).
3. With radius $\overline{OA}$, draw circle.
4. With radius OA and centre at A, intersect circle at D and F. With centre at B, intersect circle at E.
5. Draw $\overline{FD}$ and $\overline{ED}$. Label intersection of $\overline{AB}$, as M and N.
6. $\overline{AM} = \overline{MN} = \overline{NB}$.
   $\therefore \overline{AB}$ has been trisected.

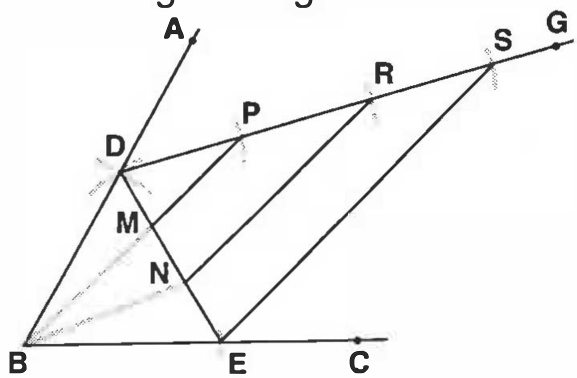
Note: This construction is similar to the construction of a regular hexagon.

Proof

Prove that $\overline{AM} = \overline{MN} = \overline{NB}$ given the above construction.

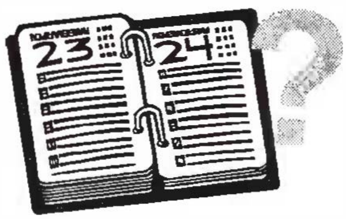
$\angle CAB = \angle CBA = 60°$	ΔABC equilateralΔ
ADBECF regular hexagon	constructed
$\angle DBE = 120°$	regular hexagon
$\angle EDB = \angle BED = 30°$	isosceles ΔDBE
ΔADB is isosceles	constructed
$\angle ADB = 120°$	regular hexagon
$\angle DBN = \angle DAB = 30°$	isosceles ΔADB
$\angle DNB = 120°$	sum $\angle$'s in Δ
$\angle SNB = 60°$	supplementary $\angle$'s
$\angle BSN = 60°$	sum $\angle$'s in Δ
$\therefore \Delta SNB$ is equilateral	$\angle$'s all 60°
similarly ΔRAM is equilateral	
and $\Delta SNB \cong \Delta RAM$	since $\Delta DBE \cong \Delta ADF$
$\angle DNB = \angle MNS = 120°$	vertical $\angle$'s
similarly	
$\angle RMN = \angle TRM = \angle UTR =$	
$\angle SUT = \angle NSU = 120°$	
$\therefore$polygon TUSNMR	
regular hexagon	
$\overline{RM} = \overline{MN} = \overline{NS}$	regular hexagon
$\overline{RM} = \overline{AM}$ and $\overline{NS} = \overline{NB}$	equilateral Δs
$\therefore \overline{AM} = \overline{MN} = \overline{NB}$	substitution

Trisecting an Angle



Trisect $\angle ABC$

1. Construct isosceles ΔBDE.
2. Draw $\overline{DG}$.
3. On $\overline{DG}$, mark off three congruent segments.
4. Construct SE.
5. Copy $\angle DSE$ at R and P; extend lines to intersect $\overline{DE}$ at M and N.
6. Construct $\overline{BM}$ and $\overline{BN}$.
   $\angle ABC$ is trisected.



Calendar Math

Arthur Jorgensen

This activity is for Grades 3–6 students to do in January 1996.

1. Continue this pattern: 0, 3, 6, 9, 12
2. How many days until Christmas Day?
3. A jar contains 550 jelly beans, consisting of 5 colors. If there are an equal number of each color, how many beans of each color are there?
4. Estimate how many students are in your school.
5. If a car can carry 5 people, how many cars will be required to haul 27 people to a hockey game in Mudville?
6. How many ways can you make change for a quarter?
7. Using 8 coins, make 46¢. Name the coins.
8. A farmer has 9 animals. They are either pigs or chickens. Together they have 30 legs. How many pigs and chickens does the farmer have?
9. How many sides has a pentagon?
10. When does 10 + 4 = +2 ?
11. Graph favorite colors.
12. How many Wednesdays are there in January?
13. Numbers such as 2, 8, 14, 96, 100 are called ________ numbers. Why?
14. How are squares and triangles different?
15. Continue this series: 21, 20, 18, 15, 11
16. Graph the eye colors of students in your class.
17. How could you determine the thickness of this sheet of paper?
18. Estimate how many 2s there are on page 27 of your textbook.
19. What do the following numerals have in common? 94, 76, 922, 553, 409, 1183
20. I took a handful of gumdrops from a bag. In my hand, I had 1 yellow, 2 green and 3 red gumdrops. If there are 60 gumdrops in the bag and the ratio is constant, how many of each color are there?
21. A Δ is worth 5¢, a ❑ is worth 10¢, and a ○ is worth 25¢. Using these shapes, draw a picture worth $1.40.
22. The straight line passing through the centre of a circle from side to side is called the ________?
23. Tom is 3 years older than Jane, but 2 years younger than Nomsa. If Nomsa is 12 years old, how old are Tom and Jane?
24. Make up a problem that has 3 for an answer.
25. If January 3 falls on a Wednesday, what day does January 23 fall on?
26. Place the numbers 1 to 6 on the sides of a triangle so that all sides have the same sum.

    ○

    ○ ○

    ○ ○ ○

27. Hotdogs cost $1.07, and colas cost 85¢. In my pocket, I have $3.90. Do I have enough money for 2 of each?
28. Willy has 3 bikes, and his sister has 3 trikes. How many wheels are there altogether?
29. The temperature at 8 a.m. was -7°C. By noon, it had risen 10°C. What was the temperature at noon?
30. Find the sum of the odd numbers between 1 and 10.
31. The following numerals are all "Bozos." What do they have in common?
    63, 270, 441, 1233, 900, 621
    Write two more "Bozos."

A good way to effectively develop numerous problems or activities for calendar math is to assign a particular date to each student, and ask him or her to bring in a challenge problem or activity for that date. In this way, students are likely to bring in problems they can relate to.

Mathematical Codes

Louise M. Lataille

Deciphering codes is an excellent exercise in mathematical reasoning, sound thinking and solid logic. Almost any code will provide this practice. Here is a simple one that can be adapted and reused as student abilities change or increase.

2	9	4
7	5	3
6	1	8

1. ⊏ + □ =
2. └ × ⊓ =
3. ⊔ ÷ ⊐ =
4. ⊐ - └ =
5. ┘ × □ =
6. ⊏^└ =
7. ┐ + ┌ =
8. (⊐^┘ + ⊓) ÷ □ =
9. ┌ × (┐ - └) =
10. ⊏ + ⊔ × ┘ =
11. ┌ ÷ ┘ + (┌ - ⊏) =
12. □ × ┐ - □ × ⊓ =
13. ⊐ × ┌ ÷ (⊏ + └) =
14. (┐^⊏ - □ × ⊔) ÷ ⊔ =

Enhancing Mathematics Teaching in the Context of the Curriculum and Professional Standards of the National Council of Teachers of Mathematics

Klaus Puhlmann

This article briefly reviews the curriculum and professional standards of the National Council of Teachers of Mathematics (NCTM). These standards have driven and directed curriculum development, teaching, evaluation and professional development in mathematics since their publication in 1989. This article provides readers with an overview of the curriculum and professional standards for mathematics.

Inherent in the standards is a consensus that all students need to learn more, and often different, mathematics and that instruction in mathematics must be significantly revised. The need for standards for school mathematics is clearly evident in that they ensure quality, indicate goals and promote change. The NCTM considers all three reasons equally important. Schools, and in particular school mathematics, must reflect the important consequences of the current reform movement in mathematics if students are to be adequately prepared to live in the 21st century. Today's society expects schools to ensure all students have an opportunity to become mathematically literate, are capable of extending their learning, have an equal opportunity to learn, and become informed citizens capable of understanding issues in a technological society.

Educational goals for students must reflect the importance of mathematical literacy. Toward this end, the K–12 standards articulate five general goals for *all* students:

- That they learn to value mathematics
- That they become confident in their ability to do mathematics
- That they become mathematical problem solvers
- That they learn to communicate mathematically
- That they learn to reason mathematically

These goals imply that students should be exposed to numerous and varied related experiences that encourage them to value the mathematical enterprise, to develop mathematical habits of mind and to understand and appreciate the role of mathematics in human affairs; that they should be encouraged to explore, to guess and even to make and correct errors so that they gain confidence in their ability to solve complex problems; that they should read, write and discuss mathematics; and that they should conjecture, test and build arguments about a conjecture's validity.

The mathematics classroom must be permeated with these goals and experiences so that they become commonplace in students' lives. Exposing students to the experiences outlined in the standards will ensure that students gain mathematical power. This term denotes an individual's abilities to explore, conjecture and reason logically, as well as his or her ability to use various mathematical methods effectively to solve nonroutine problems. This notion is based on the recognition of mathematics as more than a collection of concepts and skills to be mastered; it includes methods of investigating and reasoning, means of communication and notions of context. In addition, for each individual, mathematical power involves development of self-confidence.

The NCTM (1989, 1991) has presented 78 standards divided among eight categories: Grades K–4 curriculum, Grades 5–8 curriculum, Grades 9–12 curriculum, evaluation of students, teaching mathematics, evaluation of the teaching of mathematics, professional development of teachers, and support and development of mathematics teachers and teaching.

Curriculum Standards

Curriculum standards for school mathematics are value judgements based on a broad, coherent vision of schooling derived from several factors: societal goals, student goals, research on teaching and learning, and professional experience. Each standard starts with a statement of what mathematics the curriculum should include, followed by a description of the student activities associated with that mathematics and a discussion that includes instructional examples. Three features of mathematics are embedded in the standards. First, "knowing" mathematics is "doing" mathematics. Doing mathematics is different from mastering concepts and procedures. That is not to say that informational knowledge has no value, only that its value lies in the extent to which it is useful in the course of some purposeful activity. Students clearly must know the fundamental concepts and procedures from some branches of mathematics; established concepts and procedures must be relied on as fixed variables in a setting in which other variables may be unknown. However, instruction should persistently emphasize "doing" rather than "knowing that."

Second, some aspects of doing mathematics have changed in the last decade. The computer's ability to process large sets of information has made quantification and the logical analysis of information possible in such areas as business, economics, linguistics, biology, medicine and sociology. Because mathematics is a foundation discipline for other disciplines and grows in direct proportion to its utility, the mathematics community believes that the curriculum for all students must provide opportunities to develop an understanding of mathematical models, structures and simulations applicable to many disciplines.

Third, changes in technology and the broadening of the areas in which mathematics is applied have resulted in growth and changes in the discipline of mathematics. More than half of all mathematics has been invented since World War II. The new technology not only has made calculations and graphing easier but also has changed the very nature of the problems important to mathematics and the methods mathematicians use to investigate them.

Because technology is changing mathematics and its uses, the NCTM believes

- appropriate calculators should be available to all students at all times,
- a computer should be available in every classroom for demonstration purposes,
- every student should have access to a computer for individual and group work and
- students should learn to use the computer as a tool for processing information and performing calculations to investigate and solve problems.

Access to this technology is no guarantee that any student will become mathematically literate. Calculators and computers for users of mathematics are tools that simplify, but do not accomplish, the work at hand. Similarly, the availability of calculators does not eliminate the need for students to learn algorithms. Students should be aware of the choices of methods when calculating an answer to a problem. When an approximate answer is adequate, students should estimate. If a precise answer is required, students must be capable of choosing an appropriate procedure. Many problems should require students to conduct mental calculations or use paper and pencil. For more complex calculations (for example, long division or column addition), students should be able to use calculators. Finally, if many iterative calculations are needed, a computer program should be written or used to find answers (for example, finding a sum of squares).

With respect to mathematical content, the standards represent the minimum that all students will need to be productive citizens. The standards do not specify alternative instructional patterns prior to Grade 9. For Grades 9–12, the standards have been prepared in light of a core program for all students, with explicit differentiation in terms of depth and breadth of treatment and the nature of applications for college-bound students. There is an implied expectation that all students have an opportunity to encounter typical problem situations related to important mathematical topics.

Student activities are the second aspect of each standard. Two general principles have guided the description of these activities: first, activities should grow out of problem situations; and second, learning occurs through active as well as passive involvement with mathematics. Traditional teaching emphases on practice in manipulating expressions and practising algorithms as a precursor to solving problems ignore the fact that knowledge often emerges from the problems. Thus present strategies for teaching may need to be reversed: knowledge often should emerge from experience with problems. Furthermore, students need to experience genuine problems regularly. A genuine problem is a situation in which, for the individual or group concerned, one or more appropriate solutions have yet to be developed. However, instruction should vary and include opportunities for

- appropriate project work,
- group and individual assignments,
- discussion between teacher and students and among students,
- practice on mathematical methods and
- exposition by the teacher.

Another premise of the standards is that problem situations must keep pace with the mathematical and cultural maturity and experience of the students. For example, the primary grades should emphasize the empirical language of the mathematics and whole numbers, common fractions and descriptive geometry. In the middle grades, empirical mathematics should be extended to other numbers and the emphasis should shift to building the abstract language needed for algebra and other aspects of mathematics. High school mathematics should emphasize functions, their representations and uses, modeling and deductive proofs.

The standards specify that instruction should be developed from problem situations. Situations should be sufficiently simple to be manageable but sufficiently complex as to provide for diversity in approach. They should be amenable to individual, small-group or large-group instruction; involve a variety of mathematical domains; and be open and flexible as to the methods to be used.

The first three standards for each grade level are problem solving, communication and reasoning, although these vary between the levels on what is expected of students and of instruction. The fourth curriculum standard at each level is mathematical connections. This label emphasizes the belief that, although it is often necessary to teach specific concepts and procedures, mathematics must be approached as a whole. Concept, procedures and intellectual processes are connected. Thus the curriculum should include deliberate attempts, through specific instructional activities, to connect ideas and procedures among different mathematical topics and with other content areas. Following the connections standard, nine or ten specific content standards are stated and discussed. Some have similar titles, which reflects that a content area needs emphasis across the curriculum; however, others emphasize specific content that needs to be developed at that level.

Student Evaluation Standards

These standards are viewed and presented in three categories. The first set consists of three evaluation standards and discusses general assessment strategies related to the curriculum standards. These standards present principles for judging assessment instruments, including alignment, multiple sources of information, and appropriate assessment methods and uses.

The second set contains seven standards under student assessment and focuses on providing information to teachers for instructional purposes. They closely parallel the curriculum standards of problem solving, communication, reasoning, mathematical concepts and mathematical procedures, in addition to two separate standards on mathematical disposition and mathematical power. These seven standards are to be used by teachers to judge students and their mathematical progress.

The final set of four standards falls under program evaluation and addresses the gathering of evidence with respect to the quality of the mathematical program. The standards are indicators of program evaluation, curriculum and instructional resources, instruction and evaluation team. These standards are to be used by teachers, administrators and policymakers to judge the quality of the mathematics program and the effectiveness of instruction.

Standards for Teaching Mathematics

Central to the curriculum and evaluation standards is the development of mathematical power for all students. Mathematical power includes the ability to explore, conjecture, reason logically, solve nonroutine problems, communicate about and through mathematics, and connect ideas within mathematics and between mathematics and other intellectual activities. Mathematical power also involves the development of personal self-confidence and a disposition to seek, evaluate and use quantitative and spatial information in solving problems and in making decisions. Students' flexibility, perseverance, interest, curiosity and inventiveness also affect the realization of mathematical power.

To reach the goal of developing mathematical power for all students requires the creation of a curriculum and an environment, in which teaching and learning are to occur, that are very different from much of our current practice. The image of mathematics teaching needed includes elementary and secondary teachers who are more proficient in

- selecting mathematical tasks to engage students' interest and intellect (that is, worthwhile mathematical tasks);
- providing opportunities to deepen their understanding of the mathematics being studied and its applications;
- orchestrating classroom discourse in ways that promote the investigation and growth of mathematical ideas;
- using, and helping students use, technology and other tools to pursue mathematical investigations;
- seeking, and helping students seek, connections to previous and developing knowledge; and
- guiding individual, small-group and whole-class work.

The professional standards for teaching mathematics are a major shift away from the current practice to mathematics teaching for student empowerment. Current practice is characterized by a predictable sequence of activities. First, answers are given for the previous day's assignment. Difficult problems are worked on by the teacher or students at the chalkboard. This is followed by the introduction of a new concept and the assignment of homework. Students work on the homework for the remainder of the class period, with the teacher moving around the room to answer questions.

The professional standards for teaching mathematics call for a change in the environment of mathematics classrooms. We need to shift toward

- classrooms as mathematical communities and away from classrooms as simply a collection of individuals;
- logic and mathematical evidence as verification and away from the teacher as the sole authority for right answers;
- mathematical reasoning and away from merely memorizing procedures;
- conjecturing, inventing and problem solving and away from an emphasis on mechanistic answer finding; and
- connecting mathematics, its ideas and its applications and away from teaching mathematics as a body of isolated concepts and procedures.

The phrase "all students" is used throughout the standards. This means that schools and communities must accept the goal of mathematical education for every child from Kindergarten to Grade 12. This does not mean that every child will have the same interests or capabilities in mathematics. It does mean that we will have to examine our fundamental expectations about what children can learn and can do and that we will have to strive to create learning environments in which raised expectations for children can be met.

Standards for the Evaluations of the Teaching of Mathematics

This section presents eight standards for evaluating the teaching of mathematics organized under two categories. The first category describes the process of evaluation and includes standards dealing with the evaluation cycle, teachers as participants in evaluation and sources of information.

The evaluation process should reflect that the overall intent is to improve instruction, that it should be a dynamic and continual process, that teachers should be an integral part of that process, and that because of the complexity of teaching, it should involve a variety of sources of information gathered in various ways. The standards emphasize that teachers should be encouraged and supported to engage in self-analysis and to work with colleagues in improving their teaching. When evaluation involves supervisors or administrators, their relationship with teachers should be collegial with the intent to improve instruction.

The second category of standards in this section describes the foci of evaluation and includes five standards dealing with mathematical concepts, procedures and connections; mathematics as problem solving, reasoning and communication, mathematical disposition; assessing students' mathematical understanding; and learning environment.

These standards, particularly the standard dealing with mathematical disposition, emphasize the importance of significant mathematics when reevaluating mathematics teaching. Through encounters with significant mathematics, students develop mathematical power. But attaining mathematical power requires more. It requires a disposition to do mathematics and an environment in which the processes of doing mathematics are continually emphasized.

This can occur only when teachers present stimulating tasks and create an environment in which problem solving, reasoning and communication are valued and promoted. Further, the message teachers send students should not be limited to instruction alone; it must also include what and how mathematical learning is assessed. Through assessment, we communicate to our students what mathematical outcomes are valued.

A consistent message throughout the standards for the evaluation of teaching is the importance of teachers reflecting on their teaching and working with colleagues and supervisors to improve their teaching. While the standards provide a focus for improvement, such improvements will occur only when teachers consciously decide to engage in ongoing professional development. This, in turn, requires support and encouragement at all levels.

Standards for the Professional Development of Teachers of Mathematics

Five standards are presented in this section: experiencing good mathematics teaching, knowing students as learners of mathematics, knowing mathematical pedagogy, developing as a teacher of mathematics and the teacher's role in professional development.

Mathematics teachers must have good role models during their preservice and continuing inservice training. Teachers often teach the way they have been taught. Therefore, preservice instructors need to address the major components of teaching: tasks, discourse, environment and analysis of teaching.

The education of mathematics teachers should develop their knowledge of the content and discourse of mathematics. Teachers' comfort with, and confidence in, their own knowledge of mathematics affects what they teach and how they teach it. Their conceptions of mathematics shape their choice of worthwhile mathematical tasks, the learning environments they create and the discourse in their classrooms. Knowing mathematics includes understanding specific concepts and procedures as well as the process of doing mathematics. Mathematics involves the study of concepts and properties of numbers, geometric objects, functions and their uses—identifying, counting, measuring, comparing, locating, describing, constructing, transforming and modeling. The relationships and recurring patterns among these objects and the operations on these objects lead to building such mathematical structures as number systems, groups or vector spaces and to studying the similarities and differences among these structures.

Such knowledge ought not to be developed in isolation. The ability to identify, define and discuss concepts and procedures; to develop an understanding of the connections among them; and to appreciate the relationship of mathematics to other fields is critically important.

The standards clearly state that, to sufficiently understand the mathematical topics specified for each level, teachers teaching mathematics should have not less than nine semester hours of coursework in content mathematics at the K–4 level, fifteen semester hours of coursework at the 5–8 level, and the equivalent of a major in mathematics at the 9–12 level.

The preservice and continuing inservice training of teachers of mathematics should provide multiple perspectives on students as learners of mathematics by developing teachers' knowledge of research on how students learn mathematics; the effects of students' age, abilities, interests and experience on learning mathematics; the influence of students' linguistic, ethnic, racial and socioeconomic backgrounds and gender on learning mathematics; and ways to affirm and support full participation and continual study of mathematics by all students.

Knowing mathematical pedagogy is an important standard that can be achieved through preservice and continuing inservice training of teachers of mathematics by developing

- teachers' knowledge of and ability to use and evaluate instructional materials and resources, including technology;
- ways to represent mathematics concepts and procedures;
- instructional strategies and classroom organizational models;
- ways to promote discourse and foster a sense of mathematical community; and
- means for assessing student understanding of mathematics.

Developing as a teacher of mathematics is really at the heart of teaching. Teachers of mathematics must have ongoing opportunities to examine and revise their assumptions about the nature of mathematics, how it should be taught, and how students learn mathematics; observe and analyze a range of approaches to mathematics teaching and learning, focusing on the tasks, discourse, environment and assessment; work with a diverse range of students individually, in small groups and in large-class settings with guidance from and in collaboration with mathematics education professionals; analyze and evaluate the appropriateness and effectiveness of their teaching; and develop dispositions toward teaching mathematics.

Essentially, being a teacher of mathematics means developing a sense of self as such a teacher. Such an identity grows over time and is built from many different experiences with teaching and learning. Further, it is reinforced by feedback from students that indicates they are learning mathematics, from colleagues who demonstrate professional respect and acceptance, and from a variety of external sources that demonstrate recognition of teaching as a valued profession.

Teachers develop as professionals on an ongoing basis. The standard regarding the teacher's role in professional development suggests that teachers of mathematics should take an active role in their own professional development by accepting responsibility for experimenting thoughtfully with alternative approaches and strategies in the classroom; reflecting on learning and teaching individually and with colleagues; participating in workshops, courses and other educational opportunities specific to mathematics; participating actively in the professional community of mathematics educators; reading and discussing ideas presented in professional publications; discussing with colleagues issues in mathematics and mathematics teaching and learning; participating in proposing, designing and evaluating programs for professional development specific to mathematics; and participating in school, community and political efforts to effect positive change in mathematics education. Schools and school systems must support and encourage teachers in accepting these responsibilities. What is essential is that teachers of mathematics view themselves as agents of change, responsible for improving mathematics education at many levels: the classroom, the school, the district, the region and even the nation.

Standards for Supporting and Developing Mathematics Teachers and Teaching

Professional Standards for Teaching Mathematics (NCTM 1991) presents a vision of teaching that calls for a teacher who is educated, supported and evaluated in ways quite different from current practice. To create a teaching environment as described in the standards, teachers must have access to educational opportunities over their entire professional lives that focus on developing a deep knowledge of subject matter, pedagogy and students.

Teachers can, and do, implement successful mathematics programs with little help or encouragement. However, sustaining them or expecting that they flourish without adequate support is not reasonable. The changes called for by the curriculum and evaluation and the professional teaching standards need the support of policymakers in government, business and industry; school administrators, school board members and parents; college and university faculty and administrators; and leaders of professional organizations.

Policymakers in government, business and industry should take an active role in supporting mathematics education by accepting responsibility for

- participating in partnerships at the national, provincial and local levels to improve the teaching and learning of mathematics;
- supporting decisions made by the mathematics education professional community that set directions for mathematics curriculum, instruction, evaluation and school practices;
- providing resources and funding for, and assistance in, developing and implementing high quality school mathematics programs that reach all students, as envisioned in *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989) and *Professional Standards for Teaching Mathematics* (NCTM 1991).

School administrators and school board members should take an active role in supporting teachers of mathematics by accepting responsibility for

- understanding the goals for the mathematics education of all students set forth in the NCTM standards and the needs of teachers of mathematics in realizing these goals in their classrooms;
- recruiting qualified teachers of mathematics, with particular focus on the need for the teaching staff to be diverse;
- providing a support system for beginning and experienced teachers of mathematics to ensure that they grow professionally and are encouraged to remain in teaching;
- making teaching assignments based on the qualifications of teachers;
- involving teachers centrally in designing and evaluating programs for professional development specific to mathematics;
- supporting teachers in self-evaluation and in analyzing, evaluating and improving their teaching with colleagues and supervisors;
- providing adequate resources, equipment, time and funding to support the teaching and learning of mathematics as envisioned in the standards;
- establishing outreach activities with parents, guardians, leaders in business and industry, and others in the community to build support for quality mathematics programs; and
- promoting excellence in teaching mathematics by establishing an adequate reward system, including salary, promotion and conditions of work.

College and university administrators need to actively support mathematics and mathematics education faculty by accepting responsibility for establishing an adequate reward system, including salary, promotion and tenure, and conditions of work, so that faculty can and are encouraged to

- spend time in schools working with teachers and students;
- collaborate with schools and teachers in the design of preservice and continuing education programs;
- offer appropriate graduate courses and programs for experienced mathematics teachers;
- provide leadership in conducting and interpreting mathematics education research, particularly school-based research;
- cooperate with pre-college educators to articulate the K–16 mathematics program; and
- make concerted efforts to recruit and retain teacher candidates of quality and diversity.

The leaders of professional organizations need to take an active role in supporting teachers of mathematics by accepting responsibility for

- promoting and providing professional growth opportunities for those involved in mathematics education,
- focusing attention of the membership and the broader community on contemporary issues dealing with the teaching and learning of mathematics,
- promoting activities that recognize the achievements and contributions of exemplary mathematics teachers and programs, and
- initiating political efforts that effect positive change in mathematics education.

Summary

In *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989) and *Professional Standards for Teaching Mathematics* (NCTM 1991), the argument is made that what is needed is a design change strategy. This means that new ways of doing things within the system—new roles for teachers and students, new goals, new structures—must be explored to find solutions to persistent problems that result in students failing to become mathematically powerful.

While the standards have directed us to some degree to the issues that need to be addressed, they are not a prescription for what must done at each grade level. However, if we make a long-term commitment to the standards set forth, if we endeavor to persevere and if we continue to modify our course as new knowledge comes to the fore, we will make progress toward the goal of developing mathematical power for all students. Such massive change as is proposed in the standards will take time and much work and dedication from teachers and many others.

First, we must challenge all those charged with responsibility to teach mathematics in our schools to work collaboratively in using the curriculum and evaluation and professional standards as the basis for change so that the teaching and learning of mathematics in our schools is improved.

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When patterns are broken, new worlds can emerge.

—*Tuli Kupferberg*

When you reread a classic, you do not see more in the book than you did before, you see more in you than there was before.

—*Clifton Faidman*

Math Hunt: Providing Children with Real-World Problem Solving Experiences

Connie K. Varnhagen, Fran Yeske, Zenia Nemish and Marcia Groves

The theme of providing opportunities for real-world math problem solving runs through every issue of *delta-K*. Allowing children to experience real-world problem solving is an essential pedagogical exercise because many children simply do not make the link from the set of rules and procedures learned in the classroom to their application in everyday experience (NCTM 1989). To provide experiences for real-world problem solving, we need to find problems that are *real* to students (Swenson 1994) and that challenge them to apply procedural knowledge. The Math Hunt, a scavenger hunt for answers to real-world math problems in the school and surrounding community, provides a real-world experience that *is* real to students.

Finding Problems in the Real World

We designed a series of problems that made use of information available at a nearby strip mall, as well as in and around the school. The goal was for the children to solve various everyday math problems with the information found at the various locations or stations. We also wanted to introduce students to common problems that allow for a range of problem solving strategies to help students develop multiple, flexible strategies. Our target group was Grades 2–3 students. The children were given a clipboard with problems to solve at different locations or stations (for example, the nearby convenience store, drug store, bank, gas station or school playground). Each group was made up of two Grade 2 and two Grade 3 students. The group was given a bag of resources to help in their problem solving, including a calculator, clock board and a tax table.

We identified a number of target concepts from the Grades 2–3 math curriculum, including identifying information necessary for problem solution, estimation, comparing results, graphing and computation. Each problem required information to be obtained from the specific station (hair salon, convenience store, school parking lot) to solve the problem.

We made every effort to develop problems relevant to the children and for which they had appropriate background knowledge. At the same time, we wanted the problems to provide a unique challenge to the children and allow them to discuss mathematics while problem solving (Hauk 1995). To this end, we developed problems for which a variety of problem solving strategies were necessary. For example, one question asked at the neighborhood restaurant (Appendix 1), "Is it cheaper to buy a cheese burger and onion rings (O'rings) or a cheese donair and french fries?", required identifying appropriate information from the menu, computation and comparison. We broke down more complex problems into solution components and asked separate questions for each stage. For example, a problem regarding the height of a slide on the playground was broken down into questions regarding estimation of the slide's height, possible problem solving strategies for determining the height, actual height calculation and comparison of the estimated and calculated height.

In addition, we developed problems whose nature and solutions ranged from relatively concrete (How much would it cost you if each of you bought a 1-L bottle of pop?) to more abstract (How could you figure out how many children can stand under the overhang to the back entrance of the school?). We developed 11 problems that we anticipated would take 5–10 minutes each to complete.

Going on the Math Hunt

Appendices 1–3 show examples of the children's solutions. At a small restaurant, the children selected meals and determined the cost. The example in Appendix 1 indicates that the children decided on an 8-inch pizza. They then identified the price on the menu, determined the GST from the tax table and added up the costs. At the neighboring convenience store, children determined prices, calculated tax and determined change for items posted in the window. The children in the group that completed the sheet in Appendix 2 determined the costs for hot dogs and bottles of pop; they then discussed whether it would be cheaper to buy a meal at the convenience store or next door at the restaurant.

At a local drug store, a poster in the window displayed photo developing prices. The children had to identify appropriate prices, determine price per picture to develop two different rolls of film, compare these prices and determine the most economical type of film to develop. Interestingly, the small roll of film worked out to be more economical to print, and the children discussed why this might be so.

On the school grounds, children estimated height and length of a playground slide, then had to determine how to measure the distances with a metre (yard) stick and a ball of yarn as optional supplies. Although most groups unrolled the ball of yarn along the slide and measured the length of the yarn, one group formed a human chain along the slide and then measured each other's heights. The children also graphed colors of cars in the parking lot (Hitch and Armstrong 1994) and discussed color popularity (Appendix 3). Color groupings ranged from basic categories, such as red, brown, blue, and white, to more exotic colors.

An adult (parent volunteer, teacher or principal) accompanied each group as a problem-solving resource. The guidelines for parent volunteers assisting the children are shown in Appendix 4. Some children needed extensive adult intervention, including assistance in identifying important information, selecting appropriate procedures, writing out the problem solution from dictation and checking the results. Some children required little adult assistance.

Evaluating the Math Hunt Experience

After the Math Hunt, the children completed individual written evaluations of their experience. Appendix 5 shows the evaluation provided by a Grade 2 child. In response to the first item requesting something new the child had learned, the child described a potent problem solving strategy of estimation. Other children's responses included general revelations, such as "That math is wherever you go," "When shopping, use math" and "Math is used in many things," as well as specific skills, for example, "How to do GST (tax)," "I learned to estimate," "How much a litre (of gas) costs" and "I learned that you can use string to measure things." We were amazed by the number of children (over 50 percent) who commented that they discovered during the Math Hunt how rules and procedures they learned in class had relevance for operating in the world outside of the school.

Children generally liked a station either because it was challenging to them ("You were allowed to do lots of problems," "And I got to add the GST" and "Because it took the longest") or because of the open-ended nature of some problems ("Because I liked when we could choose the food").

Similarly, children were more likely to *dis*like a station because the problem was not sufficiently challenging ("Because it didn't take that long"; "They were too easy"). One child's least favorite station was the playground slides where the children estimated and then measured the height and length of the slide: "Because I had to wind up the wool."

Suggestions from the children for another math hunt mostly consisted of elaborating on the problems they had been given. One child provided an elaborate problem: "Go into the store and look for good food and put all the good candies together and add their prices up."

Although our evaluation of the Math Hunt was almost as positive as the children's, we did identify some concerns. For example, some parent volunteers needed more preparation for guiding the children in problem solving than the handout shown in Appendix 4; one parent admitted she did not know how to figure GST herself so she was not sure how to help the children figure out the tax. We also needed more time to complete 11 stations; some groups did not complete all the problems in the time allotted (1.5 hrs). Given the diverse nature of the problems we developed, next year we may have separate Math Hunt events for measurement, time and money concepts.

Summary: How to Organize a Math Hunt

Specific learning objectives must be identified. These objectives should include allowing children to solve real-world, personally relevant and intellectually challenging problems. We designed our first Math Hunt for Grades 2–3 students; problems can be developed for any curriculum level.

The children were carefully grouped so that more-experienced children could assist less-experienced children. We also made the groups small to allow all children to participate (Farivar and Webb 1994). Younger and/or less skilled children could identify the information necessary for solving the problem, for example, finding the window poster that shows hot dogs; older and/or more skilled children could complete the calculations, for example, computing the total cost of the hot dog purchase, including GST.

The school and surrounding neighborhood abound with real-world, child-relevant math problems. We need to identify these problems and help the children experience them to provide a bridge from school-based mathematics lessons to the world outside the classroom.

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Appendix 1

Duke Donair

(1) What would you like for dinner?

8inch pizza

(2) How much will your dinner cost you?

$5.00
.359
$5.35

(3) Is it cheaper to buy a cheese burger and onion rings (O'rings) or a cheese donair and fries?

$2.75 + 1.50 = $4.25

$3.90 + 1.50 = $5.40

It is cheeper to buy onion rings and a chee burger.

Appendix 2

Mac's

(1) (a) How much would it cost if each of you bought a 1 liter squeeze bottle of pop?

4 / 1.99 = 7.96

(b) Now add on the GST. What is the total cost?

56 + 7.96 = $8.52

(2) How much would it cost if each of you bought a hot dog? Don't forget to add on the GST!

3.96 + 28 = $4.24

(3) If you gave the clerk $5.00 for one 1 liter squeeze bottle and one hot dog, how much change would you get back?

2.98 + 21 $3.19

$5.00
− 3.19
$1.81

Appendix 3

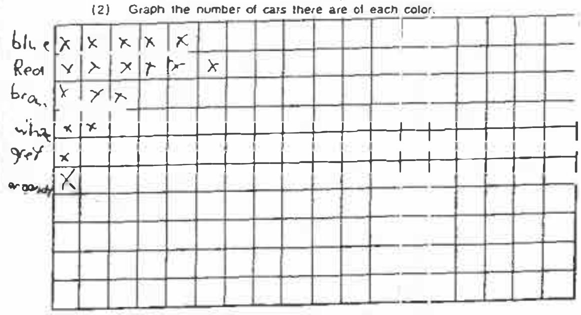
Parking Lot

(1) How many cars are parked in the parking lot? What colors are they?

18

Red white blue beige brown burgandy

(2) Graph the number of cars there are of each color.



(3) According to your graph, what is the most popular color of car?

Red

Appendix 4

Suggestions for Parent Leaders

- ★ Have the students read the problem carefully.
- ★ Remember that we want the students to solve the problem themselves. But you may need to provide some hints. Ask them questions to help them, but don't give too much help. Some questions may be the following:
  - ★ What is the problem asking?
  - ★ What information do you know?
  - ★ What do you have to find out?
  - ★ What problem solving strategy could you use?
    - ★ Could you draw a picture?
    - ★ Could you guess and check?
    - ★ Could you check for patterns?
    - ★ Could you work out the calculation?
  - ★ Did you look back to see if your answer makes sense?

Appendix 5

What did you think about the Math Hunt?

(1) Tell about something new that you learned.

I learned to try to work out the answer [illegible]

(2) What was you favorite station? Why?

my favorite station was the [illegible] slide because it's fun to work it out.

(3) What was you least favorite station? Why?

the big [illegible] because it was to hard to work out.

(4) Write down suggestions for another math hunt.

I think there should be small peces of paper with math questions on them and work them out and find them

Selected Anecdotes About Anecdotal Comments

Werner W. Liedtke

Change in content, assessment and teaching requires changes in reporting assessment results. Romberg (1995) states that authentic or trustworthy assessment requires a reporting system sophisticated enough to embrace a complex view of the learner. "No longer will a single numerical score suffice to describe the complex processes involved in engaging the kinds of mathematical activity described in the *Standards*" (p. 16).

As part of this article, a few comments will be made about each of these questions: What are the main kinds of mathematical activity described in the *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989)? What is the role of anecdotal comments as part of authentic assessment of the kinds of mathematical activity? What are some nonexamples and examples of meaningful anecdotal comments for parents? This brief discussion does not present a final word on anecdotal comments but provides information for reflection and to initiate further discussion.

The major purpose of mathematics achievement assessment is related to reporting to parents the present status of students as well as growth that has taken place as a result of instruction. Examination of the major goals of the NCTM standards reveals that it is impossible to embrace and do justice to these goals by attempting to translate related achievement into a single grade for a student that has any meaning.

To begin with, both *procedural knowledge* and *conceptual knowledge* are necessary aspects of mathematical understanding. Parents should be informed about each. An excellent pamphlet produced by the British Columbia Association of Mathematics Teachers (1995) begins by stating that the development of *mathematical power* is the central goal of any mathematics curriculum. Since that is the case, shouldn't parents be informed about their child's sense of mathematical power? Whenever possible, shouldn't authentic assessment include information about each component of mathematical power: the abilities to think mathematically, communicate mathematically, connect and solve nonroutine problems? If the answer is yes, can this be done with a single letter-grade?

Two general goals for *all* learners are that they *learn to value mathematics* and that they become *confident in their ability to do mathematics* (NCTM 1989, 5). Parents should receive information about these important goals.

Development of number sense is an important goal of the new common curriculum of the Western Canada Protocol. As teachers collect data about indicators of the presence of number sense, shouldn't these be shared with parents?

Learning mathematics includes developing a disposition toward the subject. *Mathematical disposition* (NCTM 1989, 235) includes confidence, flexibility, persevering, curiosity, reflecting on one's own thinking, valuing applications and appreciating the role of mathematics. Parents should receive information about this important aspect of mathematics too.

These aspects of mathematics learning that have been isolated and described are not mutually exclusive. However, it is obvious that achievement with respect to these major goals of mathematics learning cannot be summarized by a single grade. Anecdotal comments are also required. (For a time, teachers and schools in British Columbia had the option to use anecdotal comments as the sole method of reporting on mathematics achievement. Directions from the ministry now stipulate use of grades and anecdotal comments.)

Innovations or changes in education require professional development programs. Without such programs it is and will be difficult for some teachers to meet new standards for curriculum, evaluation and teaching. Without appropriate assistance of some sort, it may not be easy to learn how to write appropriate anecdotal achievement comments that are part of authentic assessment. The possible difficulty is illustrated in the following examples.

It is assumed that for anecdotal comments about achievement to be an important part of authentic assessment, they should refer to specific ideas, procedures and skills learned as a result of instruction in the setting students are in. These comments should not be based on what students learned in previous settings from previous teachers.

In British Columbia, anecdotal report cards became the flash point for some parents with the education system because of the tendency for the cards to be too general. A May 4, 1995, editorial in the *Victoria Times-Colonist* pointed out that "vague edu-speak like 'Johnny is an active participant in class' could mean he likes to answer questions, or he disrupts every discussion."

Some parents were happy about replacing the use of "one-dimensional" grades to describe students' achievement with meaningful anecdotal comments. In a November 9, 1994, article, a columnist for the *Saanich News* was elated over having received comments like, such as, "has mastered the nine-times table" and "has learned to divide three-digit numbers," rather than a "bare" grade.

My November 30, 1994, letter to the editor which appeared in *Saanich News* included the following questions and comments about some ideas in the earlier column:

> Are these statements as meaningful as they might sound?
>
> The intent of the questions and comments that follow is to cause reflection and reaction. At the same time these ideas may hint at the new goals of mathematics teaching and learning.
>
> What does mastery mean to different people? How is it defined? Since the definition of this term can be very subjective, a statement that makes use of it may not provide any more information than a "bare" grade. Teachers as well as parents know that what is taught can be forgotten. Wouldn't it be nice for parents to know that children are able to "recall most/all of the nine-times table and have thinking at their disposal for retrieving or re-constructing forgotten facts."
>
> What does the statement about having learned to divide three-digit numbers mean? (Which number has three digits?) How is this different [from] or the same [as] previously learned division tasks? Are students able to recite steps or rules to get an answer? Do they know whether or not a calculated answer is reasonable? Can they detect an error and correct it on their own? Do they know who would want to solve these types of tasks? When? Why? Can they explain or write out a solution procedure in their own words? Can they provide reasons for all of the "mental moves" that are recorded on paper in order to arrive at an answer?
>
> I think that in order for anecdotal comments to be meaningful, they need to go beyond the "specific examples" cited by Norbury.

The May 4, 1995, *Victoria Times-Colonist* editorial included the following observation:

> But when the reports are done properly, everyone has a very clear idea of how the student is doing. Try, Susan is a quick study at mathematics, easily adding and subtracting numbers in the hundreds. She is on target for her grade level. But she is often in a hurry, makes simple mistakes, and needs to spend more time doublechecking her work. Compare what that tells you to a grade of B-plus.

My May 9, 1995, letter to the editor in response to this *Victoria Times-Colonist* editorial included the following questions and comments:

> My response to the challenge to "Try" left me puzzled about many statements and raised questions not only about Susan but also about the purpose of the comments that were identified to provide "clear ideas."
>
> How would "quick study at mathematics" be defined? Would everyone agree on the definition? What are some characteristics of being quick? Is reflective thinking and problem solving part of this being quick? What abilities are employed as numbers in the hundreds are "easily" added and subtracted? How is "easily" defined? What is a target or "the" target? How is "target for a grade level" defined? What is the purpose of the statements about "being in a hurry" (Is that part of being quick at mathematics?), "making simple mistakes" (What kind of mistakes?) and "needing to spend more time doublechecking work?" Is something planned to correct these difficulties? Are statements of this type really clearer than what a B-plus grade tells us?
>
> "Easily adding and subtracting numbers in the hundreds" could mean "being able to arrive at the answers without having to use pencil and paper," "having several solution strategies at one's disposal" or "being able to employ several estimation and mental computation strategies" in some classrooms and "getting many answers correct on a timed test" in others.

The questions, comments and examples included in these letters provide hints about the characteristics that I think should be part of meaningful anecdotal comments if they are to be part of authentic assessments of the various aspects of mathematics learning.

Some sample materials produced for B.C. teachers fail to show what I think anecdotal comments should communicate to parents. For example, the *Evaluation Techniques and Resources—Book II* (BCPTA 1992, 10.15) for primary teachers includes the following sample entry under "Intellectual Development":

> In math, ________ is continuing to practice number facts to 18 to increase her speed and

accuracy. She has been reviewing place value to the hundreds and has demonstrated an understanding of the concept. She will be introduced to thousands next.

Similar examples are listed. These entries are a reflection of the content being studied rather than report on what has been learned by the student whose name is to appear in the blank that is provided. Information of this sort would probably be more suitable for a newsletter rather than for anecdotal achievement reports about mathematical (or intellectual) development.

Collecting anecdotal data about students for various aspects of mathematics learning should not be more or extra work. It should be part of ongoing instruction. Learning how to translate the collected data about the important aspects of mathematics learning into meaningful statements may require some effort. I am sure that our students and their parents will not only benefit from this effort but also appreciate it.

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People need responsibilties. They resist assuming responsibility, but they can't get along without it.

—John Steinbeck

Wouldn't it be wonderful if all children behaved the way you think you acted when you were a kid?

—The Globe and Mail

"Geopardy" Géométrique

Yvette d'Entremont

Il est motivant et amusant de regarder les jeux-questionnaires rapides et d'opposer nos connaissance à celles des concurrents. Nous pouvons le faire avec des émissions comme "Jeopardy," par exemple. Ce qui donne un cachet spécial à "Jeopardy" c'est le fait que l'on fournisse aux concurrents des réponses ou des indices dans diverses catégories pour qu'ils posent la question qui correspondrait au contenu de l'indice.

L'activité suivante est calquée sur cette émission populaire; la différence est que les questions ont toutes rapport avec la géométrie. Il serait souhaitable de regarder "Jeopardy" quelques fois enfin de se familiariser avec le jeu. "Jeopardy" exige que l'on ait l'esprit vif car c'est la première personne à activer son bouton qui se voit accorder la première chance de poser une question. Si celle-ci est bonne, la valeur monétaire de l'indice sera ajoutée au total du joueur; si, par contre, la question ne correspond pas à l'indice, cette valeur en sera déduite. Plus "Jeopardy" vous sera connu, plus vous vous amuserez en jouant à Geopardy. L'intégration d'éléments de "Jeopardy" tels les indices à double valeur, la deuxième demie à double valeur et l'indice déterminant (la "Geopardy finale") piquera l'intérêt des élèves. Tout ce qu'il vous faut pour jouer à Geopardy c'est une connaissance de base de "Jeopardy," un rétroprojecteur, des acétates et de petits papiers autocollants pour cacher les indices (du genre "Post-it Notes").

Préparation au jeu

Après avoir regardé "Jeopardy" quelques fois, vous créerez des catégories ainsi que des indices et des réponses pour chacune, et vous déciderez comment intégrer les indices à double valeur, la deuxième demie à double valeur et l'indice déterminant (la "Geopardy finale") dans votre jeu. Les catégories pourraient reproduire des titres de chapitre tels que "Les lignes parallèles," "Les triangles congrus" ou même être plus créatifs comme "Des mots qui commencent par la lettre C." Décidez du nombre d'indices que vous voulez dans chaque catégorie et essayez de classer ceux-ci du plus facile au plus difficile. Inscrivez la valeur sur chaque indice et préparez une feuille-réponse sous forme de questions. La figure 1 est un exemple de feuille-indice dont on peut se servir pour cette activité.

Comment jouer à Geopardy

Les indices employés dans l'activité suivante sont basés sur la série de texte de mathématiques "Actimath," conçue pour les élèves des 7e, 8e et 9e années. Vous n'aurez besoin que des acétates reproduisant les planchettes à indices (les jeux 1, 2 et 3), un rétroprojecteur, des papiers autocollants pour cacher les indices et une personne pour marquer les points. Le but de cet exercice est de réviser et de renforcer des concepts géométriques.

Pour jouer à ce jeu dans une salle de classe de taille moyenne, il faudrait d'abord former entre quatre et six équipes avec les élèves et faire en sorte que chaque équipe ait sa propre rangée de sièges ou de pupitres. Il est préférable mais pas essentiel d'avoir le même nombre d'élèves par équipe. L'enseignant pourrait, pour se faciliter la tâche, demander à un élève de marquer les points. Avant le début de la partie, rappelez toutes les règles aux élèves.

Avant de commencer le jeu, cacher tous les indices sur les planchettes avec des papiers autocollants. Pour démarrer la partie, désignez au hasard un élève de la première rangée et demandez-lui de choisir un indice de la planchette. Si l'élève choisit l'indice "Définitions pour 500" enlevez seulement le papier autocollant qui cache cet item. Une fois l'indice révélé, les élèves assis à la première rangée auront le droit d'essayer de trouver la question correspondante. La chance de proposer une question ira au premier qui lèvera la main. Selon la justesse de la question fournie par l'élève, les points de son équipe seront augmentés ou diminués de la valeur de l'indice. Ne remettez pas l'autocollant sur un indice qui a été deviné. Quant au deuxième indice, ce sera les élèves assis à la deuxième rangée qui auront le droit de le choisir et de formuler la bonne question, et ainsi de suite pour les autres cases. Les indices à double valeur, la deuxième demie à double valeur et l'indice déterminant (la "Geopardy finale") pourront être intégrés dans votre jeu comme ils le sont dans l'émission de télévision.

Conclusion

Les jeux tels Geopardy peuvent être amusants et motivants tout en présentant un défi. En plus, ils apportent un changement à l'horaire quotidien et permettent à chaque élève de participer. Encouragez vos élèves à répondre silencieusement aux indices même quand leur équipe n'a pas le droit de répondre. La plupart des élèves le feront de toute façon, pour voir s'ils peuvent battre les autres. Geopardy se déroule à un rythme suffisamment rapide pour que les élèves ne perdent pas intérêt dans le jeu. N'hésitez pas à photocopier la planchette qui sert d'exemple (figure 1) pour créer vos propres catégories et indices. Vous pourrez également adapter l'activité à l'algèbre ou à n'importe quel autre domaine. Le jeu fournit une occasion par excellence de réviser tout en captant l'attention des élèves.

Des questions possibles

Jeu 1

Droites et angles

(100) Quelle est la mesure d'un angle droit?
(200) Quelle est la mesure d'un angle aigu?
(300) Quels sont les angles opposés?
(400) Quels sont les angles complémentaires?
(500) Quels sont les angles alternes internes?
(600) Quel est l'angle extérieur?

Définitions

(100) Qu'est ce qu'un carré?
(200) Qu'est ce qu'un sommet?
(300) Qu'est ce qu'une bissectrice d'angle?
(400) Qu'est ce qu'un hexagone?
(500) Qu'est ce qu'une perpendiculaire?
(600) Qu'est ce que le nombre 12?

Diagrammes

(100) Qu'est ce qu'un triangle rectangle?
(200) Qu'est ce qu'un triangle isocèle?
(300) Qu'est ce qu'un triangle scalène?
(400) Qu'est ce que des triangles congrus?
(500) Qu'est ce qu'un angle rentrant?
(600) Qu'est ce qu'un trapèze isocèle?

Jeu 2

Droites et angles

(100) Quelle est la mesure d'un angle plat?
(200) Qu'est ce que 45 degrés?
(300) Quelle est la mesure d'un angle obtus?
(400) Quels sont les angles supplémentaires?
(500) Quels sont les angles correspondants?
(600) Quels sont les angles co-internes?

Diagrammes

(100) Qu'est ce qu'un cube?
(200) Qu'est ce qu'un parallélogramme?
(300) Qu'est ce que des droites parallèles?
(400) Qu'est ce qu'un trapèze?
(500) Qu'est ce qu'une pyramide à base carrée?
(600) Que,est ce qu'un losange?

Symboles

(100) Qu'est ce que le symbole de pi?
(200) Qu'est ce qu'un segment de droite?
(300) Qu'est ce qu'une droite?
(400) Qu'est ce que perpendiculaire?
(500) Quelle est le symbole de la congruence?
(600) Qu'est ce que le symbole du système métrique?

Jeu 3

Diagrammes

(100) Qu'est ce qu'une hypoténuse?
(200) Qu'est ce que 180 degrés?
(300) Qu'est ce qu'un rayon?
(400) Qu'est ce qu'une médiane?
(500) Qu'est ce qu'un angle inscrit?
(600) Qu'est ce qu'un apothème?

Définitions

(100) Qu'est ce qu'un degré?
(200) Qu'est ce qu'un heptagone?
(300) Qu'est ce qu'un losange?
(400) Qu'est ce qu'un angle rentrant?
(500) Qu'est ce que le centroide?
(600) Que,est ce que l'orthocentre?

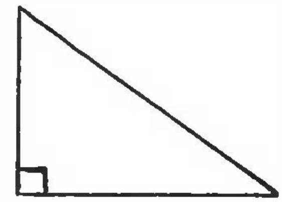
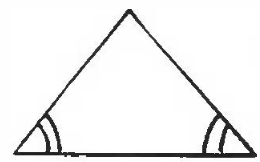
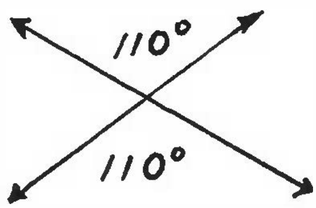
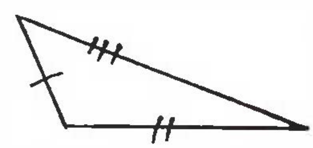
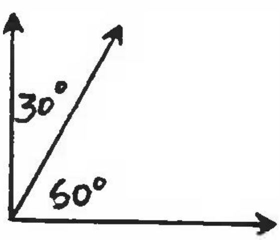
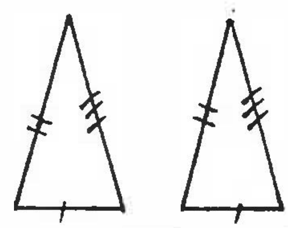
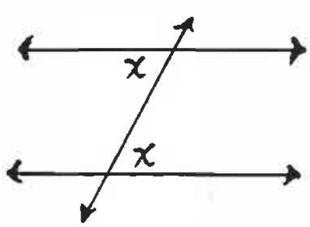
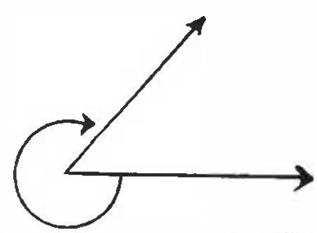
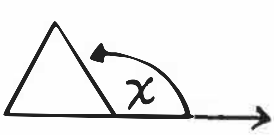
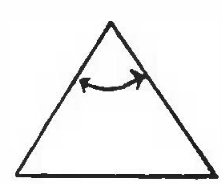
Formules

(100) Quelle est la formule pour l'aire d'un parallélogramme?
(200) Quelle est la formule pour la circonférence d'un cercle?
(300) Quelle est la formule pour l'aire d'un triangle?
(400) Quelle est la formule pour l'aire d'un cercle?
(500) Quelle est la formule pour l'aire d'un trapèze?
(600) Quelle est la formule pour le volume d'un cylindre?

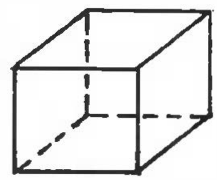
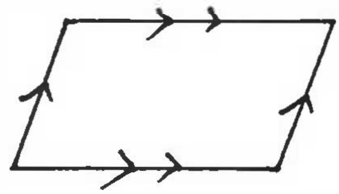
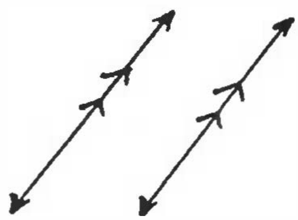
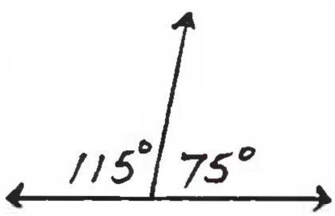
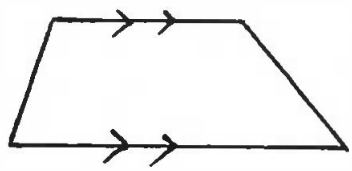
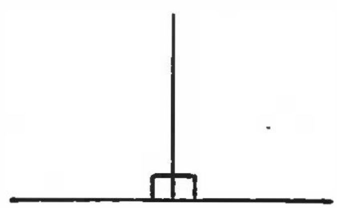
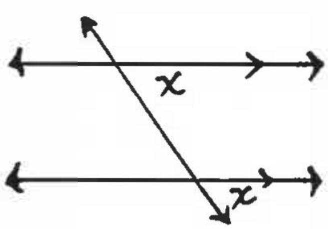
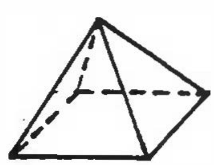
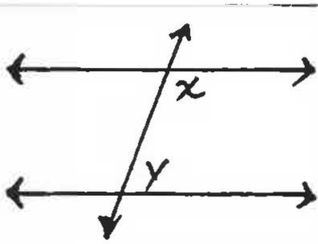
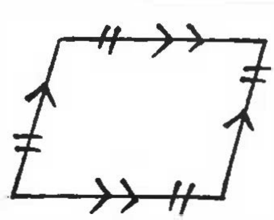
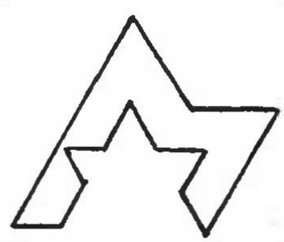
Figure 1

Valeur			
100			
200			
300			
400			
500			
600			

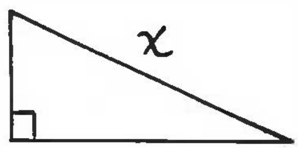
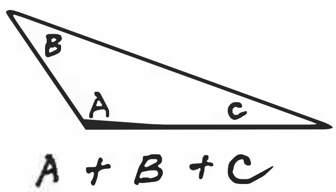
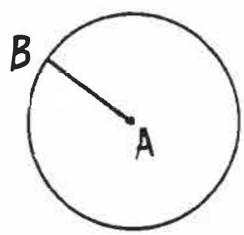
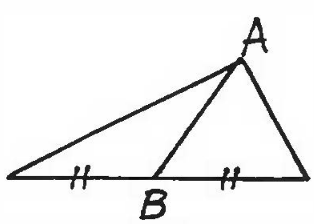
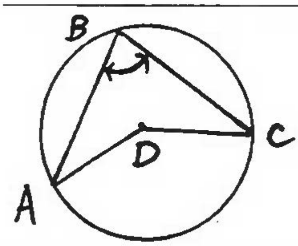
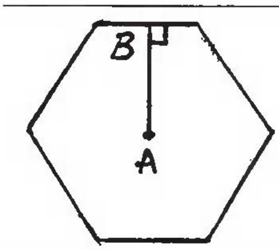
Jeu 1

Valeur	Droites et angles	Définitions	Diagrammes
100	90°	une figure ayant 4 côtés égaux et 4 angles droits	
200	< 90°	le point d'intersection des deux demi-droites d'un angle	
300		coupe un angle en deux parties égales	
400		le polygone d'un rayon de miel	
500		la relation entre nord-sud et est-ouest	
600		le nombre de côtés d'un dodécaèdre	

Jeu 2

Valeur	Droites et angles	Diagrammes	Symboles
100	180°		π
200	le nombre de degrés dans un angle droit bissecté		
300	$>90^\circ<180^\circ$		
400			
500			$\cong$
600			

Jeu 3

Valeur	Diagrammes	Définitions	Formules
100		l'unité de mesure des angles	bh
200		un polygone de 7 côtés	$2\pi r$
300		un parallélogram ayant 4 côtés égales	$\frac{bh}{2}$
400		un angle qui mesure plus que 180° mais moins que 360°	πr^2
500		le point d'intersection des médianes d'un triangle	$\frac{h(a+b)}{2}$
600		le point de rencontre des hauteurs d'un triangle	$\pi r^2 h$

Exploring Products Through Counting Digits, Part 2: Factors with an Unequal Number of Digits

W. George Cathcart

Number sense is a kind of intuitive idea about how numbers behave. Among other things, children with good number sense "recognize the relative magnitudes of numbers" and they "know the relative effect of operating on numbers" (NCTM 1989, 38). These "intuitions" enable students to make realistic estimates of computational results.

In Part 1 (Cathcart 1995), activities were suggested that would help children make generalizations about the minimum and maximum number of digits in a product of two numbers that had an equal number of digits. The number of digits in the product will be either $2n - 1$ or $2n$, where n is the number of digits in each product. Applying this understanding may help children avoid some unreasonable computational results.

But what happens if the factors do not have the same number of digits? That is the focus of this article.

One-Digit (Non-Zero) Multipliers

Have the children make a table similar to Table 1 below or provide them with the headings and nine rows (a 10 by 5 grid). Ask them to fill in the minimum product column by *mentally* doing the computation and the maximum product column by using a *calculator*. Then assign the investigation questions below for group discussion. A whole class discussion afterward would also be useful to synthesize observations and generalizations.

For Investigation

- What can you say about the minimum product? Describe the pattern that you see.
- What can you say about the maximum product? Describe the pattern that you see.
- What can you say about the number of digits in the minimum product?
- What can you say about the number of digits in the maximum product?
- What can you say about the number of digits in the product of any number and a one-digit number?
- In the case of two numbers with an equal number of digits, the number of digits in the product will be $2n$ or $2n - 1$, where n is the number of digits in each product (Cathcart 1995). How can this generalization be modified so that it is true for the case of a two-digit or greater number multiplied by a one-digit number?

Table 1
Products of One-Digit Multipliers

Factors	Minimum Product	Number of Digits in Minimum Product	Maximum Product	Number of Digits in Maximum Product
2-digit × 1-digit	10	2	891	3
3-digit × 1-digit				
⋮				
9-digit × 1-digit	100,000,000	9		

Two-Digit Multipliers

Have the children make another table similar to Table 1 with the title, Products of a Two-Digit Multiplier. In the factors column, have the children list 3-digit × 2-digit, 4-digit × 2-digit, 5-digit × 2-digit and so on to 9-digit × 2-digit.

Ask the children to

- fill in the minimum product column using mental arithmetic and
- fill in the first four rows (three-digit through six-digit) using a calculator.

For Investigation

- What can you say about the minimum product? Describe the pattern that you see.
- Examine the column with the maximum products.
  - ♦ Is there a pattern? Describe it.
  - ♦ Use the pattern to complete the rest of the maximum product column.
- Compare the maximum product column in Table 1 for one-digit multipliers and the one just completed for two-digit multipliers.
  - ♦ How are they the same?
  - ♦ How are they different?
- What can you say about the number of digits in the product of any number and a two-digit number?
  - ♦ Does the new generalization you made for one-digit multipliers in Table 1 hold for two-digit multipliers? If not, modify it so that it works for both cases.

Three-Digit Multipliers

Have students make another table like the previous ones but for three-digit multipliers. This time they will need only six rows: 4-digit × 3-digit, 5-digit × 3-digit, and so on to 9-digit × 3-digit.

Ask the children to

- fill in the minimum product column using mental arithmetic and
- fill in only the first two rows of the maximum product column using a calculator. This is all they will be able to do directly (in one step) on an eight-digit display calculator. (One possible strategy to do computations with larger numbers is outlined in Cathcart 1995).

For Investigation

- What can you say about the minimum product? Describe the pattern that you see.
- Examine the two products that you completed in the maximum product column.
  - ♦ How are these two numbers the same as the first two maximum products for two-digit multipliers?
  - ♦ How are they different?
  - ♦ Use this information to write in the rest of the products in the maximum product column for three-digit multipliers.
    - Discuss the pattern that you used with a friend. Did he or she discover the same pattern?
- Examine the pattern for minimum products in the three tables you have completed. How is the pattern the same? How is it different?
- Examine the pattern for maximum products in the three tables you have completed. How is the pattern the same? How is it different?
- Does the previous rule for how many digits there will be in a product hold for three-digit multipliers?

Larger Multipliers

Use what you have learned to

- predict the least number of digits in the product of a five-digit number and a four-digit number;
- predict the greatest number of digits in the product of a five-digit number and a four-digit number;
- write the smallest product you could get when you multiply a five-digit number by a four-digit number; and
- write the largest product you could get when you multiply a five-digit number by a four-digit number.

Conclusion

Children should be more successful at mathematics if they have good number and operation sense. Being able to see patterns is one component of number and operation sense. Explorations of the relationships and patterns suggested in this article should help children develop number and operation sense and should provide one more way for them to check the reasonableness of a product—does my answer have the number of digits that it should have?

References

Cathcart, W. G. "Exploring Products Through Counting Digits: Equal Factors." *delta-K* 32, no. 3 (August 1995): 7–9.

National Council of Teachers of Mathematics (NCTM). *Curriculum and Evaluation Standards for School Mathematics.* Reston, Va.: NCTM. 1989.

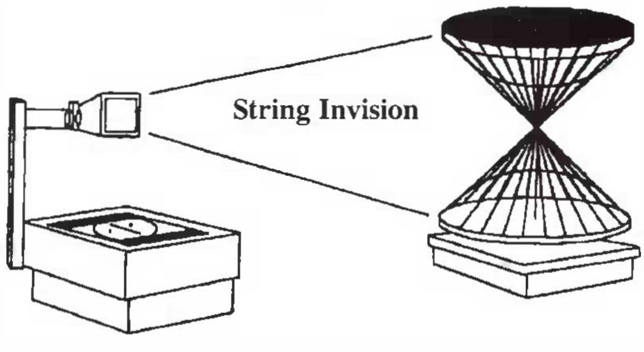
String Invision: Conics Teaching Model

Don Hillacre

The String Invision conics teaching model was designed as a classroom manipulative for Math 30 students, to enhance their ability to understand the physical properties of the conic sections with respect to the intersection of a plane and a cone. Use of the model in the classroom allows students to experiment with various conic scenarios and to demonstrate their understanding of the conic properties. The three-dimensional and interactive qualities of the model provide opportunity for hands-on learning and concept reinforcement that the two-dimensional and static models fail to do.

String Invision (Figure 1) is a tabletop model consisting of two wooden disks held apart on a centre pole (the axis of symmetry), mounted on a square wooden base. The top disk is fixed to the pole and the bottom disk is free to rotate up and down on the centre pole. The rotatable disk is suspended from the top disk by string segments, which form a cylinder with the top and bottom disks as ends. As the bottom disk is rotated, the strings form a double nappe cone with the vertex on the axis pole. An overhead projector is used to shine a light-plane onto the strings of the model. The light-plane is created by using a mask on the overhead projector that blocks out all light except for a fine line. The mask is designed to allow versatility in the placement of the light plane.

Figure 1

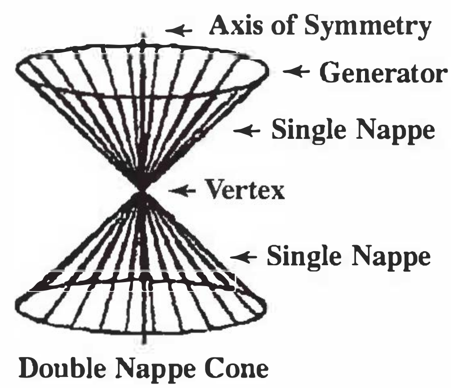


Conics Teaching Model

Because of the model's ability to demonstrate the double nappe cone and the cylinder, students can view all nine conic cross-sections and both arcs of the hyperbola. Gradually changing the angle and position of the light-plane allows discovery of how these different cross-sections are formed and how they can be changed in size and shape. The model also shows the formation of the degenerate curves. Because of the hands-on nature of the model, students are encouraged to formulate their own theories and definitions as to how the curves change with modification to the position and angle of the light-plane. Teacher-led discussion and testing promotes further refinement of these mathematical conjectures orally or in writing.

For a typical classroom lesson, the students gather around the model, taking care not to block the projection of the light-plane. The demonstration is best conducted in a semi-darkened room, using a high intensity projector or, if available, a laser light pen equipped to create a single line of light. (The Math Factor video series to be released by ACCESS Network will include a demonstration of the model, using a laser light source.) The first step of the lesson would be to familiarize the students with the parts of the model (Figure 2): generator line, axis of symmetry, vertex and both nappes of the double nappe cone. When the mask is removed from the overhead projector, the light from the projector casts a two-dimensional shadow of the model on the wall. This can be used to assist students in drawing the three-dimensional model.

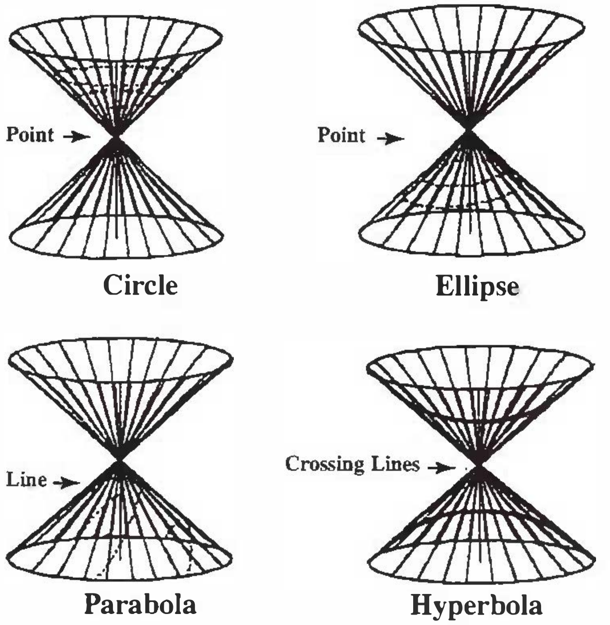
Figure 2



Double Nappe Cone

The second step in the lesson would be to project the light-plane onto the strings at various angles and to observe the four typical (basic) conic cross-section curves: the ellipse, the parabola, the hyperbola and the circle (Figure 3). The circle can be seen as the limiting position of the ellipse. At this point, the students can be encouraged to demonstrate their understanding of how the curves are formed by positioning the plane on a two-dimensional diagram and by participating in oral discussion. Definitions of the four typical conic curves can then be formally written down.

Figure 3

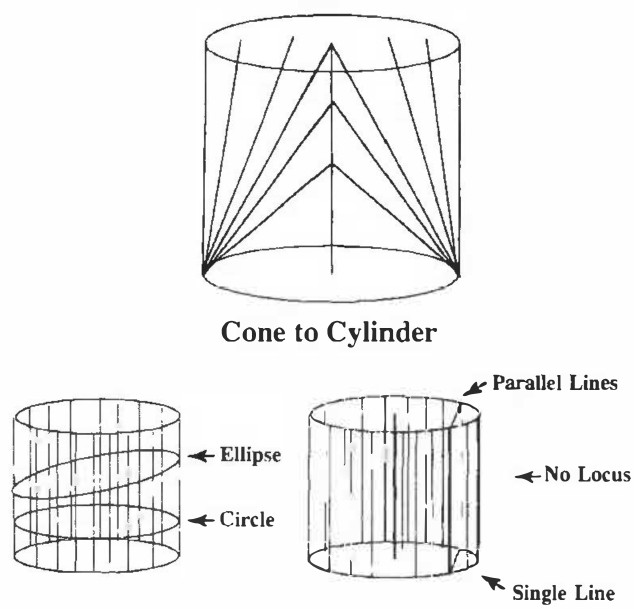


The third step will be to test these definitions using the model and then to refine them to disallow the degenerate cases. For example, an initial definition of a parabola might be stated as "the curve formed by the intersection of a plane parallel to the generator." The definition is correct to a point but needs to be made more restrictive to disallow the degenerate straight line. Another example is the hyperbola and its degenerate, the two intersecting lines. This naturally leads into discussion about degenerates and how they are formed.

The fourth step will be to observe the effects of moving each of the four typical conic curves toward the vertex and thus to their degenerates. This will show the circle and ellipse getting smaller and eventually becoming a degenerate point. The circle and the ellipse degenerate into a point since the circle is really the limiting case of an ellipse. The parabola narrows and becomes a single line. The hyperbola narrows with its vertices approaching each other and becomes two crossing lines, two V-shaped curves meeting at the vertex.

For the fifth step, begin with a recap of the four basic curves and their three degenerates, then move on to discover the remaining two curves: two parallel lines and no locus. Where are the other two curves? They cannot be observed on the double nappe cone, so the cone must be degenerated into a cylinder. This can be demonstrated by grasping the vertex in one hand and releasing the clip that holds the double nappe cone in place with the other hand. Then move the hand holding the vertex along the centre pole, allowing the string to slide through the hand. As the vertex moves up, the sides of the cone can be seen to move toward the vertical and thus at infinity would become a cylinder. The increasing slope of the walls of the lower nappe can be best observed by noting the model's shadow on the wall. At this point, the vertex can be released, and the cylinder will be created on the wall (Figure 4).

Figure 4



Replace the mask on the projector and recreate as many of the seven curves mentioned as are possible on the cylinder. Students will soon discover they are unable to create the parabola, the hyperbola, the two crossing lines, the single line through the vertex and the point. But they can now create the two parallel lines, the single line and no locus by making the light-plane parallel to the centre pole and increasing its distance from the pole. When the plane's distance is less than the radius of the cylinder, two parallel lines can be produced. At a distance equal to the radius of the cylinder, a single line is formed. Distances greater than the radius will not intersect the cylinder and thus no locus is formed. Summarize which curves can be made on the double nappe cone, which can be made on the cylinder and which can be made on both.

To reinforce and conclude the lesson, the students may choose a variety of activities, such as creating a bulletin board display, doing small-group presentations on what they learned in the lesson, answering the questions in the teacher's resource manual and so on. Students often wish to participate in further exploration of concepts such as the following:

- What happens to the original curves when the double nappe cone is converted to the cylinder with each curve on the model?
- What observations can be made in the shadow that is cast on the wall when the double nappe cone is converted to the cylinder?
- What observations can be made in the shapes of the curves if, instead of a single straight line of light, other shapes are projected on the strings such as parallel lines of varying thickness, curved lines such as circles and sine waves, or regular figures such as triangles and squares?
- Make a mobile art show as the model spins and the projector head is oscillated up and down.

The String Invision model and its attendant discovery approach to learning have been well received by teachers and students. One of the model's most noticeable effects on the students is that students can actually see in three dimensions how the conic sections are formed, as opposed to trying to grasp the concepts from seeing two-dimensional diagrams or static single nappe cones. Course-end student surveys always include positive comments and encouragement for continued use of the model in future classes. Students enjoy and benefit from the active participation and interaction that happens around the model.

The surest way to be late is to have plenty of time.

—Leo Kennedy

Yes, I'm growing older. But the important word is not "older." It is "growing."

—Joan Sutton

Games in the Primary Mathematics Classroom

A. Craig Loewen

A fundamental aspect of the new Alberta elementary mathematics program is that mathematics learning should occur within a dynamic and active learning environment. Such an environment is created and maintained through the implementation of a wide variety of fast-paced activities that encourage exploration, manipulation and problem solving. Teachers need to include a selection of activities, such as manipulative explorations, problem solving tasks, games, applications, and calculator and computer explorations. The need for such a spectrum of activities is especially important in the primary grade levels to account for shorter attention spans and to capitalize on natural student enthusiasm.

Using Games in Mathematics Instruction

Games represent one of the most compelling and motivating activities teachers can introduce into the primary mathematics classroom, but games have several strengths beyond their simple motivational qualities. First, games represent a form of problem solving. Problem solving may be defined as a process whereby the solver attempts to reach a goal from a set of given conditions, but the means to achieve the goal is not immediately or intuitively obvious. A game, like a problem, also has a goal (how to win or finish the game) and conditions (the rules under which the game is played), and the solver or player must find the route to the goal. Further, success in instructional games often requires application of various problem solving skills and strategies, such as estimation, constructing a list, reading a table or even looking for a pattern.

Second, games provide an interesting way to involve parents, guardians and siblings in a child's education. Games can be sent home with students with the intention that they be played with family and friends, thus facilitating the home/school connection. In this way, parents can be kept informed as to the topics being covered in school while at the same time spending some enjoyable time with their children.

Third, games provide an alternative to standard drill and practice exercises. Children need to drill and practise any new concept to achieve *automaticity* (the ability to recall and apply learned concepts and skills quickly and efficiently). Unfortunately, standard drill and practice exercises (for example, textbook pages) often quickly become mundane, and students resist completing them. To play a game, the player must repetitively apply required knowledge and skills in each turn; in other words, the player is engaging in a form of drill and practice. Games thus represent a significantly more motivational form of drill and practice.

Finally, games can be integrated easily into the instructional environment. It is important that games be related to specific objectives and goals in the mathematics curriculum, because this is how we ensure that students are receiving a quality, dynamic and active learning experience.

There are many reasons for introducing games in the primary mathematics classroom besides those listed above. Leonard and Tracy (1993) suggest several others, including that games

- allow students to apply what they know to the real world,
- create a positive mathematical environment,
- maximize student problem solving competence,
- increase ability to communicate,
- increase ability to reason mathematically,
- enhance student perception of the value of mathematics and
- develop students' self-confidence.

The benefits of using games are many, but smooth implementation can be tricky. The following hints on introducing games may be helpful.

Games in the Classroom

When introducing a new game in a math lesson, teachers may wish to consider a few variables. Such considerations will lead to less frustrating implementation and will enhance the game's learning potential.

First, teachers may hold a class discussion about winning and losing. Children should play both competitive and cooperative games and learn that sometimes you win and sometimes you lose. Children must learn to focus on the act of participating and the enjoyment of simply being part of the game. Teachers may introduce the game such that the students (as a class) compete against the teacher. This playful competition effectively and efficiently acquaints students with the game, allows for a brief discussion of strategy and enables a short discussion of winning and losing (the teacher inevitably loses and thus communicates that it is quite acceptable to lose).

Second, teachers will want to carefully consider the skills and knowledge required to participate in the game. Given that games are primarily drill and practice activities, it is important to remember that students must already possess the skills and knowledge to be rehearsed. Teachers may also want to consider the vocabulary necessary for playing the game. Such vocabulary may need to be introduced before the game begins and reinforced while it is in progress.

Third, teachers should consider how the game is to be introduced. One method (where the teacher plays against the class) has been discussed, but a second effective method is to invite one or two students to remain in the classroom at recess to try a new game. These students typically enjoy the special attention and later serve effectively as useful helpers when the entire class is exposed to the activity.

Finally, teachers should collect flexible games, those that can be easily adapted to more or less difficult concepts to account for individual learner abilities. An ideal game allows a strong student to play with or against a less strong student while both enjoy and learn from the experience. Typically, games that have a strong element of luck (for example, involve rolling dice or twirling spinners) are most effective for pairing advanced students with less advanced students.

Four games introduced early in the year to a Grade 1 class are provided below. The first two games, *Shade* and *Up to 10*, have a stronger strategy component while the other two, *Unlucky Sixes* and *Tic Tac Add*, have a larger element of chance. All games can be easily adapted to suit higher grade levels or more advanced students.

Game 1: Shade

Students shade figures to represent values rolled on the die. In essence, as students play, they are repetitively building models or sets of a size specified by the die. This game builds in excitement as it is played and especially when students realize the gameboard is shrinking as squares are shaded. Teachers will want to discuss and model appropriate ways to shade (discussed in more detail below) as some students may find this aspect confusing at first. As a fun and interesting adaptation, allow students to construct their own gameboards of varied shapes and sizes!

Objective

Represents numbers by creating sets, using manipulatives and diagrams. Describes a number using different arrangements and combinations of objects.

Materials

Shade gameboard (Figure 1), pencil crayons, 1 six-sided die

Number of Players

Two or more

Goal

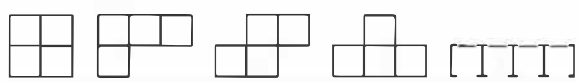
To be the last player in the game, the others having been eliminated when they were unable to shade a number of adjacent squares as rolled on the die.

To Play

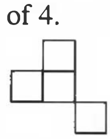
- Players decide who will go first by rolling the die. Lowest roll starts.
- The first player rolls the die and shades the same number of adjacent squares as the value rolled on the die.
- The second player now takes a turn.
- If a player rolls a 6, he or she will skip a turn.
- The players continue taking turns rolling and shading until one player rolls a value greater than the spaces available to shade (for example, there are only three spaces left and the player rolls a 4 or 5). This player drops out of the game.
- If there are more than two players, play continues until there is only one player left, and this player is the winner.

Rules

- Spaces that are shaded *must* be touching each other on at least one side. These are possible arrangements for a roll of 4.



- This is *not* an acceptable shade pattern for a roll of 4.



- If a player rolls the correct value such that he or she shades in the last space, remaining players must roll a 6 to stay in the game.

Variations

- Use a 10-sided die (that contains the values 0 to 9).
- Change the shape of the gameboard.
- Each player uses his or her own gameboard. The first player to fill the gameboard completely and exactly wins.

Game 2: Up to 10

Up to 10 is patterned after the familiar game of Nimh, the major difference being that students are counting up to a target rather than down using markers. Teachers may wish to play both versions with their students. This game has a large element of strategy, and it is surprising to see how many Grade 1 students intuitively look for this strategy.

Objective

Writes the numbers 0 to 10 in order.

Materials

Paper, pencil

Number of Players

Two players

Goal

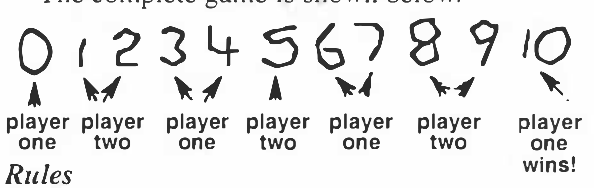
To be the player who writes the number 10

To Play

- Players decide who will go first.
- The first player may write either the first number or the first two numbers when counting from 0 to 10 (that is, the student may write either a zero or both a zero and one).
- The second player may write either the next consecutive number *or* the next two consecutive numbers when counting from 0 to 10.
- Players continue writing one or two numbers on a turn until one player writes the number 10. This player is the winner.

Examples

- Assume the first player writes in just a zero.
- The second player may now choose to write in either a one or both a one and a two.
- The complete game is shown below.



Rules

- A player must write at least one number on a turn.
- A player may write in both nine and ten on a turn to win the game.

Variations

- Change the rules so that the player who writes the 10 loses.
- Extend the game to include larger numbers (up to 20 or 100).
- To make the game a little easier, use a gameboard that has the values written in, so that students merely cross off one or two consecutive numbers.
- Change the rules so that players may add either one, two or three numbers on a turn.
- Change the rules so that players will write the numbers all the way to 100, but they score a single point for each zero that they write (for example, score one point for any multiple of 10 to 90, and two points for 100). The player with the most points when all the numbers have been written wins.

Game 3: Unlucky Sixes

The main objective of this game is to involve students in constructing pictures and models of simple addition and subtraction facts that sum to five. This game includes a large portion of chance in the rolling of the die, but this seems to add great excitement for Grade 1 students. This game may be easily adapted to include larger sums or even multiple addend equations.

Objective

Adds or subtracts numbers to 5.

Materials

Unlucky Sixes gameboard (Figure 2), 1 six-sided die, pencil crayons, pencil

Number of Players

Two players

Goal

To be the player to construct and model the greatest number of addition sentences before rolling three 6s.

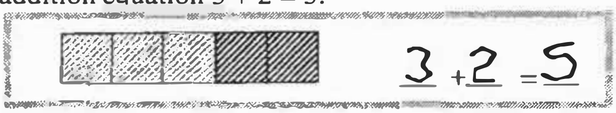
To Play

- Players rolls the die to see who goes first. The player with the highest roll starts.
- The first player plays on the left side of the gameboard; the second player plays on the right side.
- The first player rolls the die. If he or she rolls a 6, then he or she must cross out one of the circled sixes at the bottom of his or her side of the gameboard.
- If the first player rolls a value other than six, he or she can shade that number of spaces on any *one* row on his or her side of the gameboard.

- The die is now passed to the other player who also rolls and shades either a six or the appropriate number of spaces on any row.
- The players continue taking turns rolling the die, shading the rows, constructing addition equations and/or crossing off 6s until both players have crossed off all three 6s. The player with the greatest number of completed rows wins.

Example

Assume a player rolls a 3 and shades the first three boxes on a row. To complete that row, that player must now roll a 2 (on any later turn) to construct the addition equation 3 + 2 = 5.



Rules

- A player must use two or fewer rolls to complete a row.
- A roll of 5 automatically fills any row, and the player can enter either equation: 5 + 0 = 5 or 0 + 5 = 5.
- Once a player has crossed off all three 6s, the other player will continue to roll on his or her own until he or she rolls three 6s or completes the gameboard.
- Any player to complete the gameboard automatically wins.

Variations

- Adapt the game to model subtraction equations.
- Adapt the game to use great sums, for example, sums to 10.
- Change the game so that a player merely passes his or her turn if he or she rolls a 6. Instead, the first player to complete his or her side of the gameboard wins.

Game 4: Tic Tac Add

This game has a larger element of luck than strategy, but students enjoy the game's simplicity and its format. Students are usually familiar with the game Tic Tac Toe and find this simple extension easy and enjoyable to play. This game can be readily adapted to include more difficult addition sentences or even a mixture of addition and subtraction sentences. Tic Tac Add has a strong drill and practice dimension.

Objective

Adds and subtracts numbers to 5.

Materials

Tic Tac Add gameboard (Figure 3), overhead spinner, two types of colored markers (approximately 15 of each)

Number of Players

Two players

Goal

To be the first player to construct a straight line horizontally, vertically or diagonally on either Tic Tac Add board.

To Play

- Players decide who will go first.
- The first player twirls the spinner.
- This player may now place his or her marker on any space occupied by any addition sentence with the same sum as the value spun.
- The other player now gets his or her turn to spin and place a marker.
- Players continue to take turns spinning and placing markers until one player constructs three in a row or until all spaces on both boards have been claimed.
- The first player to achieve three in a row on either board wins.

Example

Assume a player spins a 4. This player may place his or her marker on any of the following spaces:

3+1	2+2	1+3

Rules

- Once a marker is placed, it may not be moved until the end of the game.
- If neither player can make three in a row, the player who has placed the greater number of markers wins.

Variations

- Change the addition sentences to subtraction sentences.
- Mix addition and subtraction sentences.
- Enlarge the range of numbers spun (and the addition sentences on the Tic Tac Add boards) to include sums to 10.

Bibliography

Leonard, L. M., and D. M. Tracy. "Using Games to Meet the Standards for Middle School Students." *Arithmetic Teacher* 40 (1993): 499–503.

Loewen, A. C., and B. J. Firth. *Mathematical Games Made Easy.* Barrie, Ont.: Exclusive Educational Products, 1994.

Figure 1

Shade Gameboard

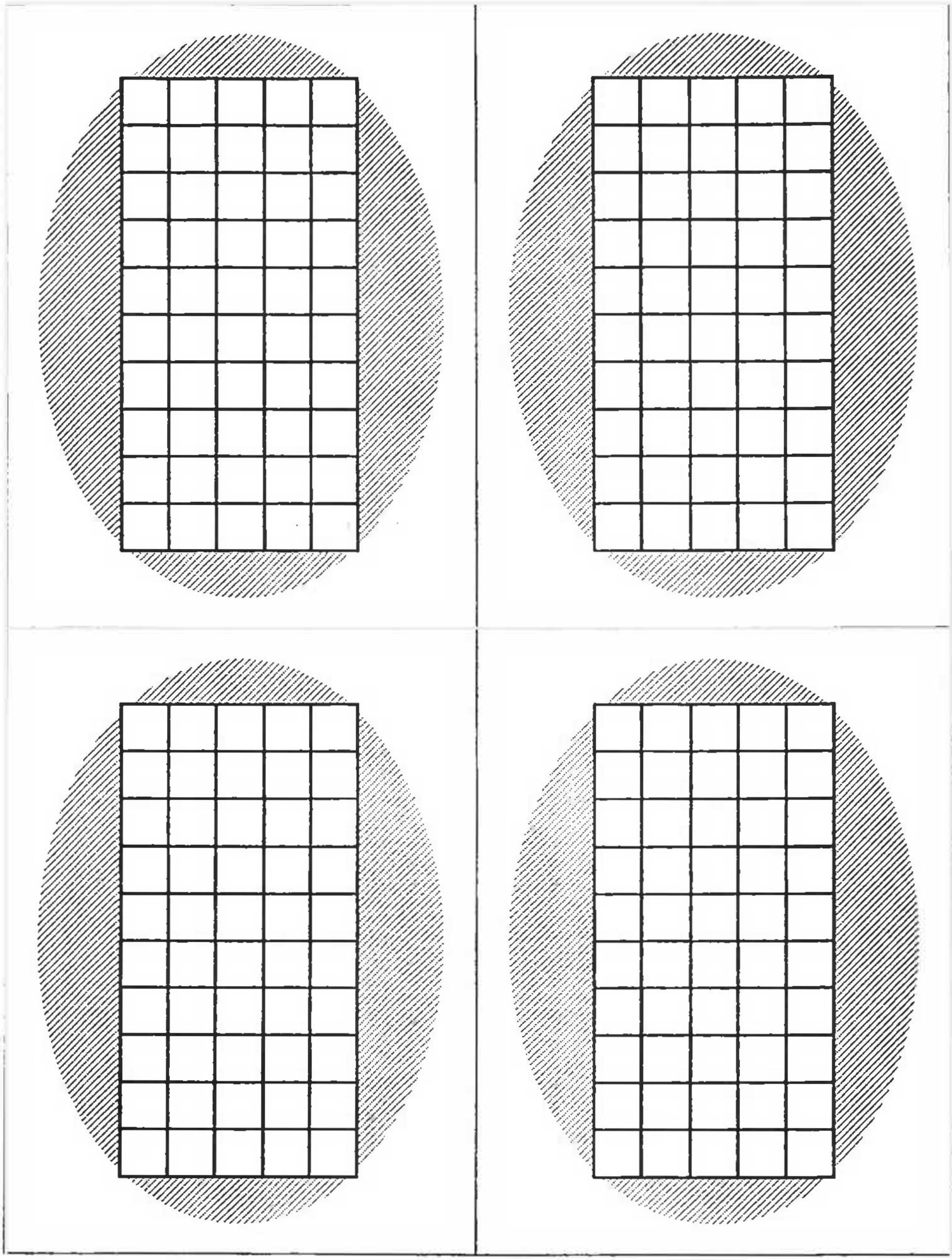


Figure 2

Unlucky Sixes Gameboard

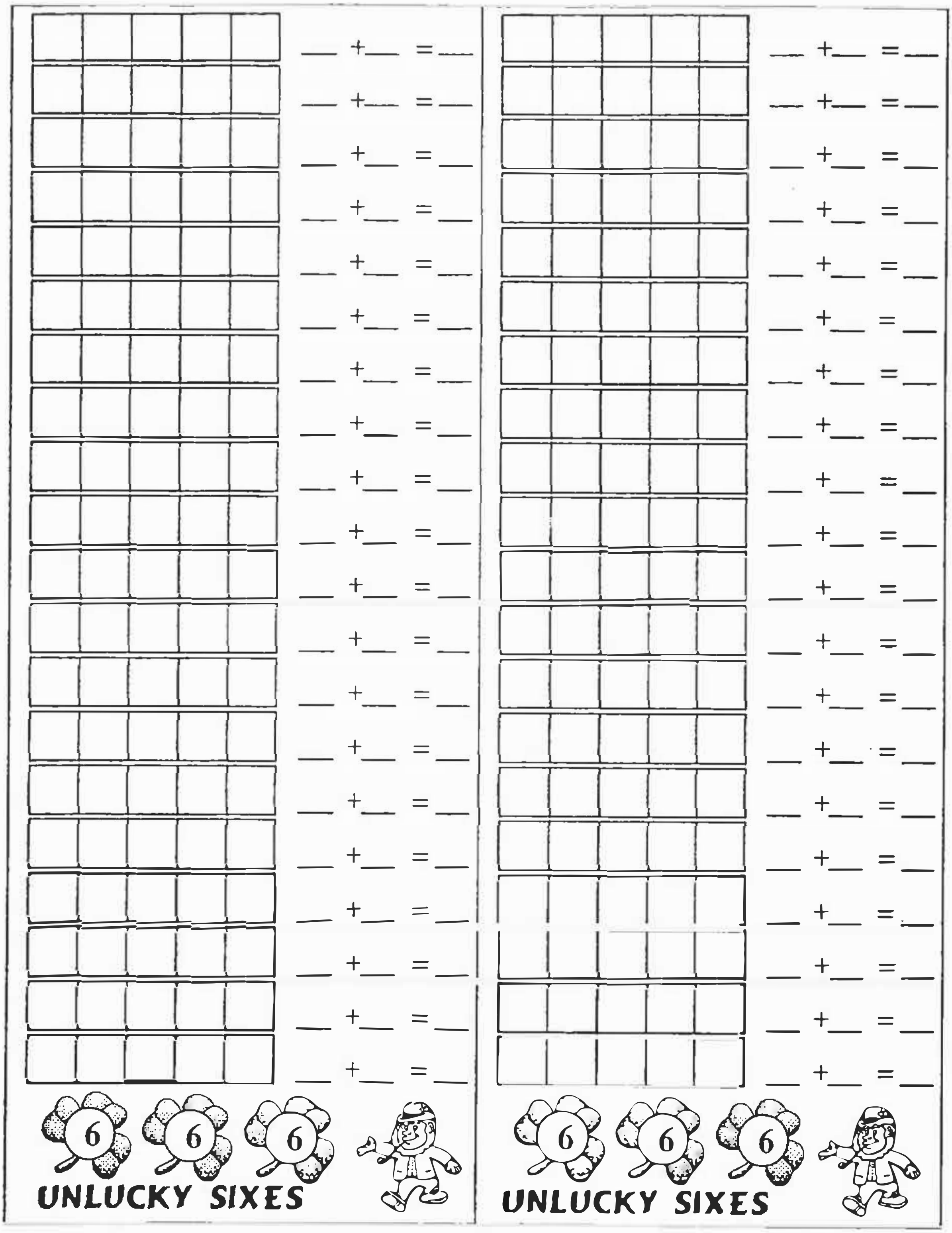
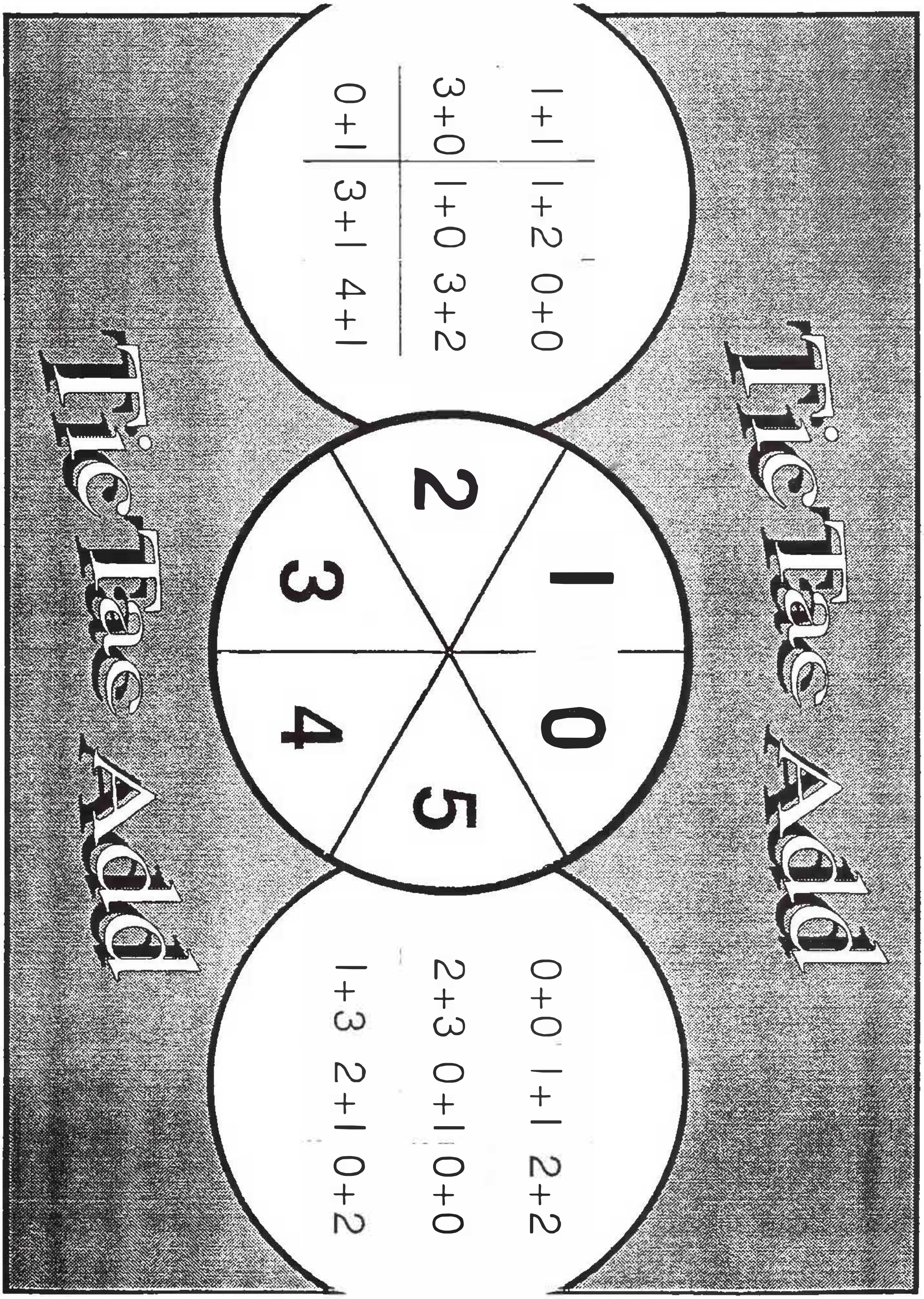


Figure 3

Tic Tac Add Gameboard



Coping with the Vortex of Change

Dan Jelinski and Jack Kelly

This article is reprinted from the *Religious and Moral Education Council Newsletter*, volume 21, no. 2, March 1995. The points raised are relevant to all of us.

1. Have a professional partner share your ups and downs. *Let's end the teacher as Lone-Ranger model. A team is always stronger than one person alone.*
2. Know your own weaknesses and strengths. *None of us teachers can really be Superteacher. Have an adequate assessment of who you really are and what you can realistically do.*
3. Give yourself an occasional knock on the head. *Force yourself to see things from a different perspective. You'll change.*
4. Be a real prophet. *Speak in "what if" instead of complaining about "what is."*
5. Correct as much as possible on the spot. *Manual correction of multiple choice is quicker than using machines.*
6. Communicate on paper. *Send ideas back and forth to colleagues on paper. Don't let rambling conversations get forgotten.*
7. Update your mark books constantly. *Make each course worth 1,000 points of work—you'll never need a calculator to figure a final grade.*
8. Fill out forms immediately. *Move things from mailbox to mailbox immediately.*
9. Differentiate your attendance. *Why must everybody come to every class?*
10. What is, is. *You have to accept some things.*
11. Have partner contacts. *Good students can share learning.*
12. Investigate alternative delivery methods. *Is the timetable a friend or a curse to education?*
13. Be good at what you do. *A real professional carries a lot of weight.*
14. Invite your administrators to be part of your journey. *Often, they are lonely and lost and want to be included if you just ask.*
15. Ride the coattails of administrators. *Know your strengths, work with them.*
16. Let administrators ride your coattails too! *When you win, they win too!*
17. Be in touch with yourself. *What are your feelings?*
18. Be holistic. *Take note of the whole person you are.*
19. Keep the big picture in mind. *One billion people in China don't really care about our problems.*
20. Live the moment. *Enough said.*

When you're young, you know a whole lot you won't know later.

—*Margaret Lawrence*

Blessed is the person who is too busy to worry in the daytime and too sleepy to worry at night.

—*Leo Aikman,*
Atlanta Journal Constitution

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Publications Coeditors
Art Jorgensen — Res. 723-5370
4411 5 Avenue — Fax 723-2414
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Calgary T2R 0H4

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Barnett House
11010 142 Street NW
Edmonton, AB
T5N 2R1