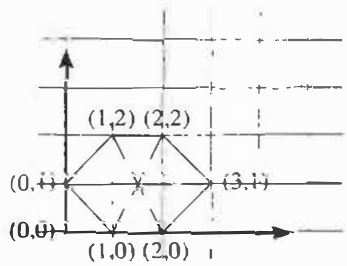
Lattice Hexagons: Pattern Discovery Activities

Bonnie H. Litwiller and David R. Duncan

Teachers are always interested in mathematical activities that encourage students to find patterns and make generalizations leading to formulas. Figures drawn on a lattice (square graph paper) provide a rich setting for this type of activity.

Figure 1 displays a hexagon drawn on a coordinated lattice. The two horizontal sides are each one unit in length; the four diagonal sides are each $\sqrt{2}$ units. In addition, diagonals are drawn connecting opposite vertices.

Figure 1

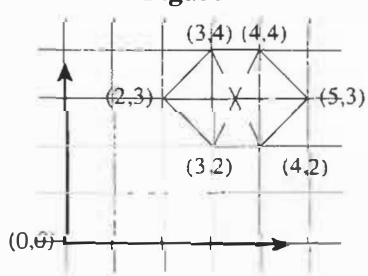


Observe the following:

1. The area of the hexagon is 4 square units.
2. The perimeter of the hexagon is $2 + 4\sqrt{2} = 2(1 + 2\sqrt{2})$ linear units.
3. Opposite vertices: (1,0) and (2,2); (2,0) and (1,2); (3,1) and (0,1).
   Sum of all coordinates: 5; 5; 5
   The sums are all constant.

Figure 2 displays the same size hexagon but is positioned differently on the lattice.

Figure 2



Observe the following:

1. The area and perimeter are, respectively, 4 units and $2(1 + 2\sqrt{2})$ units, the same as for the hexagon of Figure 1.
2. The opposite vertex coordinates again sum to a constant.

The patterns observed in Figures 1 and 2 are independent of the position of the hexagon.

Figure 3 displays a variety of hexagons drawn on a coordinated lattice.

Figure 3

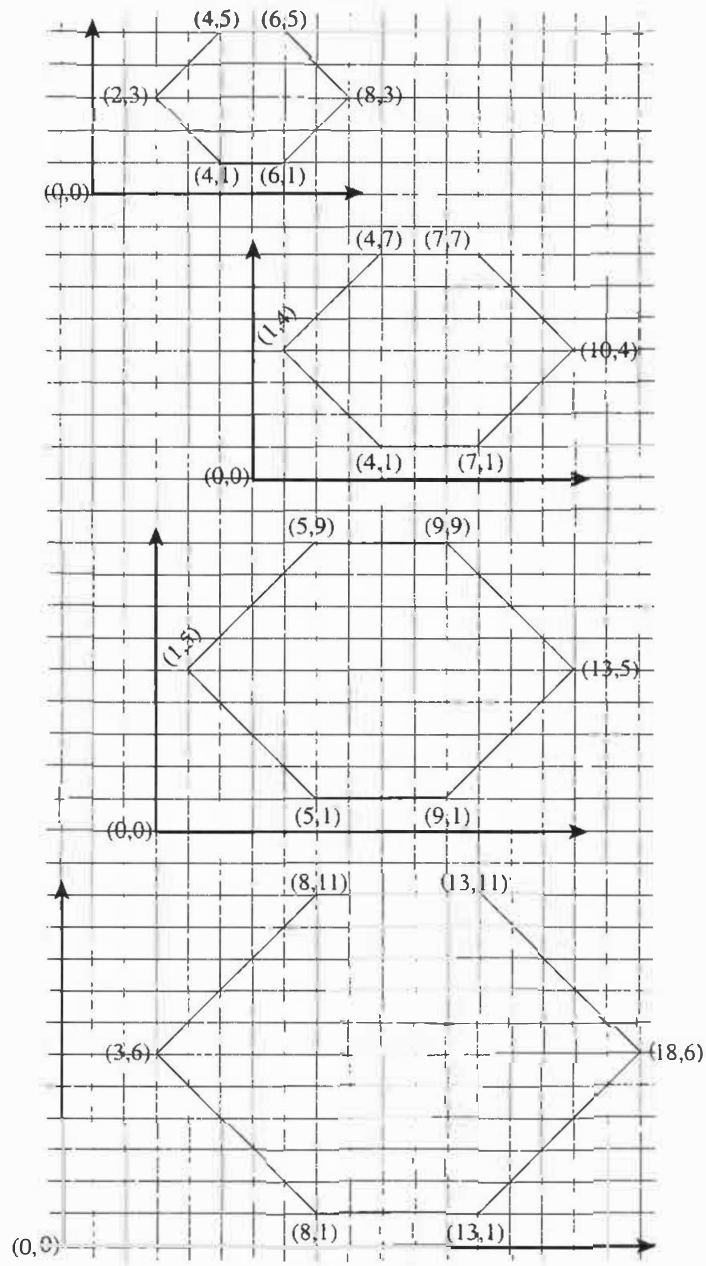


Table 1 reports the results of calculations for the hexagons of Figures 1 and 3.

Table 1

Length of horizontal side	Area	Perimeter	Sum of each pair of opposite vertices
1	4	$2 + 4\sqrt{2}$	5
2	16	$4 + 8\sqrt{2}$	16
3	36	$6 + 12\sqrt{2}$	19
4	64	$8 + 16\sqrt{2}$	24
5	100	$10 + 20\sqrt{2}$	33

Several conjectures are suggested by Table 1:

1. Area
   Observe that the sequence of areas is composed of the squares of the consecutive even numbers; we conjecture that for a hexagon having horizontal sides of length n (an n-hexagon), the area is $(2n)^2$.
2. The perimeters may be written as
   $2\ (1 + 2\sqrt{2})$
   $4\ (1 + 2\sqrt{2})$
   $6\ (1 + 2\sqrt{2})$
   $8\ (1 + 2\sqrt{2})$
   $10\ (1 + 2\sqrt{2})$
   The coefficients are the even natural numbers, so we conjecture that the perimeter of an n-hexagon is $2n(1 + 2\sqrt{2})$.
3. Although the actual sums depend on the placement of the hexagon, we conjecture that the constant opposite vertex sum property holds for any n-hexagon.

These activities connect algebra, geometry and arithmetic in a setting new to most students.

Challenges

1. Check the patterns by drawing hexagons in varying positions and sizes.
2. All figures were drawn in the first quadrant. Do the same patterns hold if other quadrants are used?
3. Examine other geometric figures drawn on a lattice. For example, try the octagon. Do similar patterns hold?
4. Can any of these patterns be replicated on an isometric lattice?
5. Find the other novel settings in which connections can be made and patterns discovered.