An Imaginary Postal Service

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Mathematics is fun. Unfortunately, some mathematics classes are not. Steven imagines himself in such a class with Todd. All the children are bored by the mechanical drills. They pass notes to one another so often that Todd opens a post office. He issues his first stamp, the penny black.

Business is booming. So Steven opens a rival post office across the classroom. He issues a set of stamps and a rate chart.

Funny Notes	1
Silly Notes	2
Naughty Notes	1+2

Now this is a very imaginative idea. There are exactly three connected blocks in this set of stamps. Each has a different value, and all the values are consecutive starting from 1 cent. It is a perfect design.

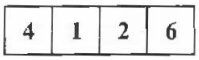
Todd points out that his penny black is a perfect design too, though not a particularly interesting one. He then comes up with the following.

NOTES	half-page	full-page
Funny	1	3
Silly	2	3+2
Naughty	1+3	1+3+2

Not to be outdone, Steven looks at the 1 by 4 set of stamps to see if it can be turned into a perfect design. He counts 10 connected blocks in the 1 by 4 set of stamps, 4 singles, 3 pairs, 2 triples and the whole set. Therefore, if it can be made into a perfect deign, the values of the blocks must be 1, 2 and so on, up to 10 cents.

Clearly, the values of the individual stamps must be 1, 2, 3 and 4 cents in some order. To get a 9-cent block, the 1 must be at one end. To get an 8-cent block, the 2 must be at the other end. To get a 6-cent block, the 4 must now be next to the 2, with the 3 filling in the gap between the 1 and the 4. Unfortunately, there are no 5-cent blocks.

So no one can make the 1 by 4 set into a perfect design. Steven now tries to see if he can get consecutively up to 9 cents. He starts with his first perfect design, where a 1 is next to a 2. This also gives him a 3-cent block. Next, he puts a 4 on the other side of the 1. This gives him a 5 and a 7-cent block. Finally, by putting a 6 on the other side of the 2, he has the missing 6, 8 and 9-cent blocks. There is also a bonus 13-cent block, even though it is not part of the consecutive values .



While not perfect, this is as good a design as anyone can get out of the 1 by 4 set. Still, Steven is annoyed that he has not succeeded in finding a perfect design with 4 stamps.

Suddenly, Steven has an inspiration. There is after all a way to achieve his goal. Instead of a 1 by 4 set of stamps, Steven uses a 2 by 2 set. By bending his latest design into this shape, he turns it into a perfect one. He is so excited that he issues the set right away, along with an expanded rate chart.

NOTES	half-page	full-page	oversize
Funny	1 - - -	1 2 - -	- - - 6
Silly	- 2 - -	1 - 4 -	- 2 - 6
Naughty	- - 4 -	1 2 4 -	1 2 - 6

PARCELS	Uninsured	Insured
Inedible	- - 4 6	- 2 4 6
Edible	1 - 4 6	1 2 4 6

Todd is very impressed with this. He goes back to the drawing board, and it does not take him long to modify his perfect 1 by 3 set into the following perfect design.

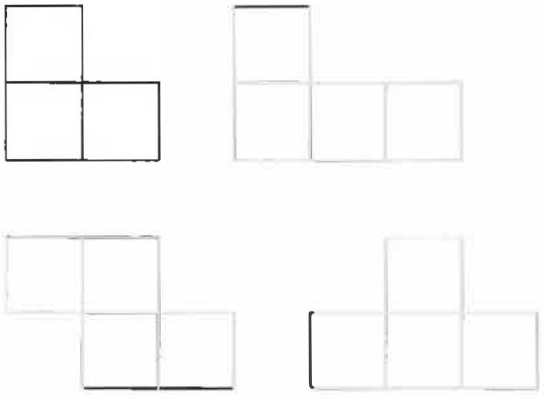
1	3
7	2

It does exactly what Steven's can do, though in different ways. Since he would be using essentially the same rate chart, he merges his post office with Steven's. The two boys then combine their brain power to look for other perfect designs. They find four other sets containing not more than 4 stamps.

"The first one is really the same as the 1 by 3 set," says Todd, "and I have already made that into a perfect design."

"The next two are really the same as the 1 by 4 set," says Steven, "and I know that it cannot be made into a perfect design."

He shows Todd the work he has done on that, including his near perfect design.



"The last set looks different from the 1 by 4 set as well as from the 2 by 2 set," says Todd.

"It is," says Steven, "because it has 4 singles, 3 pairs, 3 triples and the whole set, so there are 11 connected blocks."

"If it can be made into a perfect design," says Todd, "the values of the individual stamps must be different and add up to 11 cents. They must be 1, 2, 3 and 5 cents."

"The 1 and the 2 cannot be in the middle, or we will be missing either a 10 or a 9-cent block."

"The 3 must be in the middle. Otherwise, there will not be a 4-cent block."

"Now we do not have a 7-cent block," says Steven. "So this set cannot be made into a perfect design."

The two boys then work independently to see if they can get consecutively up to 10 cents. Clearly, they must use 1, 2, 3 and 4. Each comes up with a near perfect design.

"I get mine by adding a 4 on top of my perfect design for the 1 by 3 set," says Todd.

"Mine has the 4 in the middle instead. We always seem to come up with different designs," says Steven, remembering their perfect designs for the 2 by 2 set.

"I will have to find a near perfect design for the 1 by 4 set different from yours," says Todd. "Aha! If I clone the 3 in my perfect design for the 1 by 3 set, I will have what I want."

1	3	3	2

"It is interesting that your stamps add up to under 10 cents, while mine add up to over 10 cents. That is why you can afford a duplicate."

The two boys are silent for a while.

"I have yet another near perfect design for the 1 by 4 set," says Todd. "The stamps add up to 11 cents."

2	5	1	3

"So have I," says Steven, "except that mine add up to 9 cents."

1	1	4	3

"Are there any others?" asks Todd.

"What about the other sets of stamps we have looked at?" asks Steven. "Have we found all the perfect or near perfect designs for them?"

Just then, the bell rings, and the imaginary class dissolves into thin air. Steven hits his alarm-clock and gets ready for his real class.

Remark:

A version of this problem was posed by Steven Laffin and Andy Liu as Problem 93-9 in the S.I.A.M. Review, under the title *All-purpose Stamp Design*.

Supplementary Problem

Exactly one of the following twelve sets of 5 stamps can be made into a perfect design. Which one and how?