A Space Interlude

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Space Station Intelligentsia received a call for help on the hyperradio from Spaceship Academia. Captain Philip said, "We are on the way home after a successful mission of promoting higher learning in distant star systems. We are surrounded by a Kleingon Fleet. We are unarmed. Please send a relief force."

"Unfortunately, there are no other Spaceships on base at the moment," said Commander Gilbert. "Can you hold out?"

"Affirmative," said Captain Philip, "but we cannot disengage. It would help if you could get a Space Cannon to us."

"No problem. We will send one over by a Space Pod."

"Hang on a minute! Oh, no! The Kleingon Fleet has just been reinforced by a Space Tetropus. It can grab one Space Pod at a time."

"I will send two Space Pods, each carrying a Space Cannon," said Commander Gilbert.

"Do not do that! Repeat! Do not do that!" Captain Philip said urgently. "If a Space Cannon falls into the hands of the Kleingons, we are history. It is too powerful even for us."

"I will get back to you as soon as possible."

Commander Gilbert consulted Lieutenant Kenneth, the scientific advisor. He said, "We can break up a Space Cannon into two component parts and send them separately. This way, the Kleingons can only get half of it, which is of absolutely no use to them." "Unfortunately, Spaceship Academia will not get too much out of the other half. However, your idea is an excellent one. If we break up two Space Cannons into two component parts in identical fashion and send them by four Space Pods, the Kleingons will still be out of luck, while Space Academia will have enough parts to reassemble a complete one."

The two officers were very pleased with their plan. However, when they tried to put it in operation, they found that there were only three Space Pods on base.

Lieutenant Kenneth thought for a while and said, "We can still do it, but we have to break up two Space Cannons into three component parts in identical fashion. Let us call them A, B and C. The first Space Pod will carry A and B, the second B and C, and the third C and A. We cannot lose both copies of any part, so that Spaceship Academia can still get a complete Space Cannon, while the Kleingons can only get two-thirds of it."

"It would be best if we do not break up the Space Cannons into too many component parts. Couldn't we still do it with only two?"

"No. Since we have four copies and three Space Pods, one of them must carry two. These must be different as there is no point in any Space Pod carrying two identical parts. If the Space Tetropus grabs this one, the Kleingons will get a complete Space Cannon."

"I guess you are right," said Commander Gilbert. "It is lucky that we have three Space Pods. Had there been only two, we could not have done anything."

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"Yes, each of Spaceship Academia and the Kleingons will get one. Either both have a chance of getting a complete Space Cannon, or neither has, which is definitely not good for us."

"Let us stop theorizing and put our plan to work. We cannot count on Spaceship Academia holding out forever against the Kleingons."

This was done, and soon words came over the hyperradio that all was well. Before long, Spaceship Academia was docking at Space Station Intelligentsia. Commander Gilbert and Lieutenant Kenneth welcomed Captain Philip's safe return.

"That was a close call," reported Captain Philip. 'The Kleingons were about to replace the Space Tetropus with a Space Octopus, which can grab two Space Pods at a time."

"This is serious," said Commander Gilbert. "Let us go to work at once and figure out a solution, rather than wait until we have to face the situation."

"To begin with," said Lieutenant Kenneth, "we have to break up three Space Cannons. This way, we cannot lose every copy of any component part. On the other hand, we do not need to break up more than three, as that will only make things easier for the Kleingons."

"Also, each Space Cannon must be broken up into at least three component parts," Captain Philip said. "If there are only two, the Space Octopus can just nab one Space Pod carrying each part, and the Kleingons will have a complete Space Cannon. If we break it up into exactly three component parts, we will need nine Space Pods so that each one will carry one part. Nothing less will do."

"We seldom have that many Space Pods on base," Commander Gilbert pointed out. "What is the smallest number of Space Pods that can carry out a successful convoy?"

"It has to be five or more. If we send only four, each side will get two, and that is bad news. This is the same argument that we use to explain why two Space Pods are not enough for getting around a Space Tetropus."

"Are five Space Pods enough though?" Commander Gilbert pressed the point.

"This is tough," said Captain Philip. "Let us consider all possible scenarios. If we number the Space Pods 1, 2, 3, 4 and 5, the Space Octopus may nab 1 and 2, 1 and 3, 1 and 4, 1 and 5, 2 and 3, 2 and 4, 2 and 5, 3 and 4, 3 and 5, or 4 and 5. So for any of these ten pairs, there must be at least one component part neither of which is carrying."

"Going back to what I said earlier," chimed in Lieutenant Kenneth, "we must have three copies of each part. Therefore, if 1 and 2 are missing part A, then 3, 4 and 5 must have it."

"This means that we must break up each Space Cannon into ten component parts, so that each of the ten pairs will be missing a different part. This will work. Let us draw a chart to show what each Space Pod should be carrying. We will call the component parts A, B, C, D, E, F, G, H, I and J."

Captured	1	1	1	1	2	2	2	3	3	4	S
Space Pods	2	3	4	5	3	4	5	4	5	5	P
Parts					E	F	G	Н	I	J	1
carried		В	С	D	_			Н	I	J	2
by each	A		С	D		F	G			J	3
of the	A	В		D	E		G		I		4
Space Pods	A	в	С		E	F		Н			5

"Wow!" the three officers looked at one another and smiled.

Acknowledgments

This project is based on "The Couriers Problem" in Dennis Shasha's *The Puzzling Adventures of Dr. Ecco.*

Supplementary Problem

What is the minimum number of parts into which each Space Cannon must be divided in order to get around a Space Octopus, if

- **\$** 3. six Space Pods are available?