Primary School Mathematics Olympiad in China

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Although mathematics competitions for high school students have occurred for more than a century, mathematics competitions for primary school students are still young. Large-scale mathematics competitions for primary school students in China date back to 1986. The Chinese Mathematical Society introduced the Primary School Mathematics Olympiad (hereafter referred to as the PSMO) in 1991. Since then we have witnessed both achievements and problems associated with these competitions. A question which we must answer is: how can mathematics competitions contribute to mathematics education in primary schools?

Some Principles

This year there are two million participants in the PSMO. Students, teachers and parents are very enthusiastic about it. There are two main reasons behind their enthusiasm:

- China continuously achieved good results in the IMO.
- Many prestigious middle schools welcome winners in the PSMO to study in their schools.

Obviously, these two factors cannot be the sole driving forces for mathematics competitions forever. We believe that the aim of mathematics competitions is to arouse students' interest and encourage them to study mathematics. It is inappropriate to give each winner too many awards or to label the winners as talented or gifted.

- A crucial part of a mathematics competition is to set problems that suit the ability of most participants. To develop genuine interest in mathematics among students, we must devote more effort to the setting of the problems. A good competition paper enables participants to demonstrate their abilities and ingenuity, which in turn influences classroom teaching in mathematics positively. With these objectives in mind, the Popularization Committee of the Chinese Mathematical Society laid down two rules on problem-setting for the PSMO: Knowledge required for answering PSMO problems must be within the syllabuses for primary school mathematics.
- Every PSMO problem must have a simple arithmetic solution.

We should motivate primary school students to think rather than to know more facts in mathematics. We must not include secondary school mathematics in mathematics competitions for primary school students. In particular, the solution of applications problems using equations should be discouraged at this level. Applications problems, if included, must have simple and quick arithmetic solutions. The PSMO emphasizes familiarity with the basic operations in arithmetic and, building upon this, the ability to skillfully solve some problems.

Problem-setting is a creative task and original problems are most welcome. Setting new problems without increasing the difficulty of the competition paper is a formidable task. However, problem setters must work toward the goal of setting interesting problems using words understandable by primary school students.

Levels of Difficulty

At present, the determination of the difficulty of PSMO problems is based on the experience of the setters, so the judgment is not always reliable. Problem setters must overcome their own prejudices and the biases brought about by their problem-solving abilities. In some sense, problem-solving is analogous to taking off a dress—the more buttons, the more difficult it is to take off the dress. A stumbling block in the problem-solving process acts like a button: a problem setter should be aware of the existence of buttons and count the number of buttons when evaluating difficulty of problems.

PSMO problems are classified according to three levels of difficulty: A (difficult), B (medium) and C (easy). Each level is further divided into three sub-levels to allow for greater precision. For example, level A is divided into $A+$, A and $A-$. To quantify the levels, the following weights are assigned to them: *C-* = *1*, *C* = 2, *C+* = 3, *B-* = *4*, *B* = 5, *B+* = 6, *A-* = 7, *A* = 8 or 9 and *A+* = 10 or 11. The sum of the weights of the individual problems represents the difficulty of the whole paper. Extra weights are accorded to the *A* and *A+* problems so as to avoid setting a paper which is far too difficult.

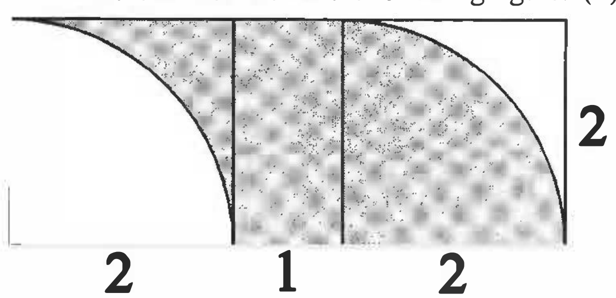
On a scale of 0 to 11, even two experts can differ widely in their assessment of the difficulty of a particular problem. To achieve some consistency, we identify three specific buttons, to continue the earlier analogy, that stand in the way of problem-solvers, and compute the weight of a problem as the sum of the following three components. While the final result also ranges from 0 to 11, experts seldom differ by more than 1 point.

- Is the problem easily understood? Is the statement too long? Are there terms which may be unfamiliar? This aspect of the problem is graded on a scale of 0 to 3.
- Are the computations, if any, very involved? Is the knowledge of specific techniques required? This aspect of the problem is also graded on a level of 0 to 3.
- Is there a natural approach to the problem? Does it require insight? How much synthesis and analysis are involved? How difficult is it to see a solution through from start to finish? This aspect of the problem is graded on a scale of 0 to 5.

We now give one sample problem of each sub-level. These are taken from past PSMOs.

C- Calculate 1991 + 199.1 + 19.91 + 1.991 (2212.001)

C Find the shaded area in the following figure. (6)



C+ There are two numbers A and B. If we move the decimal point in A two places to the left, then we get 8 of B. How many times of B is A? (12.5)

B- Among some sweets, 45% are milk sweets. If we add 16 fruit sweets, then the percentage of milk sweets changes to 25%. How many milk sweets are there? (20)

B A 7-digit number 1993*** is divisible by 2, 3, 4, 5, 6, 7, 8 and 9. Find its last three digits. (320)

B+ Five students A, B, C, D and E scored more than 91 marks each, out of 100, in an examination in which only integral scores were awarded. The average score of A, B and C is 95 while that of B, C and D is 94. A got the highest score and E got the third highest with 96 marks. Find the score of D. (97)

A- Vessel A contains 11 litres of pure alcohol. Vessel B contains 15 litres of water. Part of the alcohol in vessel A is poured into vessel B and mixed with the water. Then, part of the solution in B is poured into vessel A. Now vessels A and B contain 62.5% and 25% of pure alcohol respectively. Find the amount of solution poured from vessel B into vessel A. (6 litres)

A A car travels from town A to town B. If its speed is increased by 20%, then it will arrive at town B one hour earlier. If the car travels at the original speed for 120 km and then its speed is increased by 25%, it will arrive at town B 40 minutes earlier. Find the distance between towns A and B. (270 km)

A+ A circular track has a length of 2 kilometres. A, B and C, starting simultaneously from the same spot and in the same direction, are required to complete two laps each. A walks at 5 kilometres per hour while B and C walk at 4 kilometres per hour. All three can ride a bike at 20 kilometres per hour. There are two bikes. A starts on foot while B and C start on the bikes. Riders may get off anywhere and leave the bike for the others. Devise a scheme such that all three people and the two bikes arrive simultaneously at the destination. Then, find the shortest time required. (19.2 minutes)

There are many mathematics Olympiad schools all over China offering extracurricular instruction for mathematics competitions. Teachers in these schools teach solutions to many problems and students get practice working on problems. Even though these students are the winners in mathematics competitions, their problem solving abilities are difficult to predict.

Consider the following problem.

One bottle of soft drink can be exchanged with five empty bottles. A class of students drank 161 bottles of soft drink, some of which were obtained by exchanging with bottles emptied. Find the least number of bottles of soft drink bought. (129)

This problem has a simple solution and so we originally classified it as *C+*. However, after the competition, we agreed that it should have been at least *B+*. Ideally, a *C* problem should be solved by more than 50% of the contestants, a *B* problem by 15% to 50%, and an *A* problem by less than 15%. However, if the number of contestants increases, these percentages should be lowered.

Format

The First Round of the PSMO is held annually in March and the Final Round in April. There are 12 problems in the one-hour First Round, in which two million students participate each year. The Final Round has 12 answer-only problems to be attempted in 90 minutes. Each year there are 200,000 to 300,000 participants.

With thirty provinces in China, there is a wide range of abilities in mathematics among students across the country. In light of this, there are three versions of papers for the First Round in the PSMO: Paper A (hardest), Paper B (medium) and Paper C (easiest). The participating cities are allowed to choose the version most suitable for their students. Apart from the Han race, which is the largest ethnic group in China, there are 55 minority races. Because of historical factors, the standards of education are lower. To encourage minority students to participate in mathematics competitions and to demonstrate their attainment, a Minorities Paper (MP) has been set since 1993. The following table is an analysis of the levels of difficulty of the Papers in the first four years.

		Level C	Level B	Level A	Total Weight
	Paper C	9	3	0	37
	Paper B	6	6	0	47
	Paper A	3	9	0	53
	Final	3	6	3	59
	Paper C	7	5	0	38
	Paper B	5	7	0	44
	Paper A	4	8	0	47
	Final	4	6	2	57
	Paper MP	9	3	0	30
	Paper B	7	5	0	38
	Paper A	5	7	0	46
	Final MP	6	6	0	39
	Final	4	6	2	52

		Level C	Level B	Level A	Total Weight
	Paper MP	9	3	0	33
	Paper B	8	4	0	38
	Paper A	6	5	0	46
	Final (MP)	5	7	0	45
	Final	4	6	2	58

To adjust the standards of the First round and Final Papers so as to popularize them and provide opportunities for a small number of bright students to demonstrate their abilities, a Summer Camp has been held annually since 1993 as the Overall Final Competition. The Overall Final comprises four parts:

- First Contest (One hour) - 10 answer-only problems covering four levels of difficulty (*C+*, *B-*, *B*, *B+*) with emphasis on basic skills in arithmetic.
- Second Contest (90 minutes) - 6 problems which are more demanding and which require deeper thinking.
- Relay - Three students in a team work out six problems. Student 1 is responsible for Problems 1 and 4, Student 2 for Problems 2 and 5, and Student 3 for Problems 3 and 6. Every problem, except the first one, contains a parameter which takes the value of the answer to the previous one. Five minutes are allowed for each of Problems 1 to 5, and ten minutes for Problem 6. Students may correct the solutions to the preceding problems. The relay contest was welcomed by teachers and students.
- Calculation Contest - Some Chinese teachers and students are more interested in solving difficult problems and they neglect training on basic operations in arithmetic. A consequence is that students frequently make mistakes in calculations involved in secondary school algebra later on. Calculation is not just a skill: it also trains students to think. In this contest, students are required to do 25 problems in one hour. A relatively easy problem is

$$0.3125 \times 457.83 \times 32 = ?$$

A more difficult one is

$$\frac{10}{13} \div 2\frac{19}{22} - 1\frac{2}{5} \times \frac{11}{13} \div 7 + \frac{1}{5} \times \frac{11}{63} = ?$$

With more different forms of competitions, students can develop an interest in mathematics from different perspectives and have more opportunities to demonstrate their abilities. This also discourages students from focusing only on a particular aspect of mathematics.

Prospect

Because classroom teaching usually caters to the majority, more able students cannot be satisfied with the mathematics taught in school. In China, it is essential to organize extracurricular activities in mathematics and competition is a suitable choice. However, excessive training is a burden on teachers and students. There are too many mathematics competitions in China. Participating in several competitions in a semester is too much for students, and may lead to a loss of interest in learning mathematics. Future PSMO problems should be easier and interesting. This is a difficult task requiring the effort of many mathematicians and teachers.

As more and more Asian countries organize competitions for primary school students, it is worth considering the possibility of organizing a regional competition like the APMO. Primary school mathematics is simple but competitions at the primary level are influential in general education because of the large number of students involved.