Appendix III: A Selected Bibliography on Popular Mathematics

Part 1: Updates on the Bibliography in MfGS1

A. Martin Gardner's Scientific American Series

Two more volumes have appeared, and there will be a fifteenth and final volume. Several earlier volumes have also changed publishers. The Mathematical Association of America has acquired *Martin Gardner's New Mathematical Diversions from Scientific American, Martin Gardner's 6th Book of Mathematical Diversions from Scientific American, Mathematical Carnival, Mathematical Magic Show* and *Mathematical Circus*. The University of Chicago Press has acquired *The Scientific American Book of Mathematical Puzzles and Diversions, The 2nd Scientific American Book of Mathematical Puzzles and Diversions* and *The Unexpected Hanging and Other Mathematical Diversions*.

We are delighted that Martin Gardner has given us permission to print a comprehensive index of his Scientific American columns. This will constitute Part 2 of this Appendix.

Penrose Tiles to Trapdoor Ciphers, 1989,
W. H. Freeman.

Topics covered are Penrose tilings, Mandebrot's fractals, Conway's surreal numbers, mathematical wordplay, Wythoff's version of the game "Nim", mathematical induction, negative numbers, dissection puzzles, trapdoor ciphers, hyperbolas, the new version of the game "Eleusis", Ramsey theory, the mathematics of Berrocal's sculptures, curiosities in probability, Raymond Smullyan's logic puzzles, as well as two collections of short problems. The book also contains a surprise ending — the resurrection of Dr. Matrix!

Problem 1

Prove that at a gathering of any six people, some three of them are either mutual acquaintances or complete strangers to one another.

Fractal Music, Hypercards and More, 1992,
W. H. Freeman.

Topics covered are fractal music, the Bell numbers, mathematical zoo, Charles Sanders Peirce, twisted prismatic rings, colored cubes, Egyptian fractions, minimal sculptures, tangent circles, time, generalized ticktacktoe, psychic wonders and probability, mathematical chess problems, Hofstadter's *Gödel, Escher, Bach*, imaginary numbers, some accidental patterns, packing squares, Chaitin's irrational number Ω, as well as one collection of short problems.

Problem 2

Two players are engaged in a game of generalized ticktacktoe on an infinite board. The first player marks an X in a vacant cell, then the second player marks an O in a vacant cell, and the turns alternate thereafter. The first player wins if there is an X in each of four cells in a compact 2×2 configuration. Prove that the second player can prevent that from ever happening.

B. Raymond Smullyan's Logic Series

Satan, Cantor, and Infinity, 1992, Alfred A. Knopf.

In this book, a remarkable character known as the Sorcerer makes his debut. He escorts the readers on a wonderful guided tour, visiting familiar ground, such as the domains of the knights and the knaves, and those bordering on the land of Gödel. There are also ventures into new territories, including an island where intelligent robots create other intelligent robots that can continue this process *ad infinitum*. This eventually leads to the pioneering discoveries on infinity of the great mathematician, Georg Cantor. The readers may be amused to discover how Satan got into the picture.

Problem 3

Hilbert's Hotel has infinitely many rooms—one for each positive integer. The rooms are numbered consecutively—Room 1, Room 2, Room 3, and so on. Currently, each room is occupied by one person. A new guest arrives and wants a room. Neither she nor any of the other guests is willing to share a room, but the others are all cooperative in that they are willing to change their rooms, if requested to do so. How can the new guest be accommodated?

C. Oxford University Press Series on Recreations in Mathematics

The Mathematics of Games, by John D. Beasley, 1989.

This book mathematically analyzes some card and dice games, nim-type games, a version of John Conway's "Hackenbush", as well as providing a mathematical model for the study of some sports games. The principal techniques are counting, probability and game theory. Some mathematical puzzles are also considered.

Problem 4

You draw a card from a standard deck of 52 and claim to have a picture card, that is, a King, a Queen or a Jack. You can bet either \$5 or \$1. If your opponent concedes, she pays you the amount. If she challenges, then whoever is wrong pays the other double the amount. Your strategy is to bet \$5 whenever you have a picture card or an Ace, and \$1 otherwise. Do you win or lose on the average, and by how much?

The Puzzling World of Polyhedral Dissections, by Stewart T. Coffin, 1990.

This is a labor of love from an expert craftsman. Starting with two chapters of two-dimensional geometric puzzles, the author eases the readers gently into the third dimension and soon launches into his specialty, the burrs, which are assemblies of interlocking notched sticks. The book is profusely illustrated with black-and-white line drawings and photographs. It concludes with a chapter on woodworking techniques.

Problem 5

The surface of a $1 \times 2 \times 2$ block may be divided into 16 unit squares. Two such blocks are glued together so that the region common to both consists of 1, 2 or 4 unit squares. Find all the different solids that may be formed this way, and use a copy of each to build a solid rectangular block.

More Mathematical Byways, by Hugh ApSimon, 1990.

This book contains fourteen chapters. The first three form a sequence but the others are independent of each other. Unlike the earlier volume by the same author, the problems are of uneven difficulty levels, ranging from the relatively simple Alphametics to others which require considerable amounts of what the author calls "slog". One of the chapters, titled *Potential Pay*, is not really a problem but a commentary on a classic paradox.

Problem 6

There are four large trees in the plain: an oak, a pine, a quince and a rowan. Ceiling church lies directly between the pine and the rowan, and directly between the oak and the quince. The curate of Ceiling recommends two walks in the neighborhood. Each starts and finishes at the church. One visits in turn the oak, the pine and the quince. The other visits in turn the rowan, the quince and the pine. They are along straight paths, apart of course from turning the corners at the trees. The two walks are of the same total length. The oak and the quince are further apart than are the pine and the rowan. Which of the oak and the rowan is nearer to Ceiling church?

D. Dolciani Mathematical Expositions Series of the Mathematical Association of America

More Mathematical Morsels, by Ross Honsberger, 1991.

This is a collection of 57 problems, almost all of which are taken from the Canadian Mathematical Society's journal *Crux Mathematicorum*, plus further "gleanings" from its famed *Olympiad Corner*.

Problem 7

In a certain multiple-choice test, one of the questions was illegible, but the choice of answers, given below, was clearly printed. What is the right answer?

(a) all of the below

(b) none of the below

(c) all of the above

(d) one of the above

(e) none of the above

(f) none of the above

Old and New Unsolved Problems in Plane Geometry and Number Theory, by Victor Klee and Stan Wagon, 1991.

This book is divided into two halves, as suggested by the title, though the second half also covers problems about some interesting real numbers. Each half consists of two parts: in the first, twelve problems are presented, giving the statement, known results and background information; in the second, the same twelve problems are re-examined for further results and extensions. Each half concludes with a comprehensive bibliography. Although the problems are unsolved, and therefore difficult, it is not impossible for them to yield to an inspired attack. Even if this does not happen, gifted students who are willing to attempt them will find their mathematical talent enhanced.

Problem 8

What is the minimum number of points in the plane, no three on a line, such that some four of them will form the vertices of a convex quadrilateral?

Problems for Mathematicians Young and Old, by Paul Halmos, 1992.

The fourteen chapters of this book are titled Combinatorics, Calculus, Puzzles, Numbers, Geometry, Tilings, Probability, Analysis, Matrices, Algebra, Sets, Spaces, Mappings and Measures. The author, a ranking mathematician and master expositor, wrote this book for fun, and hopes that it will be read the same way.

Problem 9

Cucumbers are assumed to consist of 99% water. If 500 kilograms of cucumbers are allowed to stand overnight, and if the partially evaporated substance that remains in the morning is 98% water, how much is the morning weight?

Excursions in Calculus, by Robert Young, 1992.

The subtitle of this book is *An Interplay of the Continuous and the Discrete*. Using calculus as a unifying theme, the author branches into number theory, algebra, combinatorics and probability. The book contains a large collection of exercises and problems.

Problem 10

Suppose there are finitely many prime numbers and the largest is p. Let $M = \frac{2}{1}\cdot\frac{3}{2}\cdot\frac{5}{4}\cdots\frac{p}{p-1}$, the product ranging over all prime numbers. Prove that $M = 1 + \frac{1}{2} + \frac{1}{3} + \cdots$, the sum ranging over all positive integers. Deduce from this that there are infinitely many prime numbers.

The Wohascum County Problem Book, by George Gilbert, Mark Krusemeyer and Loren Larson, 1993.

This is a collection of problems, many of which are original compositions. It is mainly targeted at undergraduate students, but quite a few problems are accessible to high school students. The title refers to a recurrent group of problems based on country life in the fictitious locale.

Problem 11

Two ice fishermen set up their ice houses on a frozen circular lake, in exactly opposite directions from the centre, two-thirds of the way from the centre to the lakeshore. Assuming that any fish would go for the nearest lure, each has command of exactly half the lake. A third ice fisherman shows up, but the first two adamantly refuse to move. Is it possible for the newcomer to set up his ice house so that each has command of a region exactly one third the area of the lake?

Lion Hunting & Other Mathematical Pursuits, edited by Gerald Alexanderson and Dale Muggler, 1995.

The subtitle of this book is *A Collection of Mathematics, Verse, and Stories by Ralph P. Boas. Jr.*. A posthumous tribute to this great mathematician and teacher, it contains selected work of Boas and reminiscences by his friends and relatives. The title refers to a piece of mathematical folklore about various mathematical "methods" to trap a lion in a cage. The material chosen is most suitable for undergraduate students, but high school teachers and students should find it a most amusing and informative reading. Find out how the following "problem" arose.

Problem 12

Where in the world are the following places and what do the names mean?

(a) Llanfairpwilgwyngyllgogerychwyrn-drobwllllandysiliogogogoch

(b) ChargogogogmanchargogogchaubunnagungamAug

(c) Taumatawhakatangihangakoauauotamateapokai-whenuakitanatahu

The Linear Algebra Problem Book, by Paul Halmos, 1995.

The whole book is a sequence of structured problems. Like the preceding volume, most of this book is beyond high school level. However, the introductory problems are certainly not intimidating, and inquisitive students may be lured into a most rewarding exploration, laying a good foundation for their undergraduate studies.

Problem 13

If a new addition for real numbers, denoted by the temporary symbol $\oplus$, is defined by $a \oplus b = 2a + 2b$, is $\oplus$ associative?

E. New Mathematical Library of the Mathematical Association of America

In additional to the new volumes listed below, there is also a revised edition of an earlier volume, *Graphs and Their Uses*.

USA Mathematical Olympiads: 1972-1986, by Murray Klamkin, 1988.

This book collects the problems of the first fifteen USA Mathematical Olympiads. While they are presented chronologically, the solutions are grouped according to subject matters, which facilitates using this book for training sessions. There is a very useful 10-page glossary of mathematical terms and results, and an extensive bibliography.

Problem 14

Let $a_1, a_2, a_3, \ldots$ be a non-decreasing sequence of positive integers. For an integer $m > 1$, define $b_m = \min\{n | a_n > m\}$, that is, b_m is the minimum value of n such that $a_n > m$. If $a_{19} = 85$, determine the maximum value of $a_1 + a_2 + \cdots + a_{19} + b_1 + b_2 + \cdots + b_{85}$.

Exploring Mathematics with your Computer, by Arthur Engel, 1993.

The author has served as the leader and coach of the formerly West German I.M.O. team for many years, and is an acknowledged expert on problem-solving. This volume is a mathematics book, and not a programming book, even though the computer language *Pascal* is explained, and an I.B.M. diskette comes with the package. Problems in number theory, probability, statistics, combinatorics and numerical analysis are explored.

Problem 15

We have 15 boxes, each one with its own key which fits no other box. After mixing the keys at random, we drop one into each box. Now we break open 2 boxes. What is the probability that we are now able to open the remaining boxes with keys?

Game Theory and Strategy, by Philip Straffin, 1993.

This book consists of three chapters, dealing with two-person zero-sum games, two-person non-zero-sum games and N-person games. These terms are explained in details. Many applications are included, into such diverse fields as anthropology, warfare, philosophy, social psychology, biology, business, economics, politics and athletics. The book also presents concrete models such as Jamaican fishing.

Problem 16

Two people are jointly charged for a crime. If both confess, each will get a light sentence. If one confesses and the other does not, the first will receive a reward while the second will get a heavy sentence. If neither confesses, both will go free. Explain why when considering the situation individually, it is better to confess, and yet considering the situation collectively, it is better not to confess.

F. Mir Publishers' Little Mathematics Library Series

This excellent series has become an unfortunate casualty of the demise of the former Soviet Union. Lost also are *Mathematics Can Be Fun* and *Fun with Math and Physics* featured in Section I.

G. Books from W. H. Freeman & Company, Publishers

Note that the two most recent books in Martin Gardner's Scientific American Series are Freeman publications. The first of the books listed below has actually gone out of print, but fortunately, Dover Publications Inc. has decided to pick it up.

The Puzzling Adventures of Dr. Ecco, by Dennis Shasha, 1988.

The title character calls himself an omniheurist, solver of all problems (mathematical). The narrative is by a Watsonesque companion, Prof. Scarlet. Ecco's clients range from government officials, industrialists, and eccentric millionaires to no less than the president of a Latin American country. They bring him important, instructive and interesting problems in discrete mathematics, all of which Ecco solves to their satisfaction. The book concludes with the mysterious disappearance of Dr. Ecco.

Problem 17

A counselor took some campers on a wilderness trip. At 100 minutes before sunset, they were lost at a four-way cross-road. It was known that their campsite was down one of the four paths, 20 minutes away on foot. Thus there was enough time for two exploratory trips by the counselor and the campers, before they had to reassemble at the cross-road and head down the correct path. However, three unidentified campers would not necessarily tell the truth. Nevertheless, the counselor was able to deduce the correct location of the campsite. What was the minimum number of campers, counting the three clowns, and how should the counselor organize the explorations?

Codes, Puzzles, and Conspiracy, by Dennis Shasha, 1992.

In this delightful sequel, Prof. Scarlet and friends were hot on the trail of the kidnappers of Dr. Ecco. They went on a globe-trotting tour into exotic locales, where they found time to tackle mathematical problems which cropped up everywhere. The story turns into a mathematical thriller, complete with a Moriarty-like arch-villain, and a nefarious character in high places, shades of Ollie North.

Problem 18

The Amazing Sand Counter claims that if sand is put into a bucket, he can tell at a glance how many grains there are. However, he will not tell you. How can you test whether he indeed has this power, without him telling you anything which you do not already know?

New Book of Puzzles, by Jerry Slocum and Jack Botermans, 1992.

The subtitle of this book is *101 Classic and Modern Puzzles to Make and Solve*. It is the long-awaited sequel to *Puzzles Old and New* by the same dynamic duo, reviewed in **Mathematics for Gifted Students I (MfGS1)**. As with the earlier volume, it is full of superb colorful illustrations and photographs by Botermans.

Problem 19

Construct a $3 \times 3 \times 3$ cube with the following six pieces, each of unit thickness.

Another Fine Math You've Got Me Into..., by Ian Stewart, 1992.

This whimsical volume is by the current occupant of the exalted position long held by Martin Gardner, the editor of the Mathematical Games column in Scientific American. It has sixteen chapters, from *The Lion, the Llama, and the Lettuce* to *Sofa, So Good ...*, each treating a mathematical problem in some depth.

Problem 20

Seven varieties of grapes are to be arranged in plots. Each plot contains exactly three different varieties. Is it possible that any two plots have exactly one variety in common, and that any two varieties lie in exactly one common plot?

H. Books from Dover Publications, Inc.

Excursions in Number Theory,
by Stanley Ogilvy and John Anderson, 1988.

This book covers the basics of classical number theory. Topics include prime numbers, congruencies, Diophantine equations and Fibonacci numbers. The narrative style is very soothing. It concludes with 20 pages of elaboration and commentary on some finer points raised in the text.

Problem 21

Do there exist two irrational numbers a and b such that a^b is a rational number?

Excursions in Geometry, by Stanley Ogilvy, 1990.

The first half of this book is on inversive geometry, and the second half on projective geometry. These two topics are linked by the concept of cross-ratio and the study of the conic sections. It is in the same style as the preceding volume.

Problem 22

Given are two parallel lines and a segment on one of them. Construct the midpoint of this segment using a straight-edge but without using a compass.

Excursions in Mathematics, by Stanley Ogilvy, 1994.

The original title of this volume was *Through the Mathescope*. The opening chapter is titled *What Do Mathematicians Do?* It is followed by lively tours of number theory, algebra, geometry and analysis. The last chapter is titled *Topology and Apology*.

Problem 23

You toss a fair coin 2n times, hoping to get n heads and n tails. What is the value of n for which your probability of success is the highest?

I. Individual Titles

Selected Problems and Theorems in Elementary Mathematics has been acquired by Dover and renamed *The USSR Olympiad Problem Book*. Dover has also picked up *The Moscow Puzzles. All the Best from the Australian Mathematics Competition* has now become the incipient volume in a new series. The Mathematical Association of America has published *Five Hundred Mathematical Challenges*, comprising the first five booklets of the project, *1001 Problems in High School Mathematics*.

The Canadian Mathematical Olympiad,
1969—1993, edited by Michael Dobb and Claude Laflamme, Canadian Mathematical Society, 1993.

This book combines two earlier volumes, *The First Ten Canadian Mathematics Olympiads, 1969–1978* and *The Canadian Mathematics Olympiads, 1979–1985. The contests from 1986 to 1993, each consisting of five questions also appear.*

Problem 24

A number of schools took part in a tennis tournament. No two players from the same school played against each other. Every two players from different schools played exactly one match against each other. A match between two boys or between two girls was called a *single* and that between a boy and a girl was called a *mixed single*. The total number of boys differed from the total number of girls by at most 1. The total number of singles differed from the total number of mixed singles by at most 1. At most, how many schools were represented by an odd number of players?

J. Addresses of Publishers

The Canberra College of Advanced Education has now become the University of Canberra. The publication of books is now done by the Australian Mathematics Trust. The address is still P.O. Box 1, Belconnen, ACT 2616, Australia

Part 2: Index of Martin Gardner's Scientific American Columns

Anthologies

1. The Scientific American Book of Mathematical Puzzles and Diversions
2. The Second Scientific American Book of Mathematical Puzzles and Diversions
3. New Mathematical Diversions from Scientific American
4. The Magic Numbers of Dr. Matrix
5. The Unexpected Hanging and other Mathematical Diversions
6. The Sixth Book of Mathematical Diversions from Scientific American
7. The Mathematical Carnival
8. The Mathematical Magic Show
9. The Mathematical Circus
10. Wheels, Life, and other Mathematical Amusements
11. Knotted Doughnuts and other Mathematical Entertainment
12. Time Travel and other Mathematical Bewilderments
13. Penrose Tiles to Trapdoor Ciphers
14. Fractal Music, Hypercards and more Mathematical Recreations

Note: The fifteenth and last volume should appear some time in 1996.

Monthly Columns

01	Dec 56	Hexaflexagon
01	Jan 57	Magic with a Matrix
01	Apr 57	Probability Paradoxes
01	May 57	The Icosian Game and the Tower of Hanoi
01	Jun 57	Curious Topological Models
01	Jul 57	The Game of Hex
01	Aug 57	Sam Loyd: America's Greatest Puzzlist
01	Sep 57	Mathematical Card Tricks
01	Oct 57	Memorizing Numbers
01	Nov 57	***Nine Problems
01	Dec 57	Polyominoes
01	Jan 58	Fallacies
01	Feb 58	Nim and Tac Tix
01	Mar 58	Left or Right
02	Apr 58	The Monkey and the Coconuts
02	May 58	Tetraflexagons
02	Jun 58	Henry Ernest Dudeney: England's Greatest Puzzlist
02	Jul 58	Digital Roots
02	Aug 58	***Nine Problems
02	Sep 58	The Soma Cube
02	Oct 58	Recreational Topology
02	Nov 58	Squaring the Square
02	Dec 58	The Five Platonic Solids
02	Jan 59	Mazes
02	Feb 59	Recreational Logic
02	Mar 59	Magic Squares
02	Apr 59	James Hugh Riley Shows, Inc.
02	May 59	***Nine Problems
02	Jun 59	Eleusis: The Induction Game
02	Jul 59	Origami
02	Aug 59	Phi: The Golden Ratio
02	Sep 59	Mechanical Puzzles
02	Oct 59	Probability and Ambiguity
03	Nov 59	Euler's Spoilers: An Order-10 Graeco-latin Square
03	Dec 59	Group Theory and Braids

***

04	Jan 60	—Dr. Matrix
03	Feb 60	***Eight Problems
03	Mar 60	The Games and Puzzles of Lewis Carroll
03	Apr 60	Board Games
03	May 60	Packing Spheres
03	Jun 60	Paper Cutting
03	Jul 60	The Transcendental Number Pi
03	Aug 60	Victor Eigen: Mathemagician
03	Sep 60	The Four-color Map Theorem
03	Oct 60	***Nine Problems
03	Nov 60	Polyominoes and Fault-free Rectangles
03	Dec 60	The Binary System

***

04	Jan 61	—Dr. Matrix
03	Feb 61	The Ellipse
03	Mar 61	The 24 Color Squares and the 30 Color Cubes
03	Apr 61	H. S. M. Coxeter
03	May 61	Mr. Apollinax Visits New York
03	Jun 61	***Nine Problems
03	Jul 61	Bridg-it and Other Games
03	Aug 61	The Calculus of Finite Differences
05	Sep 61	Knots and Borromean Rings
05	Oct 61	The Transcendental Number *e*
05	Nov 61	Geometric Dissections
05	Dec 61	Scarne on Gambling

***

05	Jan 62	The Church of the Fourth Dimension
05	Feb 62	***Eight Problems
05	Mar 62	A Matchbox Game-learning Machine
05	Apr 62	Spirals
05	May 62	Rotations and Reflections
05	Jun 62	Peg Solitaire
05	Jul 62	Flatlands
05	Aug 62	Tests of Divisibility
05	Oct 62	***Nine Problems
05	Nov 62	The Eight Queens and Other Chessboard Diversions
05	Dec 62	A Loop of String

***

04	Jan 63	—Dr. Matrix
05	Feb 63	Curves of Constant Width
05	Mar 63	The Paradox of the Unexpected Hanging
05	Apr 63	***Thirty-seven Quickies
05	May 63	Rep-tiles: Replicating Figures on the Plane
06	Jun 63	The Helix
06	Jul 63	Klein Bottles and Other Surfaces
06	Aug 63	Combinatorial Theory
06	Sep 63	Bouncing balls in Polygons and Polyhedrons
06	Oct 63	Four Unusual Board Games
06	Nov 63	***Nine Problems
06	Dec 63	Parity Checks

***

04	Jan 64	—Dr. Matrix
06	Feb 64	Sliding-block Puzzles
06	Mar 64	Patterns and Primes
06	Apr 64	Graph Theory
06	May 64	The Ternary System
06	Jun 64	*** Eight Problems
06	Jul 64	The Cycloid: Helen of Geometry
06	Aug 64	Mathematical Magic Tricks
06	Sep 64	Word Play
06	Oct 64	The Pythagorean Theorem
06	Nov 64	Limits of Infinite Series
06	Dec 64	Polyiamonds

***

04	Jan 65	—Dr. Matrix
06	Feb 65	Tetrahedrons
06	Mar 65	***Nine Problems
06	Apr 65	Infinite Regress
06	May 65	The Lattice of Integers
06	Jun 65	O'gara, The Mathematical Mailman
06	Jul 65	Op Art
06	Aug 65	Extraterrestrial Communication
07	Sep 65	Piet Hein's Superellipse
08	Oct 65	Polyominoes and Rectification
08	Nov 65	***Nine Problems
07	Dec 65	Magic Stars and Polyhedrons

***

04	Jan 66	—Dr. Matrix
07	Feb 66	Penny Puzzles
07	Mar 66	Aleph-null and Aleph-one
07	Apr 66	The Art of M. C. Escher
07	May 66	Cooks and Quibble-cooks
07	Jun 66	How to Trisect an Angle
07	Jul 66	The Numerology of Dr. Fliess
07	Aug 66	The Rising Hourglass and Other Physics Puzzles
07	Sep 66	Mrs. Perkins' Quilt and Other Square-packing Problems
07	Oct 66	Card Shuffles
07	Nov 66	Hypercubes
07	Dec 66	Pascal's Triangle

***

04	Jan 67	—Dr. Matrix
07	Feb 67	Jam, Hot and Other Games
07	Mar 67	***Nine Problems
07	Apr 67	Calculating Prodigies
07	May 67	Tricks of Lightning Calculations
08	Jun 67	Polyhexes and Polyaboloes
07	Jul 67	Sprouts and Brussels Sprouts
08	Aug 67	Factos

***

04	Jan 69	—Dr. Matrix
09	Feb 69	Boolean Algebra
09	Mar 69	Fibonacci and Lucas Numbers
09	Apr 69	***Seven Problems and Ten Quickies
09	May 69	Random Walks and Gambling
09	Jun 69	Random Walks on the Plane and in Space
09	Jul 69	Matches
09	Aug 69	Simplicity
09	Sep 69	Mascheroni Constructions
04	Oct 69	—Dr. Matrix
09	Nov 69	Patterns of Induction
09	Dec 69	Dominoes

***

09	Jan 70	The Abacus
09	Feb 70	***Nine Problems
09	Mar 70	Cyclic Numbers
09	Apr 70	Solar System Oddities
09	May 70	Optical Illusions
09	Jun 70	Elegant Triangles
10	Jul 70	Diophantine Analysis and Fermat's Last Theorem
09	Aug 70	Palindromes: Words and Numbers
10	Sep 70	Wheels
10	Oct 70	The Game of Life, Part 1
10	Nov 70	***Ten Problems
10	Dec 70	Nontransitive Dice and Other Probability Paradoxes

***

04	Jan 71	—Dr. Matrix
10	Feb 71	The Game of Life, Part 2
10	Mar 71	Alephs and Supertasks
10	Apr 71	Geometrical Fallacies
10	May 71	The Combinatorics of Paper Folding
09	Jun 71	Can Machines Think?
10	Jul 71	***Thirty-six Quickies
10	Aug 71	Ticktacktoe Games
10	Sep 71	Plaiting Polyhedrons
10	Oct 71	The Game of Halma
10	Nov 71	Advertising Premiums
10	Dec 71	Salmon on Austin's Dog

***

10	Jan 72	Nim and Hackenbush
04	Feb 72	—Dr. Matrix
10	Mar 72	Golomb's Graceful Graphs
10	Apr 72	***Nine Problems
10	May 72	Chess Tasks
10	Jun 72	Slithers, $3x + 1$, and Other Curious Questions
10	Jul 72	Mathematical Tricks with Cards
11	Aug 72	The Binary Gray Code
11	Sep 72	Polycubes
11	Oct 72	Coincidence
11	Nov 72	Bacon's Cipher
11	Dec 72	Doughnuts: Linked and Knotted

***

11	Jan 73	Sim, Chomp and Race Track
11	Feb 73	Elevators
11	Mar 73	Napier's Bones
11	Apr 73	Napier's Abacus
11	May 73	***Nine Problems
11	Jun 73	Crossin Numbers
11	Jul 73	Newcomb's Paradox
04	Aug 73	—Dr. Matrix
11	Sep 73	Point Sets on the Sphere
11	Oct 73	Look-see Proofs
11	Nov 73	Worm Paths
11	Dec 73	Waring's Problems

***

11	Jan 74	The I Ching
11	Feb 74	Cram, Bynum and Quadraphage
11	Mar 74	Reflections on Newcomb's Paradox
11	Apr 74	***Nine Problems
12	May 74	Time Travel
04	Jun 74	—Dr. Matrix
12	Jul 74	Hexes and Stars
12	Aug 74	Tangrams, Part 1
12	Sep 74	Tangrams, Part 2
12	Oct 74	Nontransitive Paradoxes
12	Nov 74	Combinatorial Card Problems
12	Dec 74	Melody-making Machines

***

12	Jan 75	Anamorphic Art
08	Feb 75	Nothing
12	Mar 75	***Eight Problems
12	Apr 75	Six Sensational Discoveries
12	May 75	The Csaszar Polyhedron
12	Jun 75	Dodgem and Other Simple Games
12	Jul 75	Tiling with Convex Polygons
12	Aug 75	Tiling with Polyominoes, Polyiamonds and Polyhexes
04	Sep 75	—Dr. Matrix
xx	Oct 75	Extrasensory Perception by Machine

12	Nov 75	Curious Maps
12	Dec 75	***Five Problems

***

12	Jan 76	Magic Squares and Cubes
12	Feb 76	Block Packing
12	Mar 76	Induction and Probability
xx	Apr 76	Snarks and Boojums
08	May 76	Everything
12	Jun 76	Catalan Numbers
12	Jul 76	Fun with a Pocket Calculator
12	Aug 76	Tree-plant Problems
13	Sep 76	Conway's Surreal Numbers
13	Oct 76	Back From the Klondike and Other Problems
04	Nov 76	—Dr. Matrix
13	Dec 76	Mandelbrot's Fractals

***

13	Jan 77	Penrose Tiling
13	Feb 77	The Oulipo
13	Mar 77	Wythoff's Nim
13	Apr 77	***Eight Problems
13	May 77	Mathematical Induction and Colored Hats
13	Jun 77	Negative Numbers
13	Jul 77	Cutting Shapes into *n* Congruent Parts
13	Aug 77	Trapdoor Ciphers
13	Sep 77	Hyperbolas
13	Oct 77	The New Elussis
13	Nov 77	Ramsey Theory
04	Dec 77	—Dr. Matrix

***

13	Jan 78	From Burrs to Berrocal
13	Feb 78	Sicherman Dice, Kruskal Count and Other Curiosities
13	Mar 78	Raymond Smullyan's Logic Puzzles
14	Apr 78	White, Brown, and Fractal Music
14	May 78	The Tinkly Temple Bells
14	Jun 78	Mathematical Zoo
14	Jul 68	Charles Sanders Peirce
14	Aug 78	Twisted Prismatic Rings
14	Sep 78	The Thirty Color Cubes
14	Oct 78	Egyptian Fractions
14	Nov 78	Minimal Sculpture
04	Dec 78	—Dr. Matrix

***

14	Jan 79	Tangent Circles
14	Feb 79	***Six Problems
14	Mar 79	Does Time Ever Stop? Can the Past be Altered?
14	Apr 79	Generalized Ticktacktoe
14	May 79	Psychic Wonders and Probability
14	Jun 79	Mathematical Chess Problems
14	Jul 79	Douglas Hofstadter's Godel, Escher, Bach
14	Aug 79	Imaginary Numbers
14	Sep 79	Pi and Poetry: Some Accidental Patterns
14	Oct 79	Packing Squares
14	Nov 79	Chaitin's Omega
xx	Dec 79	***Seven Problems

***

xx	Jan 80	Checkers
xx	Feb 80	The Coloring of Unusual Maps
xx	Mar 80	Graphs that can Help
xx	Apr 80	Fun with Eggs
xx	May 80	Combinatorial Designs
xx	Jun 80	The Monster Group
xx	Jul 80	Science and Technology in the Planiverse
xx	Aug 80	The Pigeonhole Principle
04	Sep 80	—Dr. Matrix
xx	Oct 80	The Mathematics of Elections
xx	Nov 80	Taxicab Geometry
xx	Dec 80	The Strong Law of Small Numbers

***

xx	Feb 81	Gauss's Congruence Theory
xx	Apr 81	***Seven Problems
xx	Jun 81	Geometrical Symmetries of Scott Kim
xx	Aug 81	The Abstract Parabola Fits the Concrete World
xx	Oct 81	Euclid's Parallel Postulate
11	Dec 81	The Laffer Curve

***

xx	Aug 83	Tasks You Cannot Help Finishing
xx	Sep 83	The Topology of Knots
xx	Jun 86	Casting a Net on a Checkerboard

Note:
Columns denoted by "xx" have not yet been anthologized in this series.

Part 3: An Extension of the Bibliography in MfGS1

K. Mathematics Competition Enrichment Series

As mentioned earlier, *All the Best from the Australian Mathematics Competition* has become the incipient volume of this new series, published by the Australian Mathematics Trust. The AMT was established by the late **Peter O'Halloran**, one of his most significant accomplishments among many outstanding achievements in a distinguished career. The Australian Mathematics Competition has become the model for many around the world. This series, under the leadership of **Graham Pollard** and **Peter Taylor**, will consolidate the Trust's leadership role in the world of mathematics competitions.

Mathematical Tool chest, edited by A. W. Plank and N. H. Williams, 1992.

This is a compilation, started by the late **Jim Williams**, of the basic results in mathematics most often called upon in competitions. It is organized into the following nine chapters: combinatorics, arithmetic & number theory, algebra, inequalities, analysis, plane geometry, solid geometry, analytic & vector geometry, and geometric transformations.

Problem 25

Nine real numbers are arranged in a circle. Their sum is 90. Prove that the sum of a block of four adjacent numbers is at least 40.

International Mathematics Tournament of the Towns: 1984-1989, edited by Peter Taylor, 1992.

For information about the Tournament, see Appendix II, which also features a sampling of Junior problems. This book contains one of the best collections of problems of all time. With a few exceptions, the solutions are worked out independently of the Russian proposers, with Edmonton student **Calvin Li** contributing quite a few.

Problem 26

Inside a rectangle is inscribed a quadrilateral, which has a vertex on each side of the rectangle. Prove that the perimeter of the inscribed quadrilateral is not smaller than double the length of a diagonal of the rectangle.

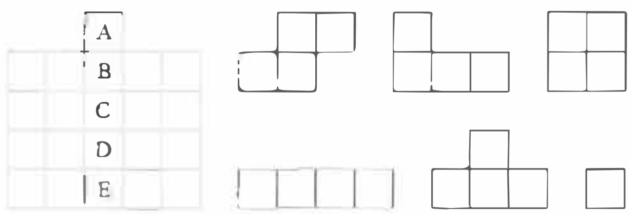
More of All the Best from the Australian Mathematics Competition, edited by Peter O'Halloran, Graham Pollard and Peter Taylor, 1992.

The earlier volume covers the AMC from 1976 to 1984, and the present one from 1985 to 1991. Here, the 483 problems are also grouped by subject.

Problem 27

A toy in six pieces, which can be turned over, came in a box as shown in the diagram. The unit square was in one of the marked spaces. This space was marked by the letter

(A) A (B) B (C) C (D) D (E) E



Problem Solving via the Australian Mathematics Competition, by Warren Atkins, 1992.

This book takes 149 problems from the Australian Mathematics Competition for 1978—1991 and develops strategies for their solutions. The problems are about Diophantine equations, counting techniques, speed, time and distance, and geometry. In each area, problems are presented, along with discussions and some solutions, with further solutions deferred to an appendix.

Problem 28

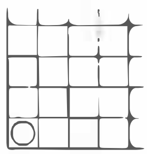
Warren and Naida have a straight path, 1 metre wide and 16 metres long, from the front door to their front gate. They decide to pave it. Warren brings home 16 paving stones, each 1 metre square. Naida brings home 8 paving stones, each 1 metre by 2 metres. Fortunately, they can get refunds on the unused paving stones. Assuming that they would consider using all rectangular or all square paving stones, or some of each, how many patterns are possible in paving the path?

International Mathematics Tournament of the Towns: 1980-1984, edited by Peter Taylor, 1993.

This volume appeared after the volume on the Tournaments from 1984 to 1989 because much time and energy were required to compile the older contests from the archives. The effort was amply rewarded with this book which is of the highest quality.

Problem 29

The entire first quadrant of the coordinate plane is divided into unit squares and serves as a checkerboard. The diagram shows a single counter inside the region consisting of six squares designated as home squares. In each move, we may replace a counter by two others, occupying the square to the north and to the east of its current position, provided that those squares are vacant. Is it possible to find a finite sequence of moves after which no counters remain on any home squares?



International Mathematics Tournament of the Towns: 1989-1993, edited by Peter Taylor, 1994.

This third volume on the Tournament covers only four years because the number of problems per year has increased. It is thicker than either of the other two books. Edmonton students **Jason Colwell, Peter Laffin, Steven Laffin, Calvin Li** and **Matthew Wong** all contributed solutions.

Problem 30

Find ten different positive integers such that each of them is a divisor of their sum.

The Asian Pacific Mathematics Olympiad, 1989-1993, edited by Hans Lausch, 1994.

This regional contest was another brainchild of the late **Professor Peter O'Halloran**. From a modest beginning with Australia, Canada, Hong Kong and Singapore in 1989, the number of participating countries had grown to thirteen in 1993. In addition to the problems, the book gives a detailed account of the organization of the contest.

Problem 31

Suppose there are 997 points given in the plane. If every two points are joined by a line segment with its midpoint colored red, prove that there are at least 1991 red points in the plane. Can you find a special case with exactly 1991 red points?

L. Enrichment Series from the Center for Excellence in Mathematics Education

The Center is the creation of **Alexander Soifer**, who emigrated from the former Soviet Union to the United States in 1978. He brought with him the tradition of the famed Mathematics Circles, and his own expertise in problem solving, particularly in combinatorial geometry.

Mathematics as Problem Solving, by A. Soifer, 1987.

This book has five chapters. The first one is on general techniques and useful tools in problem solving. The remaining four give illustrations from Number Theory, Algebra, Geometry and Combinatorics.

Problem 32

The numbers a, b and $\sqrt{a}+\sqrt{b}$ are all rational. Prove that so are $\sqrt{a}$ and $\sqrt{b}$.

How Does One Cut a Triangle, by A. Soifer, 1990.

Two principal problems are discussed in this book. The first is to find all positive integers n such that every triangle can be cut into n triangles congruent to each other. In the second, "congruent" is replaced by "similar". Necessary tools are developed along the way, and other related problems are raised.

Problem 33

Given nine points in a triangle of area 1, prove that three of them form a triangle of area not exceeding $\frac{1}{4}$.

Geometric Etudes in Combinatorial Mathematics, by V. Boltyanski and A. Soifer, 1991.

This book contains four chapters. The first one is on tiling rectangles with polyominoes. The second is on the application of the Pigeonhole Principle in Geometry. The third is on the Theory of Graphs. The fourth, which is quite substantial and more advanced, deals with Convex Sets and Combinatorial Geometry.

Problem 34

The V-tromino is a shape obtained from a 2×2 square by removing one of the four unit squares. Show how to pack 15 copies of the V-tromino into a 5×9 box.

Colorado Mathematical Olympiad, by A. Soifer, 1994.

The first part of the book contains the problems of the first ten olympiads, plus personal reminiscences of its early history. The second part contains the solutions and further explorations. Some of the problems are Russian folklore, while others are the creations of the author and his wide network of friends.

Problem 35

On a 3×3 chessboard, two white knights are on adjacent corners and two black knights are on the other two corners. Is it possible to move them so that the two knights of each color occupy opposite corners?

M. Birkhäuser Mathematics Series

This series contains three volumes to be published later on geometry, calculus and combinatorics. The first two volumes were written in Russian more than twenty-five years ago, and were the notes of a mathematics correspondence school in the former Soviet Union. This School was organized by **I. M. Gelfand** and continues to be directed by him to this date.

Functions and Graphs, by I. M. Gel'fond, E. G. Glagoleva and E. E. Shnol, 1990.

The important concept of a function is carefully introduced via examples. Graph sketching is explored using little more than high school algebra. The main part of the book is the study of the polynomial and rational functions, built up one step at a time, starting with the linear and quadratic functions. There are many exercises. Hints and answers are given to the more difficult ones.

Problem 36

Two roads intersect at a right angle. Two cars drive towards the intersection, one on the first road at a speed of 60 kilometres per hour and the other on the second road at a speed of 30 kilometres per hour. At noon, both cars are 10 kilometres from the intersection. At what moment will the distance between the cars be least? Where will the cars be at this moment?

The Method of Coordinates, by I. M. Gel'fond E. G. Glagoleva and A. A Kirilov, 1990.

Algebra and geometry, which most students today consider completely different subjects, are in fact quite closely related. The method of coordinates transforms geometric images into algebraic formulae. In the first part of the book, coordinate systems on the line, in the plane and in space are studied. In the second part, four-dimensional spaces are discussed.

Problem 37

A four-dimensional cube has 16 vertices. How many edges, two-dimensional faces and three-dimensional faces does it have?

Algebra, by I. M. Gelfand and A. Shen, 1993.

Starting from basic arithmetic, many important properties of the number system are molded into general results in algebraic terms. In seventy-two sections, the book takes the readers through the essential parts of high school algebra, including exponents, factorization, progressions, polynomial equations, and the often neglected topic of inequalities.

Problem 38

Let $p_1, p_2, \dots, p_n, q_1, q_2, \dots, q_n$ be real numbers. The famous Cauchy-Schwarz Inequality states that $(p_1^2 + p_2^2 + \cdots + p_n^2)(q_1^2 + q_2^2 + \cdots + q_n^2) \geq (p_1 q_1 + p_2 q_2 + \cdots + p_n q_n)^2$. Give a proof by considering the quadratic expression $(p_1 + q_1 x)^2 + (p_2 + q_2 x)^2 + \cdots + (p_n + q_n x)^2$.

N. Cambridge University Press Puzzle Books

Mathematical Amusement Arcade, by Brian Bolt, 1984.

This book contains 130 mathematics puzzles, many of which are taken from the popular literature. The statements are presented with attractive story lines, clear diagrams and amusing cartoons. The solutions are given in red, and comprise half the book.

Problem 39

Four points in the plane determine six distances. In six configurations, there are only two different values among these distances. One of them consists of the centre and the three vertices of an equilateral triangle. Find the other five configurations.

Mathematical Funfair, by Brian Bolt, 1989.

This book contains 128 mathematics puzzles.

Problem 40

Arrange the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9 in some order to form a nine-digit number such that it is divisible by 9, the number formed of the first 8 digits is divisible by 8, the number formed of the first 7 digits is divisible by 7, and so on.

Mathematical Cavalcade, by Brian Bolt, 1992.

This book contains 131 mathematical puzzles.

Problem 41

Sixteen points are given in a 4×4 array. Draw a broken line consisting of six line segments joined end to end, passing through each of the points at least once.

A Mathematical Pandora's Box, by Brian Bolt, 1993.

This book contains 142 mathematical puzzles.

Problem 42

Make a single straight cut through some of five squares each of area 1 and arrange the resulting pieces into a square of area 5.

Challenging Puzzles, by Colin Vout and Gordon Gray, 1993.

After 18 starter puzzles, the reader goes on an adventure with Snow White and the Seven Dwarfs (20 problems), visits the Isle of Maranga (28 problems), consults the Martian Dating Agency (18 problems) and finishes up by playing some sports and games (16 problems). Hints, solutions and further explorations are provided in separate sections.

Problem 43

Jock, Doc, Grumpy and Bossy were playing cards, one of which was lying on the table. "Who led that?" asked Grumpy. Bossy muttered something. Doc said, "Bossy led it." Jock said that Bossy had told him that Grumpy had led it. "No, it wasn't me or Jock," said Grumpy. "As I said before, it was Doc," said Bossy. In fact, only one of the dwarfs had been speaking truthfully. So who led that card?

O. The Spectrum Series of the Mathematical Association of America

This consists of a number of independent titles written for or acquired by the Mathematical Association of America. It contains over twenty volumes, including five titles from Martin Gardner's Scientific American Series.

The Last Problem, by Eric Temple Bell, 1991.

This book was originally published by Simon and Schuster in 1961, a year after the death of the author. While the central theme is Pierre Fermat and his Last Problem or Theorem, there are many historical references to western civilization in general and related mathematical problems in particular.

Problem 44

Find all positive integer solutions of $x^2 + 12 = y^4$.

Journey into Geometries, by Marta Sved, 1991.

In Part 1 of the book, the reader embarks on a geometric tour with Alice, Lewis Carroll and Dr. Whatif. The company is amicable, the conversation lively and the mathematics engaging. In Part 2, the tour is reexamined in a more formal setting, and solutions to the problems raised in Part 1 are given.

Problem 45

The power of a point P with respect to a circle with center O and radius r is defined as $OP^2 - r^2$. Prove that if a line passing through P cuts the circles at A and B, then the product of PA and PB is equal to the absolute value of the power of the point P.

Polyominoes, by George Martin, 1991.

While there is some overlap with Solomon Golomb's classic of the same title, reviewed later (see Section P), there is also interesting new material in this book, which can be read independently or as a supplement to the Golomb text.

Problem 46

Can a 6×6 square be tiled with 18 dominoes so that it is impossible to divide it into two subrectangles without splitting any dominoes?

The Lighter Side of Mathematics, edited by Richard Guy and Bob Woodrow, 1994.

This book reprints the proceedings of the Eugene Strens Memorial Conference on Recreational Mathematics and its history, held in 1986 at Calgary on the occasion of Richard Guy's seventieth birthday. This occasion also marked Guy's acquisition of the Strens collection of material on recreational mathematics on behalf of the University of Calgary. The contributed papers are grouped under Tiling & Coloring, Games & Puzzles, and People & Pursuits.

Problem 47

A county has five towns, A, B, C, D and E. There is a road connecting every pair of them. None of the roads meet except at the towns. Five are one-way paved roads, going from C to D, D to E, E to A, A to B and B to D. The other five are one-way country roads, going from D to A, A to C, C to E, E to B and B to C. A tourist is lost in a snow storm in one of the towns, and there is no one in sight. He cannot find any sign with the name of the town. The directions of the four roads are clearly marked, but they do not say where the road leads. The tourist phoned the Travel Bureau which is in D. The operator gives the sequence of instructions: "From wherever you are, sir, take the paved road leading out of town. When you get to the other end, take the country road leading out of that town, ..." Complete the sequence so that the tourist will end up in D, even though he may already be there or pass through it while following the instructions.

Circles: A Mathematical View, by Dan Pedoe, 1995.

This book was originally published by Pergamon Press in 1957 and later picked up by Dovers. In this third version, the author starts off with a new Chapter 0, introducing basic terminology and results about the circle which unfortunately have become increasing foreign to the current generation of students. The four chapters of the original work then follow, investigating in-depth the fundamental properties of this most pleasing curve, with explorations into inversive geometry, complex numbers and non-Euclidean geometry. A new appendix reprints an engaging story about the tragedies and triumphs of the short life of Karl Wilhelm Feuerbach. It was written by Laura Guggenbuhl in 1953 but is not generally known in the mathematics circle.

Problem 48

Using a compass without the straight-edge, construct the midpoint of a segment given only its two endpoints.

P. More Individual Titles

The first part of this section describes books related to mathematics competitions and Olympiads, and the second decribes books on popular mathematics.

Index to Mathematical Problems: 1980–1984, by Stan Rabinowitz, Math Pro Press, 1992.

This monumental volume contains all problems posed in a large number of mathematics journals with problem sections, as well as many mathematics competitions, in that five-year period. In this book only the statements are given, and they are classified systematically by subject. The book contains many useful indices with well-constructed cross-references. Among other things, they can be used to track down the solutions to the problems. The price for this large and valuable book is extremely reasonable.

Problem 49

Grandpa is 100 years old and his memory is fading. He remembers that last year — or was it the year before? — there was a big birthday party in his honor, each guest giving him a number of beads equal to his age. The total number of beads was a five-digit number x67y2, but to his chagrin he cannot recall what x and y stand for. How many guests were at the party?

An Olympiad Down Under, by the late Peter O'Halloran, Australian Mathematics Trust, 1988.

This is the report on the 29th International Mathematical Olympiad, held in Australia in conjunction with their bicentennial celebrations in 1988. It is one of the crowning achievements of the author, and shows an enormous organizational effort. It contains the six contest problems, all other problems that were submitted, as well as solutions.

Problem 50

In a multiple choice test, there were 4 questions and 3 possible answers for each question. A group of students was tested and for any 3 of them, there was a question which the 3 students answered differently. What is the largest possible number of students tested?

Chinese Mathematical Olympiads, 1986-1993, by C. Li and Z. Zhang, Chiu Chang Mathematics Publishers, 1994.

This English translation was donated by the publishers to the Organizing Committee of the 1994 International Mathematical Olympiad in Hong Kong, as gifts to the leaders and deputy leaders of all participating teams. This book can be obtained from the publishers directly as well as from the Australian Mathematics Trust.

Problem 51

PQRS is a convex quadrilateral inside triangle ABC. Prove that the area of one of the triangles PQR, PQS, PRS and QRS is not less than ¼ of the area of triangle ABC.

Mathematical Challenges, edited by the Scottish Mathematical Council, Blackie and Sons, 1989.

Since 1975–76, **David Monk** has run a Mathematical Challenge for Scottish students. It is a problem solving contest conducted by correspondence. The problems are at varying levels of difficulty, but all are very attractive. This book contains the first twelve contests.

Problem 52

Prove that the sum of the squares of five consecutive integers is never a perfect square.

Mathematical Challenge!, by Leroy Mbili, Mathematics Digest, 1978.

This little booklet contains, as suggested by its subtitle, *100 problems for the Olympiad enthusiast*, with a healthy dose of geometry.

Problem 53

AB and EF are equal segments on a line parallel to CD. AC intersects BD at P, and CE intersects DF at Q. Prove that PQ is parallel to CD.

Cariboo College High School Mathematics Contest Problems, 1973–1992, edited by Jim Totten, University College of the Cariboo, 1992.

This book contains over 1000 problems for students in Grades 8 through 12. There is a companion Solution Manual by Leonard Janke and Jim Totten.

Problem 54

ABFE, BCHG and CADJ are squares constructed outside triangle ABC. The combined area of the first two squares is one half that of the hexagon DEFGHJ. Prove that the area of ABC is equal to that of BFG and find $\angle ABC$.

The Art of Problem Solving - Volume 1, by Sandor Lehoczky and Richard Rusczyk, Greater Testing Concepts, 1993.

The book was written when the authors were senior undergraduate and beginning graduate students. The style is that of an informal discussion, with plenty of opportunities for on-hand experience in problem solving. The twenty-nine chapters cover most topics in the standard high school curriculum, along with many others that are often neglected. There is a companion Solution Manual by the same authors.

Problem 55

Let l, m and n be positive integers. Prove that their product is equal to their greatest common divisor times the least common multiple of lm, mn and nl.

The Art of Problem Solving - Volume 2, by Sandor Lehoczky and Richard Rusczyk, Greater Testing Concepts, 1994.

This book continues where the previous one left off, with another twenty-six chapters covering more advanced topics. There is also a companion Solution Manual by the same authors.

Problem 56

Let x, y and z be real numbers with $xyz = 1$.

Evaluate $\frac{1}{1+x+xy}+\frac{1}{1+y+yz}+\frac{1}{1+z+zx}$.

Problem Solving Through Problems, by Loren Larson, Springer-Verlag, 1983.

The first two chapters cover general methods in problem solving, and the remaining chapters cover their application in arithmetic, algebra, infinite series, introductory analysis, inequalities and geometry.

Problem 57

Let S be a set and $\otimes$ be a binary operation on S such that $x \otimes x = x$ for all x in S and $(x \otimes y) \otimes z = (y \otimes z) \otimes x$ for all x, y and z in S. Prove that $x \otimes y = y \otimes x$ for all x and y in S.

The Green Book, by Kenneth Williams and Kenneth Hardy, Integer Press, 1985.

The subtitle of the book is *100 practice problems for undergraduate mathematics competitions*, but many problems are at the high school level. Some are original compositions while others are compiled from various sources. Hints and solutions are provided.

Problem 58

Prove that the equation $x^4 + y^4 + z^4 - 2y^2z^2 - 2z^2x^2 - 2x^2y^2 = 24$ has no solution in integers x, y and z.

The Red Book, by Kenneth Williams and Kenneth Hardy, Integer Press, 1988.

This book has the same subtitle as the Green Book. There are no geometry problems in either.

Problem 59

Prove that there exist infinitely many positive integers which are not expressible in the form $n^2 + p$ where n is a positive integer and p is a prime.

Principles and Techniques in Combinatorics, by C. C. Chen and K. M. Koh, World Scientific, 1992.

Although intended principally as an undergraduate textbook on combinatorics, the book can also be used for a problem solving course in the subject. It has a wide selection of problems from various journals and contests.

Problem 60

Six scientists are working on a secret project. They wish to put the documents in a cabinet with many locks, all of which must be open before the document can be retrieved. Each scientist is given a number of keys, such that the cabinet can be opened if and only if at least three scientists are present. What is the smallest number of locks required, and what is the smallest number of keys each scientist must carry?

The Japanese Temple Problem Book, by Hidetosi Fukagawa and Dan Pedoe, Charles Babbage Research Center, 1989.

This book contains a selection of problems in Euclidean geometry displayed on *Sangaku*, which are mathematical tablets which were hung under the roofs of shrines or temples in Japan during the Edo period (1603—1867). Solutions to selected problems are given in the second part of the book, which also contains photographs of some Sangaku, plus other historical references.

Problem 61

Two circles touch each other externally and an external common tangent touches them at the points A and B respectively. Prove that AB^2 is equal to four times the product of the radii of the circles.

Polyominoes — Puzzles, Patterns, Problems and Packings, by Solomon W. Golomb, Princeton University Press, 1994.

This is the long awaited reprinting of the definitive treatise, originally published in 1965 by Charles Scribners Sons. It contains two new chapters, two new appendices and a vastly expanded bibliography section. Written by the founder of the subject, this book is a must for all who are interested in polyominoes, either for recreation or as a subject for serious research work.

Problem 62

It is well known that if two opposite corners were removed from a chessboard, the remaining part cannot be covered by 31 dominoes. This is because the two cells removed are of the same color. If two cells of opposite colors are removed, can the remaining part of the chessboard always be covered by 31 dominoes?

Dictionary of Curious and Interesting Numbers, by David Wells, 1986.

The first entry is -1 and i, and the remaining entries are real numbers in increasing order. The second entry 0 is followed immediately by Liouville's number $10^{-(1!)} + 10^{-(2!)} + 10^{-(3!)} + \cdots$. Each entry contains useful information about each number's mathematical properties as well as its place in the history of mathematics.

Problem 63

Let $\sigma(n)$ denote the sum of all positive divisors of the positive integer n. It is easy to show that neither 2 nor 5 is equal to $\sigma(n) - n$ for any positive integer n. Find the next integer with this property.

Dictionary of Curious and Interesting Geometry, by David Wells, 1991.

The entries are mostly geometric objects and theorems named after famous mathematicians, such as the Euler line and Menelaus' Theorem. Terms are clearly defined and relevant information is provided, with references where appropriate. The whole book is profusely illustrated with well-drawn line diagrams. It is a fertile ground for exploration in geometry.

Problem 64

Prove Viviani's Theorem, which states that in an equilateral triangle, the sum of the perpendiculars from any point to the sides is equal to the altitude of the triangle.

Book of Curious and Interesting Puzzles, by David Wells, 1992.

This is a compilation of 568 puzzles from various sources, including quite a few ancient civilizations from around the world. The second half of the book contains the answers and solutions.

Problem 65

Some rabbits and chicken have among them 35 heads and 94 feet. How many of each kind of animal are there?

Mathematics in Education, edited by Themistocles Rassias, University of La Verne Press, 1992.

This volume is published as part of the centennial celebration of the University of La Verne. It contains 10 articles, 8 research notes plus a problem section.

Problem 66

Prove that for any integer $n > 1$, there exists a permutation $(a_1, a_2, \ldots, a_n)$ of $\{1, 2, \ldots, n\}$ such that for $1 \le k < a_1 + a_2 + \cdots + a_k$ is divisible by a_{k+1}.

Hoffmann's Puzzles Old and New, edited by Edward Hornden, L. E. Hornden, 1993.

This is a modern edition of the original work published in 1893 by Frederick Warne and Company. It was the first book which featured mechanical puzzles prominently, and it attempted to catalog all that were known at the time. The new edition adds color pictures of many of the now rare puzzles.

Problem 67

Arrange nine counters so that there are ten rows each with three counters.

The Book of Ingenious and Diabolical Puzzles, by Jerry Slocum and Jack Botermans, Times Books (a division of Random House), 1994.

In this third book by the same two authors, the puzzles are grouped into chapters according to an earlier version of Jerry Slocum's Taxonomy reproduced later in this issue (see Appendix IV).

Problem 68

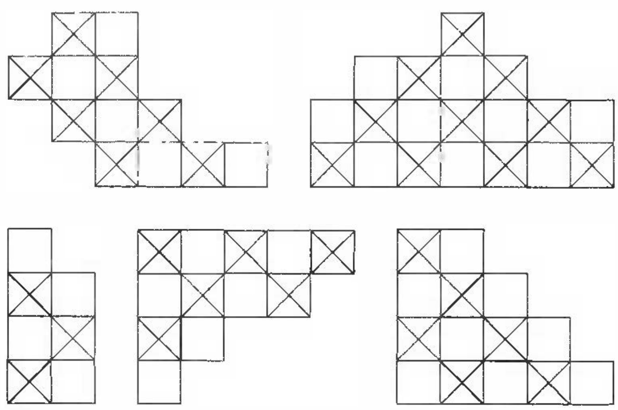
There are 10 pentacubes which consist of more than one layer, have no planes of symmetry and do not fit in a $2 \times 2 \times 2$ box. Use them to construct a $2 \times 5 \times 5$ box.

Compendium of Checkerboard Puzzles, by Jerry Slocum and Jacques Haubrich, Slocum Puzzle Foundation, 1993.

This is a comprehensive listing of dissection problems organized according to the number of pieces into which the checkerboard is divided. The number of distinct solutions to each problem is given, and if it is not too large, all solutions are featured.

Problem 69

Use the following five pieces to form a checkerboard.

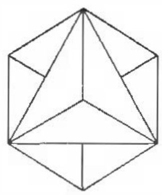


Unit Origami, by Tomoko Fusé, Japan Publications (Canadian distributors Fitzhenry & Whiteside), 1990.

This book introduces a new twist to the popular pastime of origami, or paper folding. The underlying idea is to build many copies of relatively simple building blocks, which are then put together like Lego pieces into fantastically complex designs. Full instructions are given, illustrated with line drawings, and there are also color photographs of many finished products. The book is also carried by Key Curriculum Press.

Problem 70

The diagram shows what appears to be three interlocking squares, defining the skeleton of a regular tetrahedron. Construct it using two pieces of origami paper of each of three colors, all six of the same size and folded in an identical manner. No cutting or gluing is allowed.



Exploring Math Through Puzzles, by Wei Zhang, Key Curriculum Press, 1996.

This book describes a number of mechanical puzzles and explains how they can be constructed and used in the classroom setting.

Problem 71

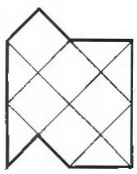
Find all hexominoes which can be folded into empty cubes.

Super-Games, by Ivan Moscovitch, Hutchinson, 1984.

This book consists mainly of dissection puzzles, and is full of superb color illustrations.

Problem 72

Dissect the octagon in the diagram into four pieces congruent to one another, in two different ways.



Q. More Addresses of Publishers

Birkhäuser,
675 Massachusetts Avenue,
Cambridge, MA, USA, 02139.

Charles Babbage Research Center,
P.O. Box 272,
St. Norbert Postal Station,
Winnipeg, MB, R3V 1L6.

Center for Excellence in Mathematics Education,
885 Red Mesa Drive,
Colorado Springs, CO, USA, 80933.

Chiu Chang Mathematics Publishers,
4/F, #3, Alley 15, Lane 147,
Section 3, Shin-Yi Road,
Taipei, Taiwan.

Fitzhenry & Whiteside,
195 Allstate Parkway,
Markham, ON, L3R 4T8.

Greater Testing Concepts,
P. O. Box A—D, Stanford,
CA, USA, 94309.

Hutchinson Publishing Group,
17—21 Conway Street,
London, W1P6JD,
United Kingdom.

Integer Press,
P.O. Box 6613,
Station J, Ottawa, ON, K2A 3Y7.

Key Curriculum Press,
P.O. Box 2304,
Berkeley, CA, USA, 94702.

L. E. Hornden,
Cane End House,
Cane End, Reading,
RG49HH, United Kingdom.

Math Pro Press,
P.O. Box 713,
Westford, MA, USA, 01886-0021.

Mathematics Digest,
Department of Mathematics,
University of Cape Town,
7700 Rondebosch, South Africa.

Penguin Books,
10 Alcorn Avenue,
Toronto, ON, M4V 3B2.

Slocum Puzzle Foundation,
257 South Palm Drive,
Beverly Hills, CA, USA, 90212.

Springer-Verlag,
175 Fifth Avenue,
New York, NY, USA, 10010.

University College of the Cariboo,
Box 3010,
Kamloops, BC, V2C 5N3.

University of La Verne Press,
1950 Third Street, La Verne,
CA, USA, 91750.

World Scientific,
Suite 1B, 1060 Main Street,
River Edge, NJ, USA, 07661.