In this new section, we will share your points of view on teaching mathematics and your responses to articles. We appreciate the interest and value the views of those who write. The December 1995 issue of delta-K *(vol. 33, number 1) contained a student submission entitled "Linda's Trisection" (p. 18) in which a student, Linda Chiem, demonstrated the trisection of an angle using only a compass and straightedge. This prompted the following reaction from Professor Michael G. Stone.*

Examining the Impossible

Michael G. Stone

"Trisecting the angle does not compute."
—*Old Vulcan saying*

One of the most well-known impossibilities in mathematics is the trisection of an angle with compass and straightedge. In particular, it is impossible to trisect a 60° angle using only these tools. Roughly speaking, this is because the available operations will only allow us to construct from a unit length only all of those lengths which can be obtained by arithmetic operations and square roots. For a fascinating, yet simple, account of this and other mathematical impossibilities, see John Paulos' (1991) *Beyond Numeracy: Ruminations of a Numbers Man*. For a more detailed account, see Howard Eves' (1976) wonderful *An Introduction to the History of Mathematics*.

Linda Chiem's (December 1995) article, "Linda's Trisection," provides an excellent forum to promote classroom discussion of what is meant by the impossibility of such a construction. Certainly, some angles can be trisected easily (for example, 90°), although not by the method here! But not all angles can be trisected, and the construction given here, in particular, fails to do so. Perhaps the easiest way to do this is to add to the figure used in Linda's Trisection of the angle with centre B and radius BM. Then consider the triangles BD′M, BMN and BNE′, where D′ and E′ are the points where BD and BE meet the circle. If the angle were truly trisected by the given construction, these angles would all be congruent (side/angle/side = side/angle/side). However, they are clearly not congruent in general, as a little experimentation with very large angles will reveal.

Can you find the angles for which Linda's Trisection will not work? (Hint: What is the sum of the angles in triangle BMD′? Note that triangle MD′D is similar to BM′D.)

Don't be discouraged, Linda, by finding a flaw in your proof. Every working mathematician has had similar experiences! Each time we discover an error in our reasoning there is an opportunity to learn something new which strengthens our intuition. Here there is something to be learned about the way that arc and span are related. To extend this discussion to an analysis of a proof that you cannot [sic] bisect an angle see Eric Chandler's article in *Fallacies, Flaws, and Flim Flam*, an issue of *College Mathematics Journal* (1995) edited by Ed Barbeau. For more about capitalizing on errors to turn these opportunities into learning experiences, read "Capitalizing on Errors as 'Springboards to Inquiry' " by Raffaella Borasi.

References

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Editor's note: The trisection of an angle is also discussed in D. E. Smith's History of Mathematics *(New York: Dover: 1958, pp. 297–300). The problem of trisecting any angle with straightedge and compass alone was proved impossible by P. L. Wantzel in 1847. Any angle can be trisected, however, in several ways, for instance, by the use of a protractor, the limaçon of Pascal (that is, Etienne Pascal, the father of Blaise Pascal), the conchoid of Nicodemes or the trisectrix of Maclaurin.*