Apply the Curriculum Standards with Project Questions

Richard T. Edgerton

A goal of the Curriculum and Evaluation Standards for School Mathematics (NCTM 1989) is facilitating "mathematical power" in students. The curriculum standards use problem solving, communication, reasoning and connections as organizing principles. One way to apply these principles in the classroom is with the use of "project questions."

A project question is a real-world problem on which a student or group of students work over the course of many days. Different from an exercise, a project question

- · has multiple solution methods,
- has no obvious solution sequence,
- · requires some hands-on data collection, and
- affords an opportunity for students to explore various topics within and outside mathematics.

I currently use project questions as one of many ways students may earn extra credit. When students unavoidably miss a quiz or think that their grade needs bolstering, we discuss the possibility of using a project question. After a question is agreed on through student-teacher dialogue, a contract is signed. (See Figure 1 for a sample contract.) I require all written submissions to be accompanied by the signature of the supervising parents or guardians and of all students who expect to collaborate on a question.

Project-Question Evaluation

Evaluation of project questions involves the examination of the *process* that students take in exploring the problem rather than just their product. Frequent discussions between teacher and student are important. I usually meet with students about three times while they work on their question. This approach helps the students stay on track and permits me to monitor their progress. I base credit for project questions on the knowledge students gain in the process rather than the correctness of their final product. I expect students to learn a great deal by working on a project question even if they never fully solve it. Because I check their progress and have them explain their results to me before their presentation, my students always earn full credit.

Figure 1 Sample Contract

"Project questions" are intended to give an opportunity to explore concepts of classwork by working on and, it is hoped, solving a real-world problem. The nature of the problem as well as the due date and point value are negotiated in advance.

To receive credit, the student must perform the bulk of the work on the problem himself or herself (or themselves if working in a group). No one outside the student's household who has been enrolled in college classes may help with the question. All persons helping with the question must be listed in the report, which is to be submitted as the answer. A written report must accompany an oral presentation of the findings on the due date. Your written report must include

- answers to the questions on the "Project-Question Assessment Sheet" (to be filled out after your presentation),
- your procedure for gathering data,
- · your approach to analyzing the data,
- your conclusion,
- each person's contribution to the project,
- things you would do differently next time on a project of this kind and
- the way in which credit should be divided among the group members.

Student(s):
Question:
Point value:
Date given:
Date of formative discussion:
Date of first results:
Date of final draft:
Date due:
Describe how you plan to present your results:
Signatures
Student(s):
Supervising parents or guardians:
Teacher:

The final step in working on a project question is the presentation of results. A short oral report is given to the class after which the class has the opportunity to ask questions concerning the project. A written report is submitted at the time of the oral report. The written report must include

- (a) an outline of each participant's role in the solution process,
- (b) outside resources (such as parents, friends and neighbors) used,
- (c) conception and refinement of the question,
- (d) data-gathering techniques,
- (e) data-analysis techniques,
- (f) the apparent solution and
- (g) things the group would do differently on future questions.

The time of presentation is usually one of celebration for the solution team and the class. At this time I also have the presenter and students complete a short evaluation form regarding the presentation (see Figure 2). Both groups typically enjoy the presentations: they are a break from their usual class activities, they are informative and they demonstrate some usefulness for mathematics.

Figure 2			
Project-Question Assess	sment \$	Sheet	

Name:	_
Date:	_
Project question:	_
Presenter(s):	_

1. What did you learn while observing this presentation?

2. Please circle the numeral that best represents your assessment of the presenter(s) and his/ her/their performance.

The presenter(s) was (were):

1 2 3 4 5 6 7 8 9 10				
poorly prepared well prepared				
The pace of the presentation was:				
1 2 3 4 5 6 7 8 9 10				
too slow/fast good				
The discussion was: 1 2 3 4 5 6 7 8 9 10				
unclear clear				
The question was: 1 2 3 4 5 6 7 8 9 10				
explained poorly explained well				
The answer was: 1 2 3 4 5 6 7 8 9 10				
explained poorly explained wel				
Do you want your responses to be kep				
confidential? Yes No				

Bob's Question

Bob was enrolled in a first-year integrated mathematics class at a four-year high school. After observing another student's success with a project question, Bob asked for one. From previous discussions, I knew Bob enjoyed baseball, so I asked him if he wanted his question to relate to baseball in some way. He was delighted but had no idea how baseball and mathematics could mix. After a few dead ends with such topics as batting averages, we centred on properties of a baseball itself. I asked Bob if he knew at what angle to throw a baseball to make it travel the farthest. Bob had no idea but guessed that a maximum distance would result from throwing at an angle of elevation of 20 degrees. After further discussion, Bob agreed to make this investigation his project question. The initial discussions that resulted in Bob's questions took about 30 minutes.

Bob and I discussed how he might make a slingshot from surgical tubing to ensure uniformity of throws. To obtain a better estimate, we agreed that he should measure each "throw" at least three times. Bob signed a contract to answer the question for a value equal to that of a major quiz and agreed to complete the question within two weeks.

The next day, Bob returned with a completed table of data (see Table 1) and his answer. I noticed from his data table that he performed four trials instead of the suggested three, about which he said, "I figured that doing four would make a better average." Bob spent the next two weeks writing the results of his experiment. During his oral report, Bob described the study well and included two very descriptive graphs. The first graph (see Figure 3) plotted the ball's distance against the angle of the launch. This graph adequately showed the relationship that he found from the data. Since Bob's reports were handwritten, the tables and graphs herein have been modified for clarity. Only the basic appearance of the graphs and tables has been changed.

Bob's second graph was more interesting than the first. Although this tactic was not previously discussed, Bob graphed the relative flight distances along with the angle of the launch (see Figure 4). Rays were drawn at their respective launch angle to show the relative distances that the ball traveled. This clever view was an even more meaningful representation of the data because each ray represented two pieces of information. During Bob's oral presentation, I made a point of telling the class how his innovative graph clarifies the data of his experiment. Table 1

Bob's Table of Horizontal Distance Traveled by a Baseball Launched at Different Angles

Degrees	0	30	45	60	65	70	80	90
Trial 1	9	16.8	19.7	22.8	28.2	24.8	9.3	1.7
Trial 2	8.7	17.6	18.6	23.3	27.8	25.3	9	2
Trial 3	9.2	17.3	19.2	23.8	28.3	25.7	8.7	1.5
Trial 4	8.6	17	19.4	23.7	28.7	26	9.4	2.2
Average	8.9	17.2	19.2	23.4	28.3	25.5	9.1	1.9

One fact that I did not tell the class about Bob's answer was that it was not significantly close to the answer derived by either ballistics or physics. Although I am not sure what the correct answer should have been, I am certain that neither discipline would support his answer of 65 degrees. I chose not to mention that the "correct" answer should be closer to 55 degrees (45 degrees if no air resistance were involved) because Bob's process was correct-he just needed more accurate instrumentation and a stronger sling to increase the distance. Bob's data also showed horizontal distance for both 0 degrees and 90 degrees, which could not have been true in reality. I felt that if I were to point out the weaknesses of his experiment, I would reduce the importance what he had done and reinforce the conventional notion of the existence of one correct answer known by the teacher. Bob learned how to test a hypothesis scientifically and how to report experimental findings. To me, his process was worth much more than being significantly close to an ideal solution.

Summary

The project question was a powerful application of the NCTM's curriculum standards for Bob. He had an enjoyable time working on his question and gained confidence in his mathematical understanding. I believe that his classmates also benefited from hearing about the process and seeing new ways to think about mathematics.

The autonomy that Bob enjoyed helped him to experience mathematics in a participatory, occasionally frustrating mode. Project questions do not lend themselves to the neatness of traditional approaches, but guidance from the teacher during the problemsolving process helps to keep students' frustration under control while still allowing students to work independently. In evaluating project questions, the process is more important than the product. Figure 3

Bob's Graph of Travel of a Baseball Launched at Different Angles

(The line segments have been added only to make the relationship between the discrete data points clearer.)



Figure 4 Bob's Graph of a Ball's Flight Relative to the Ground



Here are a few concluding thoughts about the use of project questions:

- Have students record their initial guess for a question, including hypothetical tables and graphs if possible.
- Start small by involving only a few students at a time.
- Be certain that students and parents know at the beginning the question and expectations for its solution.

- Be sure to detail the people who may be involved and to what extent.
- Indicate possible trivial solutions and ways of achieving the detail you want.
- Let other teachers in your school know your plans.
- Work to keep the lines of communication open among yourself, your students and their parents.

Reflecting on my initial use of project questions, I plan to increase the level of students' involvement in various ways. I will eventually have all students work on project questions as part of their coursework. I will have students define the problem and take more responsibility in writing the actual question.

Project questions give students opportunities to exercise their powers of reasoning, create critical mathematical connections, communicate mathematics with others and experience problem solving in a natural setting. Such questions are an ideal way to apply the NCTM's curriculum standards in the mathematics classroom.

Sample Project Questions

- What is the relationship between the wattage of light bulbs and their luminosity?
- What is the relationship between the length of a person's forearm and the length of his or her foot?
- How fast does hair grow?
- Which is steeper—the stairs in the school or the route to the summit of Mount Everest?
- If the Earth were a solid rock, into how many grains of sand would it be split?
- What is the next day for which an object's shadow is equal to the object's height at noon? (Note: not all latitudes will have a solution to this question; the latitude at which solutions begin is an interesting investigation, and the actual "critical latitude"

is different from the mathematical result because of the refraction of light by air.)

- At what speed do raindrops fall?
- How much energy do you use each day?
- How much trash do you produce each day?
- How long will it take for a class computer to print each number from one to one million?
- Which variety of firewood heats most efficiently?
- How many BBs would be required to make a life-sized statue of yourself and how much would that statue weigh?
- How many names are in the white pages of the local telephone book?
- How many times will your heart beat during your lifetime?
- How unusual is it for a person your age to have exactly 28 natural teeth with no fillings?
- For how many hours would a person have to mow lawns to get a pile of grass clippings the same height as the world's tallest building?
- How tall is a stack of one million dollar bills?
- What is the area of your skin?
- How many sugar cubes would you need to make a scale model of your house?

Reference

NCTM. Curriculum and Evaluation Standards for School Mathematics. Reston, Va.: Author, 1989.

Reprinted with permission from the NCTM publication The Mathematics Teacher 86, no. 8 (1993): 686–89. Minor changes have been made to spelling and punctuation to fit ATA style.

A polygon with seven sides is called a hooligan.