The Probability of Winning and Losing at Craps and Roulette

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One of the most popular betting games around seems to be the game of craps. We can use this to our advantage in the classroom by teaching something in which our students will have an interest.

Craps is a challenging game because it involves finding the sum of an infinite geometric series. It is played by having the player throw two dice. If a sum of 7 or 11 appears, the player automatically wins. The player loses immediately if the total is 2, 3 or 12. But if the total is any one of the remaining six possible sums (a 4, 5, 6, 8, 9 or 10), the player neither wins nor loses on the first throw, but continues to roll the dice until he/she either duplicates the first throw or gets a sum of 7.

The total shown by the dice on the player's first throw is called his/her point. If that player throws his point next, he wins. If he throws a 7 next, he will lose.

It is well known that when craps is played in gambling houses, the house never throws the dice. We may therefore feel reasonably certain that the odds are against the player who is throwing the dice.

The set of all possibilities of getting one of the numbers is as follows:

2	3	4	5	6	7	8	9	10	11	12
(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)					
	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)				
		(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)			
			(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)		
				(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)	
					(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

We then obtain the following probabilities for the various throws:

Total of Throw	Probability
2 or 12	1/36
3 or 11	2/36 or 1/18
4 or 10	3/36 or 1/12
5 or 9	4/36 or 1/9
6 or 8	5/36
7	6/36 or 1/6

We see at once that the probability that the player throwing the dice will win on the first throw (getting a 7 or 11) is

$6/36 + 2/36 = 8/36 = 2/9.$

The probability that he/she will definitely lose (that is, get a 2, 3 or 12) is

$1/36 + 2/36 + 1/36 = 4/36 = 1/9.$

The chance that he/she will either win or lose on the first throw is then

$2/9 + 1/9$ or $3/9 = 1/3$.

Therefore, the probability that the first throw will not be decisive is 2/3.

Suppose the shooter's point is 4, for example, an event with probability 1/12. Then on the second roll the conditional probability of rolling another 4 is 1/12, the conditional probability of losing immediately by rolling a 7 is 1/6, and thus the probability of no decision on the second roll is 3/4. Hence the probability of winning, given that the shooter's point is 4 is

$$\frac{1}{12}+\frac{3}{4}\times\frac{1}{12}+\left(\frac{3}{4}\right)^2\times\frac{1}{12}+\left(\frac{3}{4}\right)^3\times\frac{1}{12}+\ldots=\frac{1}{12}\left(\frac{1}{1-\frac{3}{4}}\right)=\frac{1}{3},$$

the sum of the probabilities of winning on the 2nd, 3rd, 4th, . . . rolls of the dice.

Similarly, the probability of winning, given that the shooter's point is 5 is found to be

$$\frac{1}{9}+\frac{13}{18}\times\frac{1}{9}+\left(\frac{13}{18}\right)^2\times\frac{1}{9}+\left(\frac{13}{18}\right)^3\times\frac{1}{9}+\ldots=\frac{1}{9}\left(\frac{1}{1-\frac{13}{18}}\right)=\frac{2}{5},$$

and the probability of winning, given that the shooter's point is 6 is 5/11.

So the total probability of the shooter's winning is

P(7 or 11 on the 1st roll) +
P(4 on 1st roll) × P(win | 4 is point) +
P(5 on 1st roll) × P(win | 5 is point) +
P(6 on 1st roll) × P(win | 6 is point) +
P(8 on 1st roll) × P(win | 8 is point) +
P(9 on 1st roll) × P(win | 9 is point) +
P(10 on 1st roll) × P(win is point) =

$$\frac{2}{9}+\frac{1}{12}\times\frac{1}{3}+\frac{1}{9}\times\frac{2}{5}+\frac{5}{36}\times\frac{5}{11}+\frac{5}{36}\times\frac{5}{11}+\frac{1}{9}\times\frac{2}{5}+\frac{1}{12}\times\frac{1}{3}=\frac{244}{495}.$$

The probability of the shooter's losing is therefore

$$1-\frac{244}{495}=\frac{251}{495}.$$

The game is only very slightly unfavorable to the shooter, with the expected payoff for an even-money bet of \$1 being

$$\$1 \times \frac{244}{495} + \$(-1)\frac{251}{495} = -\$.014\overline{14}$$

—a loss of less than 1½ cents per game. The reason so much money changes hands in a craps game, of course, is that many large bets may be made for or against the shooter and the game usually requires less than a minute to reach a decision.

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The probability of calculating the odds in the game of roulette is also suitable for a student of elementary probability and will provide another interesting application for the classroom.

The roulette wheel contains 38 slots, of which two are numbered 0 and 00, and the rest are numbered from 1 through 36. Half of the numbers 1 through 36, (not including 0 and 00), are red, and the other half black.

The player can bet on red, black, odd numbers, even numbers or any of the 38 numbers.

If the player wins on red, black, odd or even, the house plays even (the amount of money the player bet). If the player wins on one of the 38 numbers, the house pays 36 times the money wagered.

The probability of winning on even is 18/38, because half of the 36 numbers are even. The probability of winning on odd is also the same. The same holds true for betting on red or black, because the 0 and 00 are not colored.

The probability of winning when betting on one of the numbers is 1/38, and this is where the "house" gets its odds because the house pays only 36 times the amount bet, not 38.

Bibliography

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A student of statistics defines a census taker: "A census taker is a man who goes from house to house increasing the population."