Assessing Mathematics Learning for Students with Learning Differences

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The *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989) advocates the alignment of the mathematics curriculum with instructional practices and assessment techniques. The authors clearly understood that this alignment would not occur without expanding the notion of assessment and making the process of assessment more meaningful for all students. Consequently, the guidelines for the evaluation of mathematics became a significant part of the curriculum standards and stated the following:

- Student assessment must be integral to instruction.
- Multiple means of assessment should be used.
- All aspects of mathematical knowledge and its connections should be assessed.

Teachers should be paying more attention to assessing what students know about mathematics and spending less time determining what they do not know. This attention is especially true for students who have experienced difficulty in learning mathematics. Constant reminders of failure only lead to low self-esteem, which can lead to lower achievement. Perhaps this downward spiral can be stopped by changing the emphasis of assessment from checking only for the correct answer to recording what students *know*, how they *think* about mathematics and how they *apply* mathematics to real-world problems.

Many students in our schools have learning problems in the area of mathematics. Many of these students are labeled handicapped and at-risk for school failure. These students may exhibit deficits in computational skills, spatial awareness, understanding of mathematical concepts, problem solving, and memory of procedures and strategies. A larger and larger percentage of these students are being taught in regular classrooms for all or most of the school day. Educators (Gartner and Lipsky 1987; Lilly 1988; Reynolds, Wang and Walberg 1987; Stainback and Stainback 1987; Wang, Reynolds and Walberg 1986) have advocated that adaptive instructional strategies be used to help these students succeed in regular classrooms. More recently, researchers have proposed that educators working with disabled students must adapt instruction in mathematics to that proposed by the curriculum standards (Cawley, Baker-Kroczynski and Urban 1992). However, adapting instruction alone is not sufficient. The methods of assessment must be adapted as well. These disabled students are penalized most by traditional paper-and-pencil tests, particularly when their performance is being compared with that of typical children at a particular grade or age. Clearly, if children learn mathematics with difficulty or use different methods of learning, standard paper-and-pencil assessments will not be sufficient to document learning and students' progress. Using alternative forms of assessment is essential to describe what a student has learned, how he or she learns best, under what conditions he or she learns, and his or her understanding of mathematical processes.

Some alternative-assessment forms appropriate for students with handicaps are observation, interviews, holistic scoring, checklists, portfolios and journals, as well as paper-and-pencil forms of assessment.

Assessing Through Observation

Many teachers have made observation an integral part of evaluation. They practise targeting one or two students at each lesson for observation. To record observations, sticky notes for computer labels are used. Names of students targeted for the lesson are written on one or two sticky notes. Blanks are available for writing spontaneous observations about other students. These observations are then pasted on a specific sheet for each student in a class notebook. Figure 1 illustrates how observations can be used to assess students' understanding of mathematics effectively.

For students with disabilities, using observation as an assessment technique gives the teacher a window to obtain student-performance information that cannot be gleaned from paper-and-pencil tests. Teachers can unobtrusively gain insight into the approach to the task as well as the persistence in completing the task. Additionally, information can be obtained about how students are constructing meaning from concrete manipulations, as in the illustration with James. All observation should be systematically recorded. The record can document achievement and

communicate a student's success with mathematics to the student and others. These documented observations, when linked to paper-and-pencil test results, the results of interviews, the record of achievement on a checklist, and other assessment data, will give a more complete picture of a student's success in mathematics.

Interviewing

Students with disabilities often have difficulty in problem solving because their lack of fluency in reading can cause a misunderstanding of mathematical concepts, poor computational skills or poor dispositions toward learning mathematics. Alternative-assessment techniques should be used to determine the areas of problem solving in which the student has strengths as well as weaknesses. One alternative technique for assessing problem-solving abilities is interviewing students while they are in a problem-solving situation. Teachers can interview students informally in conversation while monitoring seatwork, or the teacher may plan a more structured, individual interview to survey the understanding of several students. Figure 2 illustrates assessing with an interview.

Holistic Scoring

Relying on observation or interview techniques to assess problem solving is not always effective. Interviews may be too time-consuming to use for an entire class or even with targeted students. An alternative to interviews is using a holistic-scoring technique. (See Figure 3 [Hynes 1990].) Three types of holistic scoring are generally recognized for evaluating mathematics learning: analytic scoring, focused holistic

Figure 1. **Using Observation to Assess James's Understanding of Area and Factors**

Mr. Mir has been working on geometry and measurement in his class and wants to assess students' understanding of calculating the area of rectangles. He is concerned about how to assess James, a student with learning disabilities who has difficulty making transitions from the concrete to the abstract. Mr. Mir was concerned that James might not be able to distinguish perimeter from area or apply a rudimentary formula to calculate the area. Previously, James had done poorly on written tests on area. Mr. Mir believes that the written tests do not reflect James's understanding of the mathematical concepts because James shows more insight during class presentations and discussions. Mr. Mir decides to assess James's understanding of area by observing him working with manipulatives while the rest of the class works at the abstract level.

Mr. Mir requests, "Class, make a rectangle that is 4 units by 6 units. Record the area of this rectangle, and the dimensions of all rectangles that have the same area as the first rectangle, using only whole-number dimensions."

Mr. Mir observes James using square tiles to make his 4-by-6 rectangle. James makes the rectangle and records that the area of the rectangle is 20. Mr. Mir notes that James added 4 + 6 + 4 + 6 to get 20. As James tries to respond to Mr. Mir's direction to make more rectangles, he seems confused. Mr. Mir asks the class to take time out. "Each member of the class can ask someone sitting nearby two questions about the problem." James interacts with the student in front of him in an acceptable manner and asks how the other student got 24. After the class returns to work, Mr. Mir notes that James has erased his first answer and written the correct answer.

As James makes other rectangles with an area of 24, Mr. Mir writes that he seems to have grasped a concrete understanding of area but failed to show all the possible rectangles with an area of 24.

Teacher Assessment. James demonstrated some understanding of area and the ability to find the area of the rectangle using concrete materials. He is also able to represent the area of a rectangle symbolically when allowed to use concrete materials.

Recommendations. James seemed to benefit from using manipulatives. He could make the described rectangles but he seems to confuse area and perimeter. Continued use of the manipulatives will be necessary to help him make this distinction. Since James was not able to make all the rectangles for an area of 24, he may need more work on the factors of 24. He may know the factors and not be able to connect this problem and the factors of a number. More observation is needed. The written work given James was assigned to help determine if he is progressing in relating abstract number sentences to models and pictures. If time allows, interviewing James about his understanding might be helpful.

scoring and general-impression scoring. (More information on holistic scoring can be found in *How to Evaluate Progress in Problem Solving* [Charles, Lester and O'Daffer 1987].) The focus of holistic scoring is on the process rather than on the correct answer. Students are given some credit for employing all or part of the correct steps in the problem, even if they get the wrong answer. Of course, if the student executes an appropriate process and also gets the correct answer, more points are awarded. In short, students are given credit for what they know. In implementing holistic-scoring techniques, it becomes imperative that students show all written work and record the thinking processes they used to solve the problem. Since writing problem situations is often difficult for students with disabilities, these students may need to work in pairs or cooperative groups, have an adult assist in recording their work or act out the solutions while showing the abstract solutions. Figure 4 presents a solution to a problem by two students, and the teacher's assessment and recommendation are shown in Figure 5.

Checklist

When assessing progress in learning, teachers should be primarily concerned with the content of mathematics. Another dimension of learning mathematics, however, should also be assessed. Students' dispositions toward learning mathematics are important, too. Students' confidence during mathematics learning, their willingness to persevere in mathematical tasks and their inclination to monitor their own thinking and performance all are important in the evaluation process (NCTM 1989). These dispositions are usually assessed as students engage in instructional activities in the classroom. For example, as they work in cooperative groups, teachers should note the ability of students to function properly during the instructional activity.

Mrs. Locklear has been using cooperative groups in her mathematics class two to three times a week for several weeks. Groups in her class solve problems, build models, practise with activities and study for weekly tests. She has seen increases in

Figure 2. **Using Interviewing to Assess José's Ability to Solve Two-Step Problems**

Since José has reading difficulties, Ms. Ryerson presents the following problem to him orally while pointing to the important facts.

"You and your dad go fishing. Your dad catches 4 fish and throws back 2 because the fish were too small. You catch 6 good-sized eating fish. Late in the afternoon, you and your dad go home and give your mom the fish to cook for supper. How many fish did your mom have to cook for supper?"

The first step for Ms. Ryerson was to ask José to tell her about the problem. By explaining the problem, he has completed the first step in problem solving—understanding the problem. José's response indicates that he is supposed to tell how many fish were cooked for supper. The teacher then continues the interview process by asking José how he would find out how many fish should be cooked. José's response will indicate whether he can select the correct operations and plan the solution. This question will probably lead to calculating the answer or solving the problem. José explains that he will add 4 and 2 and 6. At this point, the teacher suggested that José could use some fish counters to retell the story of the fishing trip. The teacher gives José a red paper labeled "dad" and a green paper labeled "José." José is asked to retell the story, placing the fish counters on the appropriate paper. As José retells the story with the manipulatives, he takes the correct action to indicate he understood the meaning of throwing back fish. However, in continuing the abstract solution of the problem, he adds 4, 2 and 6 correctly.

Teacher Assessment. When given two-step problems, José is able to identify the question the problem is asking. Initially, he does recognize one of the correct operations; however, José does not appear to comprehend the problem. In the retelling procedure, the teacher observes that José appears to comprehend the problem, but he is unable to transfer this comprehension and the physical action to the mathematical operations.

Recommendations. José needs more experience with two-step problem-solving exercises. He appears to profit from acting out the problem. His experiences should include working with another student to share their understanding of problems and creating his own two-step problems. He also needs to improve his conceptual understanding of operations. This student needs to experience real-world situations that indicate the operations needed. He needs to experience many models of subtraction: how many more, subset and comparison, as well as the take-away model.

Figure 3. **A Sample Focused Holistic-Scoring Scale for a Problem-Solving Assignment**

0 points

1. Problem is not attempted or the answer sheet is blank.
2. The data copied are erroneous and no attempt has been made to use that data.
3. An incorrect answer is written and no work is shown.

1 point

1. The data in the problem are recopied but nothing is done.
2. A correct strategy is indicated but not applied to the problem.
3. The student tries to reach a subgoal but never does.

2 points

1. An inappropriate method is indicated and some work is done, but the correct answer is not reached.
2. A correct strategy is followed but the student does not pursue the work sufficiently to get the solution.
3. The correct answer is written but the work either is not intelligible or is not shown.

3 points

1. The student follows a correct strategy but commits a computational error in the middle, which leads to an incorrect solution.
2. The student uses a correct strategy but ignores or misunderstands some conditions and never reaches a solution.
3. The correct answer is given and the work gives some evidence that an appropriate method was used. However, the implementation of the strategy is not clear.

4 points

1. The student uses an appropriate method and implements it correctly but commits a computational error toward the end and obtains an incorrect answer.
2. The student follows a correct method and performs the necessary work but toward the end loses sight of the answer or does not label the answer appropriately.
3. The student makes an error in copying. Except for this error, the work shows complete understanding of the method and implementation, even though an incorrect answer is reached.

5 points

1. The student has followed a correct method, performed appropriate computations and labeled answers correctly.

achievement in some of the slower students in her class. Mrs. Locklear has worked with her students on such social skills necessary for cooperative-group work as negotiating, complimenting one another and accepting criticism. Although she thinks that most groups are working well together, she is concerned that a few individuals and their groups might not be responding well to this strategy. She has noticed that some of the students with learning problems may not be actively involved in their groups. A month ago, Mrs. Locklear developed a checklist to assess students while they worked in cooperative groups (Figure 6). Figure 7 highlights an assessment and recommendation of the checklist.

A checklist supplies a strategy to record data systematically. Whereas the checklist in this example is used to assess students who are working in cooperative groups, checklists have many potential uses in assessment. These devices are excellent assessment tools to support teacher observation (Charles, Lester and O'Daffer 1987). In cooperative learning, group members can use a checklist to rate their peers in the group. Checklists can also be used for self-appraisal.

Filling out a checklist every day on every student is not necessary; however, a teacher who chooses to use checklists to assess students should use them periodically to assess students' progress on the attainment of concepts and skills as well as dispositions. Repeated observation using checklists will make patterns of behavior more apparent.

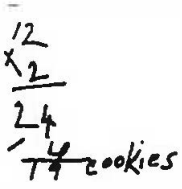
Figure 4. **Using Focused Holistic Scoring to Assess Li Wong and Sarah's Ability to Solve Multistep Problems**

Li Wong and Sarah were given the following problem in written form to solve independently.

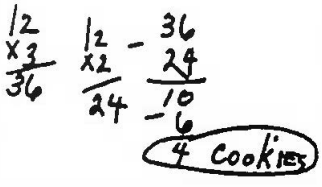
> Mother made a batch of cookies.
> She sent 2 dozen cookies to school with her daughter Ginger. Mother also gave 6 cookies to the children after school.
> If mother had made 3 dozen cookies, how many cookies will she have left for dessert at dinner?

Li Wong and Sarah's papers contained the following answers and solutions.

Li Wong



Sarah



Journals

Mathematics instruction should furnish opportunities for students to communicate their understanding of mathematics. Journal writing is a communication format that allows students to reach agreement among themselves about the use of mathematical terms and to recognize the importance of shared understanding of mathematical ideas. Writing about mathematics helps students clarify their understanding and gives teachers valuable information from which instructional decisions can be made (NCTM 1989). Figure 8 describes one use of journal writing to improve understanding of mathematical ideas.

Journals are an excellent means of self-assessment. Students with disabilities often have difficulty expressing themselves in writing. With structure and guidance they can, and should, develop reflective skills. Often students with mild handicaps are so accustomed to receiving feedback from others, particularly negative feedback, that they fail to take responsibility for their own understanding of content. Journals help develop this important skill.

Journals should be kept on a consistent basis. Ideally, daily entries should be made in the journal. Teachers of self-contained classrooms can have students keep one journal for all subjects.

Students can write in a special mathematics section or communicate in a special color. If journals are used as part of the assessment program, they should be continually monitored by the teacher to identify any instructional needs of the student. Systematically gathering and analyzing information from the students' journals is necessary. Some teachers collect three or four different journals each day to read and then respond to the students' entries. In addition to the three or four targeted students, teachers may allow one or two students voluntarily to submit their journals if they have something they want the teacher to read and respond to quickly.

Teachers who function in a departmental situation can collect two or three journals from each class. This tactic requires that the teacher do no more "grading" than if one whole class turned in an assignment for grading. When students with learning differences are in the mathematics class, monitoring the journals of these children more often than those of other students may be necessary.

Adapting Assessment Techniques

Table 1 lists characteristics that impede progress in mathematics for students with mild handicaps, along with suggested alternative-assessment procedures for these students. Even these procedures may need some adaptation to be appropriate for students with disabilities. Some suggestions for adaptation appear in the third column. As students with learning problems become more proficient in mathematics, many of the adaptations may be phased out, or less obtrusive methods may be selected.

Figure 5. **Teacher's Assessment of Student Work**

Teacher Assessment. Although Li Wong did not get the correct answer to the problem, he did convert 2 dozen to the exact number of cookies. He then subtracted 6, the number of cookies given to the children after school, to determine the number of cookies left for dinner, but he made a computational error. However, Li Wong forgot to consider the total number of cookies. Mother made 36 cookies. The teacher made a decision to give Li Wong 2 points because he used an inappropriate method, did complete the problem and reached an incorrect answer.

Using the focused holistic-scoring scale, the teacher gave Sarah 4 points. She used the correct strategy but near the end committed a computational error, which caused an incorrect answer.

Li Wong and Sarah received some points for their solutions even though they failed to obtain the correct answer. Both students attempted to apply the correct strategies in solving the problem. Thus, their ability to use the correct problem-solving processes were recognized. Analyzing the processes both students used was fairly easy, since they showed a great deal of their work.

Recommendations. Both Li Wong and Sarah may benefit from problem-solving experiences in pairs or cooperative groups. Computational errors are a barrier to completing the problem correctly for both students. By working together or with other students, Li Wong and Sarah may detect and correct these errors. Likewise, the discussion of a group may have allowed Li Wong to see another step in the problem. Additionally, both students need some practice with the algorithms for operations. If these students work on problem solving independently, they should be encouraged to use a calculator so that their computational skills do not interfere with the problem-solving process.

Figure 6. **Cooperative-Learning Checklist**

Skills	Names					
Followed directions	❑	❑	❑	❑	❑	❑
Stayed on task	❑	❑	❑	❑	❑	❑
Explained ideas clearly to others	❑	❑	❑	❑	❑	❑
Supported ideas of others	❑	❑	❑	❑	❑	❑
Developed a plan	❑	❑	❑	❑	❑	❑
Engaged in constructive criticism	❑	❑	❑	❑	❑	❑
Persisted in completing the assignment	❑	❑	❑	❑	❑	❑
Performed the following roles:						
Checker	❑	❑	❑	❑	❑	❑
Recorder	❑	❑	❑	❑	❑	❑
Leader	❑	❑	❑	❑	❑	❑
Summarizer	❑	❑	❑	❑	❑	❑

Figure 7. **Using a Checklist to Assess a Student's Disposition Toward Learning Mathematics**

While students are working in groups of four, Mrs. Locklear walks from group to group and uses the checklist to assess the individual students in the group. She has had an opportunity to assess each group two or three times.

Teacher Assessment. Mrs. Locklear notes that Tyrone, a low achiever, is not really participating in his group. He is slow to get on task and seems to be easily frustrated. He rarely supports others and does not share his ideas in the group. Although he was assigned the role of checker, he rarely assumes the responsibility. In contrast. Latesha, a student with mild mental retardation, seems to function well in her cooperative group. Latesha gets on task quickly and supports other group members. She has also assumed the role of checker, but her resource teacher had practised that role with her.

Recommendations. Mrs. Locklear feels that Tyrone would benefit from the group experience but is not taking advantage of the opportunity. She decides that tomorrow she will take aside all the students who are checkers and do some role-playing with them to give some strategies for the checker role. Asking one of the members of Tyrone's group to encourage him to participate might be helpful. Perhaps Tyrone would do better in another group. Latesha needs no adjustment at this time, only continued support and encouragement.

Figure 8. **Using Journals to Assess Students' Ability to Communicate Mathematical Ideas**

Miss Brant, a Grade 5 teacher, has been having her students write in their mathematics journals daily as a form of self-assessment. In the initial journal-writing experiences, Miss Brant instructed her students to write about what they had learned and what was difficult for them. While reading the students' journals, she realized that many were having difficulty expressing themselves in writing. For those students who needed more structure, she decided to begin the daily journal assignment using the questions that follow. Miss Brant suggested to Donny and Enrico, two students who attend the special-education resource class, that they may want to copy the problem down to answer questions 1 and 2. Miss Brant told them not to worry about spelling every word correctly. She stressed how important it was to get their ideas in writing.

1. What did you find easy in this lesson?
2. What was the most difficult for you today?
3. What new thing did you learn today?

Teacher Assessment. In reviewing Donny and Enrico's journals, Miss Brant discovered that both boys were having difficulty with equivalent fractions. On Tuesday and Wednesday, both boys said that "3/4 = ?/8" was the most difficult equation for them to solve. Enrico's response to the third question was, "I hate fractions!" Donny's response to it was "Nothin'." However, when Miss Brant asked Donny to write a sentence about what he learned that day, he responded, "The bigger the bottom number the smaller the piece."

Recommendations. Miss Brant realizes that she can gain some insight into both boys' progress by continuing to monitor their journals carefully. At the same time, she wants the boys to use the journals as a way to describe their own progress. She decided that she will spend time with some of the students in extending their journal-writing skills. Miss Brant decided that she will also begin charting the areas that students frequently identify as difficult. Miss Brant views the students' journals as valuable information about students' progress and the need for further instruction.

These assessment procedures should not be viewed as alternatives but rather as examples of appropriate assessment procedures for the student who learns mathematics differently. These assessment techniques should be used systematically if sound decisions are to be made about instructing children with disabilities.

Summary

To determine the progress of, and make appropriate educational decisions for, students with disabilities or at-risk students, teachers should use assessment techniques that accurately determine the students' progress *in spite of their learning differences.*

Table 1. **Adapting Assessment Techniques for Students with Learning Difficulties**

Examples of Learning Difficulties	Alternative-Assessment Procedures	Examples of Assessment Adaptations
Reading Difficulty	Interview Observation & questioning Journals	Students use pictures or manipulatives. Teacher presents problems orally. Students dictate journal entries to an aide who records them. Students respond by acting out interpretations of problems or solutions.
Computational difficulty	Interview Observation Journals	Students use manipulatives in skill-development activities. Students use calculators in problem solving. Teacher provides positive feedback about progress.
Difficulty translating concrete understanding to abstract level	Interview Checklists Journals	Students use manipulatives to create an abstract solution Teacher records repeated successes at the abstract level before verifying mastery. Students' entries use pictures. Teacher has students explain pictures orally.
Learns more slowly	Holistic scoring	Teacher to— provide mnemonics for steps during assessment, give fewer questions or problems on tests, test in pairs or in cooperative groups, and allow students to complete graded assignments at home.
Difficulty remembering procedural steps	Interview Observation Holistic scoring	Teacher to— color code steps on tests. remind students of self-monitoring strategies. provide mnemonics for steps during assessment.
Fear of failure	Observation Self-appraisal checklist Interview	Teacher to— remind students to use stress-reduction techniques. remind students to use "self-talk" techniques. ask easy questions initially to build up students' confidence.
Lack of such number-sense concepts as "more than," "less than," or the value of multidigit numbers	Interview Holistic scoring Observation Journals	Teacher to— encourage the use of manipulatives give partial credit in estimation exercises. color code to focus attention on place value. encourage expressions about numbers.

Alternative-assessment procedures are not only for students with disabilities but are appropriate for *all students*. However, given the learning characteristics of many students who are challenged in one way or another, it is imperative that alternative-assessment procedures be used.

The authors are not suggesting that pencil-and-paper assessments be abandoned. Information gained from pencil-and-paper assessments, however, is not sufficient to assess the mathematics learning for these students.

An assortment of appropriately applied assessment strategies should be used to gain a comprehensive view of the abilities of students with disabilities.

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A mathematical truth is in and by itself neither simple nor complicated.

—Émile Lemoine