Increasing Mathematics Confidence by Using Worked Examples

William M. Carroll

An important way to help our students learn and strengthen their knowledge of mathematics is by studying already-worked examples of problems. A considerable amount of evidence suggests that analyzing worked-out problems can be as conductive to learning as actually *solving* practice problems and that it may be more effective in helping students to recognize underlying similarities *between* problems (Sweller 1989; Ahu and Simon 1987; Sweller and Cooper 1985). Well-constructed worked examples do more than merely teach rote procedures; they illustrate mathematical principles and classes of problems.

However, worked examples are not always used effectively in instruction. Text formats vary widely, and some examples are difficult to follow (Ward and Sweller 1990). A few worked examples are generally presented a page or two ahead of practice problems, sometimes surrounded by unnecessary or difficult text. In some books, several sample problems are posed simultaneously, followed by their solutions. Because students need to search back and forth between the initial problem and the corresponding solution, this split format may interfere with learning rather than facilitate it. Whereas high-achieving students may be able to extract the important underlying mathematical principles from a few examples and then apply these principles during practice and problem solving, many others may have difficulty using a few examples isolated from problems, especially as they try to match problems to appropriate examples (Chi et al. 1989). Many students have told me that they are aware of the worked examples that precede the practice problems in their books and have difficulty relating a problem to an appropriate example. Many students have given up trying to use examples.

As a mathematics teacher, I was impressed with the results of previous studies in which worked examples were used to teach algebra to average or above-average students (Sweller 1989; Zhu and Simon 1987). As a teacher in an urban high school, I was interested in how a more extensive use of worked examples could be used to help my at-risk students. Many of the students enrolled in my algebra classes have a history of failure in mathematics and negative feelings and beliefs about their own ability. A number of students are diagnosed as having a learning disability or some other special need. Although these students might have difficulty using worked examples effectively (Chi et al. 1989), it also seemed likely that they would have much to gain from having these examples to support their learning and to help them focus on important features of a problem and its solutions. By giving sufficient meaningful examples, I hoped the students would have the necessary support to work independently at home, which in the process would lift their self-esteem concerning mathematics.

Over the past two years, I have used a worked-example format to introduce or reinforce various topics in algebra and other classes. Though not the only means of instruction, worked examples were used throughout the year on various topics, from writing and solving equations to using algebra in geometry.

Modeling Effective Use of Worked Examples

When I first began using worked examples as an instructional tool, many of my students had difficulty using them effectively. It was clear that many viewed mathematics as merely solving a group of problems with little regard for understanding. Although I asked students to spend time studying each example until they understood it, students often skipped the examples and went directly to the practice problems that followed, even when they did not know how to solve them. If they did not know how to solve a problem immediately, many would skip it or rely on the teacher for guidance.

Because one of my goals in using worked examples was to help students become more independent learners who relied less on me and more on themselves, I found it helpful in the beginning of the year to model a strategy for using worked examples. To demonstrate such a strategy, a worked example followed by similar problem or two were placed on the overhead projector or chalkboard. I asked the class to (1) study the example for understanding in a step-by-step

manner; (2) make some statements about what is taking place in the example, for instance, "Oh, I see. A variable is being used for the unknown number"; (3) proceed to the practice problem when the example is understood, referring back to the example as necessary; and (4) consider the answer in the practice problem, relating it back to the example if necessary. Not all students would *always* use this approach, but it was a place to start.

Some students did not see the point in studying a problem that was already worked out, so I also gave sets of worked-out problems in which errors occurred at various steps. Students had to identify the errors and correct the problems. Besides familiarizing students with examining worked-out problems for real understanding, I hoped that this task would also help them to remember to check their own work for errors. Two weeks of systematically modeling the use of worked examples and strategies was sufficient in my algebra classes, although some students needed occasional reminders to slow down and use the examples for understanding.

Worked-Example Formats

A typical format would show several worked examples on a page for students to study. Two to four practice problems generally followed each example, depending on the topic's complexity. To assist students, examples were matched to similar problems initially (Figure 1). As a topic was being practiced, fewer examples were presented or they were completely eliminated. Worked examples also helped students review a mix of previous topics with minimal instruction needed.

Figure 1

Examples show original format, although generally more problems were given per example.
Study each worked example. Then write an equation for the problem that follows.

1. Five less than a number is seven.

5 less than a number	is 7
$x - 5$	$= 7$

2. Three less than a number is six.
3. Six equals a number increased by three.

6 equals	a number increased by 3
$6 =$	$x + 3$

4. Two equals a number increased by twelve.
5. Twice a number is negative six.

Twice a number	is -6
$2x$	$= -6$

6. Twice a number is fourteen.
7. Negative six increased by a number is four.

-6 increased by a number	is 4
$x + (-6)$	$= 4$

8. Ten increased by a number is a negative six.

Many homework assignments also followed a worked-example format, especially on difficult topics. I hoped that by having meaningful worked examples to study along with their homework, students who had difficulty at home would be more likely to persevere. This is especially important if students are to realize that they can be successful in mathematics and problem solving.

Benefits of Using Worked Examples

Most of my students became proficient at using worked examples effectively. Students were more likely to return correct homework when they had examples as an added support. The at-risk students, who were of particular concern, profited the most. For instance, one learning-disabled student who had a high level of mathematics anxiety and a history of failure in mathematics became quite adept at solving problems, often helping other students around him, by using the examples. Another learning-disabled student showed marked improvement on classwork, homework and tests during lessons in which he had been given worked examples. The improved performance of these two students was fairly typical of at-risk students in my algebra classes. Students who were learning English seemed to learn more quickly and remember the relationship between English words and mathematical symbols better when using a worked-example format.

The improvement of the students during practice was not merely owing to rote copying of the worked examples. Most of them did much better on tests after using the examples even though they were not available during the test. Obviously more than rote learning had occurred.

Several reasons are possible why many students, especially the low achievers, gained from using these examples. First, many of them persisted longer both at home and in class because they had the worked examples for support when memory or understanding failed, which was evident by fewer problems being left undone. Second, the idea that they could study a problem, see a relationship or rule, and apply this understanding independently was highly motivating for many of them and affected their beliefs concerning mathematics and their own abilities. Many students who would typically wait for the teacher when they ran into difficulty found they could help themselves. Furthermore, it was challenging to try to

understand an example and apply it rather than simply solve a set of problems just to get through the assignment. Finally, having worked examples may prohibit the development of faulty rules and procedures and help students see similarities between types of problems. In fact, some evidence exists that students who study worked examples are quicker to recognize similarities and correct solutions for classes of problems than are those who merely practise sets of problems (Sweller 1989).

When students used worked examples, they were often working in small groups, taking part in mathematical conversions as they assisted each other and relying on their own resources when they had difficulty. Because less time was generally needed for direct instruction and practice, since we usually had fewer practice problems, more time could be spent on discussions, student explanations, problem-solving activities or various mathematical activities, including finding alternative methods for solving problems. Discussions of this worked-examples strategy led to similar discussions about problem solving, alternative solutions and the use of other heuristics.

Using worked examples need not promote the idea that only one correct way is possible to solve a problem. However, it is hoped that students will be more adept at using a successful problem-solving strategy of finding and studying a good example as a useful way to understand an underlying principle or rule. A more extensive use of worked examples, integrated with various other sound instructional formats as suggested by NCTM's *Curriculum and Evaluation Standards* (1989), can be a good way to help all students achieve success and independence as they attempt to construct meaning for their mathematical tasks.

References

Chi, M. T., et al. "Self Explanations: How Students Study and Use Examples." *Cognitive Science* (January–March 1989): 145–82.

National Council of Teachers of Mathematics (NCTM). *Curriculum and Evaluation Standards for School Mathematics.* Reston, Va.: NCTM, 1989.

Sweller, J. "Cognitive Technology: Some Procedures for Facilitating Learning and Problem Solving in Mathematics and Science." *Journal of Educational Psychology* 81 (December 1989): 457–66.

Sweller, J., and G. A. Cooper. "The Use of Worked Examples as a Substitute for Problem-Solving in Learning Algebra." *Cognition and Instruction* 2 (1985): 59–89.

Ward, M., and J. Sweller. "Structuring Effective Worked Examples." *Cognition and Instruction* 7 (1990): 1–39.

Zhu, X., and H. A. Simon. "Learning Mathematics from Examples and by Doing." *Cognition and Instruction* 4 (1987): 137–66.

Reprinted with permission from the NCTM publication The Mathematics Teacher, *Volume 88, Number 4, April 1995, pages 276–79. Minor changes have been made to spelling and punctuation to fit ATA style.*

As Fast as the Wind

A cyclist won a 1,000 m race. How many rotations did each wheel make around its own axis, when the diameter of the wheel is 685.8 mm?