Let's Solve the Problem Before We Find the Answer

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Ask a roomful of elementary mathematics teachers where their students have the most trouble, and the chorus of "problem solving" is overwhelming. Teachers reply this way because it is true. Study after study continues to cite the area of problem solving as the number-one concern in the classrooms of America as was recently well documented (Suydam 1982, 56–60). Some students who can perform arithmetical computation quite well often have problems that require the application of those same computational skills. Why is this? What is so difficult about solving word problems? The answer may lie in the discrepancy that exists between the skills taught as problem-solving skills and the critical thinking skills actually needed to solve word problems.

Problem-Solving Skills Taught in the 1960s and 1970s

In the 1960s and 1970s, many textbook authors suggested that students do the following to solve word problems:

1. Read the problem.
2. Determine what is asked in the problem.
3. Determine what facts are given.
4. Choose the operation.
5. Solve the problem.

Which step did your students find difficult? Was it step 1, "read the problem"? No, the National Assessment of Educational Progress (NAEP 1979) found through extensive research what most classroom teachers already knew, that even when problems were read aloud to students, those who were experiencing difficulties continued to find word problems difficult.

Was step 2, "determine what is asked in the problem," hard for your students? Most of us would agree that it was not. Usually the question was presented last and stated in such a way that students could readily find what must be answered in the problem. Besides, a question mark usually came with a question.

What about step 3, "determine the facts given." Couldn't your students usually pick out the facts from the problem? The main difficulty with this step in problem solving often involves finding *too many* facts as in the number "3" in the following problem:

> Jerry and Douglas went fishing 3 days last week and together they caught 12 white perch, 6 bass and 2 catfish. How many fish did they catch in all?

Do your students want to write "3 + 12 + 6 + 2 = _____" as the number sentence? Many do. Yes, students can find facts in word problems. However, they often experience difficulty eliminating unnecessary facts. Steps 2 and 3 are probably easy for most students because only recall at the knowledge level of thinking is required.

Step 4, "choose the operation," is the main area of difficulty in the preceding strategy, isn't it? Have you ever thought why this step is so difficult for many students? You have probably said, "Students won't *think* about the word problem, they just want to find an answer whether it's right or wrong."

Problem-Solving Skills Taught in the 1980s

In the 1980s, owing largely to research by Polya (1973, 8–10) and others (Carpenter et al. 1980; Herrington 1980; Marcucci 1980; Moser 1980; NAEP 1979; Zweng, Turner and Geraghty 1979), textbooks have begun to provide a more detailed plan for problem solving that is designed to promote higher-order thinking skills. The plan is similar to the following:

1. Read.
2. Plan—make a table, think backward, apply logic, draw a diagram, work a simpler problem, choose the operation, guess and test, and so on.
3. Solve.
4. Check.

Has this plan enabled more of your students to be more successful problem solvers? The additional emphasis on making tables, drawing pictures, working backward, guessing and testing and so on, will undoubtedly enable students to view problem solving more positively, but will these strategies enable

students to "determine the operation" in the traditional word problem more easily? If this skill is still a major area of difficulty for some of your students, you might like to try the procedure described in this article. The procedure involves teaching students to read word problems and then make a determination about the number(s) of groups involved. Once this determination is made, emphasis is placed on the question in the word problem to determine which of three actions is required for solving the problem. Suggestions are included to use as a guide for teaching the three actions, and diagrams of posters and bulletin boards are included to teach the process in your classroom.

Analyzing Three Important Actions in Routine Word Problems

Word problems, both one-step and multistep, are variations of three basic actions: combine, separate and compare. In the primary grades, when two or more groups are to be combined or joined, the operation to be applied is only *addition* (see problem 1, Figure 1). In the middle and upper grades, however, before combining groups, we have to look at the types of groups to be combined. If the groups are *equal* in number, the operation of *multiplication* can be used (see problem 2, Figure 1).

Figure 1

Typical routine word problems, primary through upper grade levels.

1. May picked 3 flowers. Sue picked 5 flowers. How many flowers were picked in all? (3 + 5 = 8)
2. Mother bought 3 packages of cookies for the club meeting. If each package contained 12 cookies, how many cookies did she buy? (3 × 12 = 36)
3. Fifteen children were playing basketball on the playground. Seven of the children went home. How many stayed to play? (15 – 7 = 8)
4. Fifty-six colored eggs were hidden during the egg hunt. If 8 children each found the same number of eggs and all of them were found, how many did each child find? (56 ÷ 8 = 7)
5. Susan is 12 years old and her brother, Bill, is 9 years old. How many years older is Susan than her brother Bill? (12 – 9 = 3)
6. There were 22 cars parked in the restaurant parking lot. Five of the cars were blue, 7 were black, and 10 were red. Compare the number of cars:
   a. black to blue (7 to 5 or 7:5)
   b. blue to red (5 to 10 or 5:10) simplify to (1:2)
   c. red to total (10 to 22 or 10:22) simplify to (5:11)

When a word problem contains only one group, the only action that can be taken on the group is to separate it into parts. In the primary grades, only *subtraction* is used to separate a group into parts (see problem 3, Figure 1). By the middle grades, however, if we know that the group must be separated into equal parts, we should divide (see problem 4, Figure 1).

The third action, compare, is sometimes necessary when the word problem contains two or more groups. In the primary grades, we are asked only to compare groups to *find a difference*. Number 5 of Figure 1 is an example of this problem type, and subtraction is the choice of operation to find the answer. In the upper grades, however, word problems may involve two or more groups that must be compared to *find a ratio*. Number 6 in Figure 1 is an example of this problem type. Sometimes, as in parts b and c, *division* is necessary to simplify the ratio.

Teaching Students to Understand the Three Actions

After a word problem has been read, students must reflect on the problem and make a very important decision regarding the question "How many groups are in the problem?" If counters such as beans or craft sticks are available, ask students to model the groups on their desks using the counters. This is a very important step in solving word problems so that children can understand the situation being presented. This step will help stop students from trying to manipulate the numbers quickly to find an answer. In this step, we are trying to get students to think about solving the situation presented in the word problem before finding an immediate answer.

After modeling the groups, reread the question in the problem to determine what action must be taken on the group or groups. If two or more groups must be joined to find a total, have students move their counters together forming one group and say, "We have *combined* the groups." If only one group of objects had been in the word problem and the action required part of the group to be taken away, have the children show the one group with counters and then take away the appropriate number as stated in the word problem. Say, "We have *separated* the group into parts." If two or more different groups of objects were present in the word problem and the action required comparing the groups to find a difference, model the groups using several colors of counters to distinguish between the groups. Allow students to match the groups being compared in one-on-one correspondence and count the number left

over. Say, "We have *compared* the groups to find a difference."

Primary-grade students can readily learn the words *combine, separate* and *compare* because these terms represent real-life experiences they have all had in their environment. To develop the word meanings, try these classroom activities in the primary grades where only addition and subtraction are taught.

Combine

1. Ask everyone wearing a red shirt to stand. Count the number and write it on the chalkboard. Then, ask everyone wearing a blue shirt to stand. Count only the students with blue shirts and write the number on the chalkboard. Ask the class to find the total number of children standing by combining or joining the two groups. For example, if three students wore red shirts and four wore blue, write $3 + 4 = 7$ on the chalkboard. The total number of children standing is seven.

2. Give each student a paper cup containing a few beans or grains of rice. Ask four students to go to the chalkboard, count the number of objects in their cups and write the numbers on the chalkboard. Ask them to place all their beans in one cup and emphasize that the objects have been combined. Write the appropriate number sentence on the chalkboard. Repeat the activity by calling on various numbers of students to combine their beans. Draw the generalization that two or more groups can be combined by addition.

Separate

1. Place six objects on the overhead projector and call on a student to take away four of them. Say, "We separated, or took away, four objects from the group of six. How many are left?" Write $6 - 4 = 2$ on the overhead.

2. Ask all the boys in the class to stand. Count the number in the group. Ask the boys who are wearing shoes with laces in them (such as tennis shoes) to walk to the front of the room. Emphasize that the number of boys with laces in their shoes was separated from the group of boys standing. Write the appropriate number sentence on the chalkboard. In both these examples, it is important to emphasize that when we have one group in a problem situation, the only action that can be taken on the group is to separate it into parts. At the primary-grade levels, subtraction is the operation to use when separating one group into parts.

Compare

1. Ask all the girls to walk to one side of the classroom. Count them. Ask the boys to walk to another side of the classroom and count them. The children will be able to determine quickly which of the two groups has more or fewer students. Emphasize that if we wanted to know how many more boys than girls, or vice versa, we would be comparing to find a difference between the groups. We would subtract to find the answer.

2. Place three star-shaped cutouts, five circles and seven squares on the overhead projector. Ask, "How many fewer circles are there than squares?" To demonstrate comparing, ask students to match the circles and squares in one-on-one correspondence on the overhead and count the number of squares left over. Explain that subtraction is an easier way to compare two or more groups to find a difference between groups. Write $7 - 5 = 2$ on the chalkboard or overhead and say, "Subtract to compare two or more groups to find a difference."

Students in the middle and upper grades should be encouraged to follow much the same format as suggested for primary students so that a clear understanding of the three actions can occur. However, they must consider an important step. After reading a word problem, students must again determine the number of groups in the problem, model the groups with counters and determine the action that must be taken with the group or groups. If it has been determined that the problem contains several groups to be combined, ask the students to determine if the groups are equal or unequal in size. If they are of equal size, multiplication is the choice of operation to solve the problem. If the groups are unequal, they will continue to add. If the word problem contained only one group, students must now give consideration to the size of the parts to be separated from the group. If the group is to be separated in such a way that the resulting parts will be of equal size, division is the operation choice. If the problem only required that a part be separated from the group, subtraction will continue to be used. The last problem type to be introduced in the middle and upper grades is a type of comparison problem. If it has been determined that two or more groups must be compared, we must determine if we want to find a difference using subtraction as in the primary grades or if we want to find a ratio between the groups. If a ratio is required, division is often needed to simplify, or express the ratio in lower terms. Try these classroom activities to practise the three new problem types in the middle and upper grades.

Combine Equal Groups

1. Count the number of windows in the classroom. Then count the number of window panes in each window. Suppose six windows were in the classroom

and each had eighteen panes of glass. To find out the total number of window panes, multiplication would be the appropriate solution operation, since each window contained an equal number of panes. Write $6 \times 18 = 108$ on the chalkboard. Emphasize that multiplication is appropriate because each window contained and equal number of panes. Say, "Multiply to combine equal groups."

2. Use the overhead projector to illustrate setting up chairs for a concert. If twelve rows were set up and contained twenty chairs each, find the total number of chairs in each of the twelve rows. Instead of adding, we can multiply because each of the twelve groups contains an equal number of chairs.

Although the operation of addition will still find the answer, emphasize that *multiplication* is a more appropriate operation to use, as we are combining equal groups.

Separate into Equal Parts

1. Use thirty craft sticks to represent flowers and five small boxes to represent vases. Separate the thirty "flowers" into the five "vases" so that the same number of flowers is in each vase. Since the problem states that each vase will contain the same number of flowers, this is an example of separating a group into equal parts. Write $30 \div 5 = 6$ on the chalkboard and say, "Divide to separate one group into equal parts."

2. Set up three tables of recreational reading materials in the back of the classroom. Separate the students so that an equal number of students is at each table. More than likely, one or more students will be left over. Allow these students to read independently at their desks. Write the appropriate number sentence on the chalkboard. For example, if thirty-two students were in the classroom, write $32 \div 3 = 10$ with a remainder of 2. Therefore, ten students would select a book from one of the three tables and two students would read at their own desks. Say, "Divide to separate a group into equal parts."

Compare to Find a Ratio

1. Ask all the girls to stand. Count them and write the number on the chalkboard. Ask them to sit down and have the boys stand. Count them and write that number on the chalkboard. Express the numbers in a ratio. For example, if seventeen girls and fifteen boys were in the classroom, the ratio of girls to boys would be 17 to 15, or 17:15. Since 17 and 15 have no common factors greater than 1, the ratio is in its simplest form.

2. Count the number of students in the classroom who have blond hair and write the number on the chalkboard. Count the number who have brown hair and write that number on the chalkboard. If the numbers were eight with blond hair and twelve with brown hair, the ratio of blond to brown would be 8 to 12, or 8:12. Since 4 is a common factor of 8 and 12, we must simplify the ratio to 2 to 3. Say, "Divide to express a ratio in its simplest terms."

Classroom Teaching Aids for Three Actions

Figure 2 summarizes the question model that would be appropriate to teach when addition and subtraction are the only operations that have been taught

Figure 2

Primary grades question model for routine, one-step word problems

READ the word problem.

THINK:

- Add to COMBINE two or more groups.
- Subtract to SEPARATE one group into parts.
- Subtract to COMPARE two or more groups to find a difference.

WRITE the number sentence.

SOLVE the sentence.

CHECK your answer.

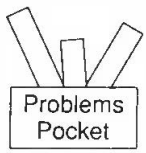
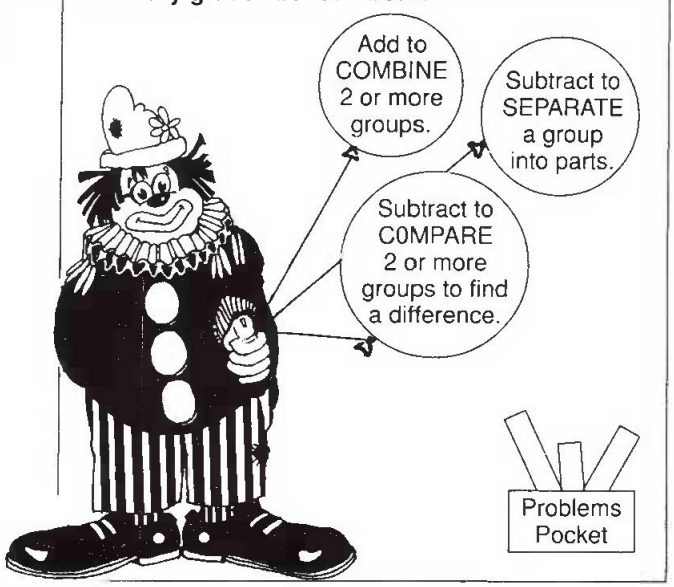


Figure 3

Primary grades bulletin board



to the students. Example problems are written on cards and placed in a "problem pocket." Students select a card and follow the algorithm outlined on the poster. After reading the problem, the student must decide the number of groups and what action is required for the groups. Then, the student writes the number sentence and solves it to find the answer to the problem. The student places the card in the appropriate pocket after solving it. This activity allows students to become actively involved with solving word problems. Young children enjoy the independence of this activity and sharing their results with the class. The answer could be written on the back of the card so that the activity is self-correcting.

Along the same lines as the poster illustrated in Figure 2, Figure 3 illustrates a bulletin board with the same directions. Examples of word problems are written on cards and placed in the appropriate balloon pocket held by the clown. Again, children become actively involved in thinking about the actions---combine, separate, compare—that are necessary to solve the problem. In a way, children are solving the problems by making the determination about actions on the groups involved before trying to find the answers.

Figure 4 is a classroom poster or bulletin-board idea appropriate in the middle and upper grades. Again, problems are written on small cards and students place them in pockets that match the problem types. As students will see from this activity, two occasions arise to use subtraction and division, but only one arises to apply the operations of addition and multiplication.

Figure 4

Middle and upper grades question model for routine, one-step word problems

READ the problem.

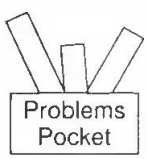
THINK:

- Add to COMBINE 2 or more unequal groups.
- Multiply to COMBINE 2 or more equal groups.
- Subtract to SEPARATE one group into unequal parts.
- Divide to SEPARATE one group into equal parts.
- Subtract to COMPARE 2 or more groups to find a difference.
- Divide to COMPARE and simplify a ratio.

WRITE the number sentence.

SOLVE the sentence.

CHECK your answer.



Summary

Using the outlined question model and suggested activities, elementary students will improve their abilities to analyze routine, one-step word problems and make a plan for the solution. The same algorithm can be applied to multistep problems by asking the questions listed on the teaching posters. This question algorithm will enable students to decide which operation—addition, subtraction, multiplication or division---will "solve" the word problem. Students can then use their arithmetic skills to "find the answer."

References

Bloom, B., ed. *Taxonomy of Educational Objectives: Cognitive Domain*. New York: David McKay, 1969.

Carpenter, T. P., et al. "Solving Verbal Problems: Results and Implications from National Assessment." *Arithmetic Teacher* 28 (September 1980): 8–12.

Herrington, J. "Experimental Analysis of Cognitive Style and Information Processing Variables in Mathematical Problem Solving." Ph.D. diss., Southern Illinois University at Edwardsville, 1980. *Dissertation Abstracts International* 41A (September 1980): 973–74.

Lerch, H. *Teaching Elementary School Mathematics: An Active Learning Approach*. Dallas, Tex.: Houghton Mifflin, 1981.

Marcucci, R. "A Meta-Analysis of Research on Methods of Teaching Mathematical Problem Solving." Ph.D. diss., University of Iowa, 1980. *Dissertation Abstracts International* 41A (December 1980): 2485.

Moser, J. M. *A Longitudinal Study of the Effect of Number Size and Presence of Manipulative Materials on Children's Processes in Solving Addition and Subtraction Verbal Problems*. April 1980. (ERIC ED 184 880)

National Assessment of Educational Progress (NAEP). *Changes in Mathematical Achievement, 1973–1978*. Report No. 09-MA-01. Denver: NAEP, August 1979.

Polya, G. *How to Solve It*. Princeton, N.J.: Princeton University Press, 1973.

Suydam, M. N. "Update on Research on Problem Solving: Implications for Classroom Teaching." *Arithmetic Teacher* 29 (February 1982): 56–60.

Zweng, M., J. Turner and J. Geraghty. *Children's Strategies of Solving Verbal Problems*. Final Report, NIE Grant No. G-78-0094. August 1979. (ERIC ED 178 359)

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