Mathematics as communication is an important Curriculum Standard and hence the mathematics curriculum emphasizes the continued development of language and symbolism to communicate mathematical ideas. Communication includes regular opportunities to discuss mathematical ideas and explain strategies and solutions using words, mathematical symbols, diagrams and graphs. While all students need extensive experience to express mathematical ideas orally and in writing, some students may have the desire—or should be encouraged by teachers—to publish their work in journals.

delta-K *invites students to share their work with others beyond their classroom. Such submissions could include, for example, papers on a particular mathematical topic, an elegant solution to a mathematical problem, posing interesting problems, an interesting discovery, a mathematical proof, a mathematical challenge, an alternative solution to a familiar problem, poetry about mathematics or anything that is deemed to be of mathematical interest.*

Teachers are encouraged to review students' work prior to submission. Please attach a dated statement that permission is granted to the Mathematics Council of The Alberta Teachers' Association to publish [insert title] in one of its issues of delta-K. *The student author must sign this statement (or the parents in the case of a student being under 18 years of age), indicate the student's grade level, and provide an address and telephone number.*

The following work, entitled "The Physics of a Classical Guitar," was written by Ryan Cassidy while he was a Grade 12 student at St. Mary's High School in Calgary. Ryan's paper is an excerpt from an extended essay that he wrote as part of his International Baccalaureate program.

Mary Chan wrote "The Ode to π" while in Grade 11 at St. Mary's High School in Calgary. This poem was written on the occasion of the annual π-day (March 14) celebration at St. Mary's High School.

The Physics of a Classical Guitar

Ryan Cassidy

I first picked up a guitar about four years ago and was captivated immediately by its complex sound and delicate tone. Since then, I have endeavored to study the instrument not only musically but also scientifically. The following is a brief synopsis of the results of one such endeavor.

The origins of the instrument are found in Egypt over 3,000 years ago, and though it became popular in all of Europe during the Middle Ages, it received most acclaim in Spain. As it evolved over the years, a number of characteristics became common.

A classical guitar is a fretted instrument composed of a wooden body over which six nylon (originally gut) strings are suspended to produce sound (the lower three strings are wrapped with a steel winding). Though different materials are used for different types of guitars (an acoustic guitar has steel strings, for example), classical strings are consistently constructed of nylon. The strings are tuned to the notes E_2 (82.41 Hz), A_2 (110.0 Hz), D_3 (146.8 Hz), G_3 (196.0 Hz), B_3 (246.9 Hz) and E_4 (329.6 Hz) from lowest to highest. They are suspended by means of tuning pegs found on the machine head.

Partial differentiation can be used to derive a formula for the speed of a transverse wave in a string. For a string with mass density μ, tension T, vertical displacement y, and horizontal displacement x over some time t, the force acting on a segment of string is given by $F = ma = (\mu\Delta x)(\partial^2 y/\partial t^2)$. As a wave does not transfer matter, only energy, the string particles do not move from side to side and are in horizontal equilibrium. The vertical component of the force acting on the string particles at x and $x + \Delta x$ (this force being the tension in the string) is given by the tension T times the slope of the string at these points: $F_{y1} = (T)(\partial y/\partial x\ (x + \Delta x))$; $F_{y2} = -(T)(\partial y/\partial x(x))$; because the string is in tension, the force at x will pull down on the segment (hence the negative sign on F_{y2}), while the force at $x + \Delta x$ will pull upwards. The total force on the segment, then, is given by

$(T)(\partial y/\partial x\ (x + \Delta x) - \partial y/\partial x(x))$, which is equal to $(\mu\Delta x)(\partial^2 y/\partial t^2)$. Dividing the left side of this equation by T, and the right side by Δx yields the following equation: $(\partial y/\partial x(x + \Delta x) - \partial y/\partial x(x))/\Delta x = (\mu T)(\partial^2 y/\partial t^2)$. Taking the limit of the left side of the equation as Δx approaches zero yields $\partial^2 y/\partial x^2$. Now recall the fundamental wave equation, $\partial^2 y/\partial x^2 = (1/v^2)\partial^2 y/\partial t^2$ (in other words, all wave functions y of x and t must satisfy the above partial differential equation, where v is the wave velocity down the string). Thus, the constant (μ/T) must be equal to $(1/v^2)$, and wave velocity is given by $v = \sqrt{\frac{T}{\mu}}$.

Modes of vibration of classical guitar strings are given by resonance formulas for fixed ends, where the length L is equal to $n\lambda/2$, where n is an integer greater than zero. The formula for the frequency of these modes is thus $f_n = nv/2L$, where $v = \sqrt{\frac{T}{\mu}}$. When a string is plucked, the resultant string vibration is a combination of a number of different modes. These modes can be isolated as harmonics, by placing a finger at select points on the neck and just "touching" the string. The first harmonic, for instance, can be enhanced by placing the finger over the 12th fret (half the string distance of 65.0 cm). As the string is now unable to vibrate in its fundamental mode (the finger prevents this from happening), the second harmonic is accentuated. Guitars can also be tuned using this method of harmonics at different frets.

The original intent of the investigation was also to apply Fourier Analysis to guitar waveforms. The Fourier Theorem states that any periodic function can be expressed as a sum of sines and cosines. It should, therefore, be possible to find a mathematical function for the guitar waveform. However, a number of factors complicated this objective. I was initially limited by the equipment available, as the sampling rate of the digital oscilloscope that I had at my disposal was too slow. The Nyquist Theorem states that in order to sample data unambiguously, the sampling rate must be twice the maximum possible frequency. The range of human hearing is about 20–20,000 Hz, so a sampling rate of at least 40,000 Hz is required to collect data, whereas the instruments available could only sample at 8,000 Hz. The waveform is also constantly changing after a string is plucked, as certain frequencies are dampened and amplified at different rates, and at different times. Taking all these limitations into account, it is easy to see why most synthesizers replicate such sounds through digitization and playback.

Undoubtedly, my future study of the guitar will be much more enjoyable as a result of the work that has engaged me during this project. To realize that the physics behind the instrument can be just as fascinating as the sound itself was illuminating.

Irrational Root

Prove that the positive root of the equation $x^2 + x = 10$ is irrational.