A Collection of Connections for Junior High Western Canadian Protocol Mathematics

Sol E. Sigurdson, Thomas E. Kieren, Terri-Lynn McLeod and Brenda Healing

We have put together "A Collection of Connections" comprising 12 uses of junior high school mathematics. These activities support the communication and connection strands of the Western Canadian Protocol mathematics curriculum. In using them, teachers may adapt them extensively. They can serve as a basis of one to three mathematics periods. We have also found that teachers need to plan if they intend to incorporate them into their teaching units. They can be used as end-of-unit activities or as focal activities in the development of a unit. We would encourage teachers to use the activities as a means of bringing mathematical skills to life. These contexts provide an opportunity to enhance a student's view of mathematics. As one Grade 9 student said at the conclusion of one of these activities, "That just proves that mathematics is everywhere."

The following are samples from the number and algebra strand.

Number (Square Roots and Powers)
The Musical Scale
The Musical Scale Student Activities
Algebra
The Clock Maker and the Pendulum
The Clock Maker and the Pendulum Student Activities

The Musical Scale

Intent of the Lesson

The mathematical basis of the musical scale is shown to have two aspects, reflected in the two parts of the lesson. In Activity 1, simple fractions determine the frequency of the notes in the *do, re, mi* scale. In Activity 2, powers and roots are used to compare frequencies and to justify the use of the black keys. The mathematics of the lesson include reciprocals, multiplying fractions and squares.

General Question

The basis of all our music is the 8-note scale:

do, re, mi, fa, sol, la, ti, do

These 8 notes get represented several times at higher and lower frequencies. We are all familiar with the piano keyboard. We notice that, in addition to the 8 white notes, we have five black notes. This is the 12-note scale because the 13th note is really the first note of the next 12 tones. The question we are asking in this lesson is how we got the 8-note scale and how the 12-tone scale works. We are going to use our knowledge of fractions to help us in this understanding.

Another interesting mathematical question about music is, if we can play only 12 different notes, how can we make so much different music? It is a mathematics problem to see how many different arrangements of 12 notes are possible. The number must be very large because we have millions of tunes. In addition to the 12 notes, we have the length of time each note sounds (1 beat, 2 beats, ½ beat, ¼ beat and so on). We also have the length of the interval between the notes. The 12 different notes, the lengths of the notes as they sound and the length of the space between the notes can be arranged in a very large number of ways to make a lot of different music.

However, this is not the question that we are dealing with today. Today we are going to investigate how we came to have 12 different notes. As you might imagine, it, like a lot of mathematics, started with Pythagoras.

Teaching Suggestion

This lesson can be taught as a whole or in two separate parts: Activity 1 and Activity 2.

If a piano keyboard and a guitar are available, they can be used effectively in this lesson. Many classes will have a guitar player and a piano player who can assist in this lesson. A knowledge of music is helpful.

Preliminary Activity

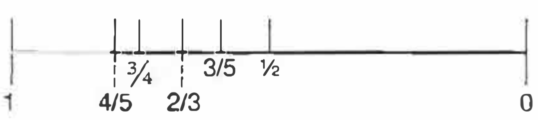
1. The pitch of a string, that is, whether it sounds high or low, depends on the frequency of its vibrations. For example, long strings vibrate slowly and therefore produce low notes. Short strings vibrate fast and produce high notes. (This idea can be illustrated nicely on a guitar.)
2. The frequency is inversely related to the length of a string.
   For example, if L (length of the string) gives a particular F (frequency of vibration) then ½ of L gives 2F (two times the frequency)
   and 2L gives ½ F
   and 1/3L gives 3 times F.
   Other examples might include 1/10 of L will give a note of 10F.
   Rule: The new frequency is found by multiplying the old frequency by the reciprocal of the change in length. If the length is doubled, the frequency is halved. If the length is ¼, the frequency is four times.
3. On particularly difficult concept is that a string which is divided in half produces that same note as the original string but twice as high. (This can be illustrated with all strings on a guitar.)

Although these concepts are not essential for the lesson, the students should have as good a grasp as possible of them. With these understandings of the physics of the vibrating string, our musical scale investigation can begin.

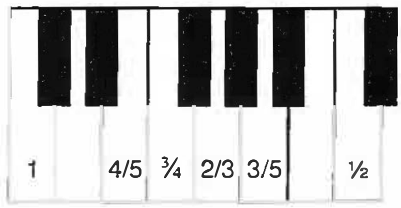
Answering the General Question

Activity 1: The Scale of Eight Notes

Pythagoras took a string and noticed how, when he divided it in half, he got the same note at twice the frequency. He wanted to divide this musical interval *do* (low) to *do* (high) into a series of notes. His first discovery was that when he divided this length of string into simple fraction ratios he got nice sounding notes. So that besides the ½ ratio, the other ratios were 2/3, ¾, 4/5 and 3/5. These are the simplest fractions we have. On a guitar string they look like this:



The teacher could have students measure the length of the guitar string from bridge to nut and find these lengths by calculating the fraction and measuring. The notes that we get from these simple fractions, *mi, fa, sol* and *la*, are shown below on the piano keyboard.



Now the teacher or a helper can find these fractional lengths on the guitar and play them on a keyboard and agree with the class that they do sound nice when compared tot eh original note.

Let us assume that the original note has a frequency of 256Hz, the frequency of middle C. Remembering our rule, we can figure out the frequency of the new notes. If the length is ½ of the original length the frequency will be two times 256 = 512. How about 2/3? We multiply 256 by 3/2. In this way, the frequencies of the other fractions of the length can be found.

Teaching Suggestion

We can figure these frequencies out by doing the fraction in two stages. First, doubling the length means ½ the frequency and taking 1/3 of that means tripling the frequency so the new frequency is 3/2 of 256 = 384.

Using the Table

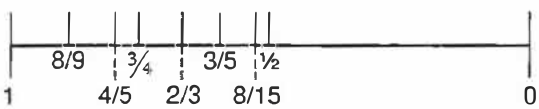
Making a table such as the one below can help keep track of these ideas. (The two blank spaces are for two additional notes which we will discuss later.) The first four notes that we have found are *mi, fa, sol* and *la*. We noticed that the frequency factor goes from 1 to 2. (In the table, the improper fractions are in parentheses, which is a useful form for comparison.)

Note	Length	Frequency Factor	Frequency
do	one unit	1	256.0
mi	4/5	(5/4) 1 ¼	320.0
fa	¾	(4/3) 1 1/3	341.3
sol	2/3	(3/2) 1 ½	384.0
la	2/3	(5/3) 1 2/3	426.7
do	½	2	512.0

These five ratios, *mi, fa, sol, la* and *do*, are simple ratios of the Pythagorean scale. To make the scale sound smoother, notes were added in the gaps between the *do* and the *mi* and between the *la* and the *do*. Two notes were added so that the first new note had a frequency 1/8 more than 1 and at the other end of the scale the new note had a frequency 1/8 less than 2. These two notes complete our eight-note scale.

Note	Length	Frequency Factor	Frequency
do	one unit	1	256.0
re	8/9	(9/8) 1 1/8	288.0
mi	4/5	(5/4) 1 ¼	320.0
fa	¾	(4/3) 1 1/3	341.3
sol	2/3	(3/2) 1 ½	384.0
la	3/5	(5/3) 1 2/3	426.7
ti	8/15	(15/8) 1 7/8	480.0
do	½	2	512.0

Charting the fractional lengths of the string on a number line, we notice that fractions of the length of the strings are not evenly spaced between ½ and 1. However these 8 fractional lengths make up the pleasant sounding musical scale: *do, re, mi, fa, sol, la, ti, do*.



Now we have an eight-note scale. By adding another eight notes and another we can have a series of notes and can play a wide range of *do*, *re*, *mi* and so on, repeated.

Activity 2: The Twelve-Tone Scale

We would like to think that in this scale the change is constant from one note because then we would have an even scale. However, the pattern of fractional lengths above suggests it is not. Let us examine how the frequency changes from note to note. How much higher is the frequency from one note to the next? That is, what do we need to multiply the frequency of *do* by to obtain the frequency of *re, re* to *mi; mi* to *fa* and so on? This number is referred to as the *multiplier*. It is the number we multiply the previous note by to obtain the next note; that is, the ratio of the note to the previous note.

Note	Length	Frequency Factor	Frequency	Multiplier
do	one unit	1	256	------
re	8/9	9/8	288	1.125
mi	4/5	5/4	320	1.111
fa	¾	4/3	341.3	1.066
sol	2/3	3/2	384	1.125
la	3/5	5/3	426.7	1.111
ti	8/15	15/8	480	1.125
do	½	2	512	1.066

As we calculate the multipliers (with a calculator) of these notes we notice that there are two (approximate) numbers. The multiplier is either 1.125 or 1.066. In fact, we note that the square root of 1.125 is approximately 1.066. Alternatively the square of 1.066 is 1.125. So the multiplier is either m or m^2 where m is 1.066.

As we go from *do* to *do*, the multipliers are: *do* × m^2 = *re*, *re* × m^2 = *mi*, *mi* × m = *fa*, *fa* × m^2 = *sol*, *sol* × m^2 = *la*, *la* × m^2 = *ti* and *ti* × m = *do*. The pattern of multipliers is m^2, m^2, m, m^2, m^2, m. This means that we *do not* have an evenly increasing scale. Between *do* and *re* the frequency increases by two jumps of m, but between *mi* and *fa* it only increased by one jump of m. A natural thing to do would be to add a note between *mi* and *fa* that only increased by one jump of m. A natural thing to do would be to add a note between *do* and *re* so that the frequency takes a jump of m to the new note and another jump of m from the new note to *re*. In this way, if we added five notes (one wherever we had an m^2 jump), the result is a 12-note scale, each note spaced a frequency m times higher than the previous.

We get the common keyboard of C, C#, D, D#, E, F, F#, G, G#, A, A#, B, C. (# means sharp.)

The advantage to the 12-note scale is that because all notes are spaced evenly apart we can start a *do-re-mi-fa-sol-la-ti-do* scale on any note. Once we start on any note, we go m^2, m^2, m, m^2, m^2, m^2, m. An m^2 jump means we go up two notes, while an m jump means we go up one note.

When we start with C we can go the usual *do*, *re*, *mi*, *fa*, *sol*, *la*, *ti*, *do* by staying on the white notes. What are we doing is starting with a frequency and following the pattern:

do, m^2, m^2, m, m^2, m^2, m^2 and arriving at *do*.

If we wanted to run the same scale beginning at D, which notes would we have to pick? Which note is m^2 above D? It is E#? Now which note is m^2 above E#? It is F#. So we see we can create a *do*, *re*, *mi*, *fa*, *sol*, *la*, *ti*, *do* pattern. In fact, we can see that we can do it by starting on any note.

What is the exact value of m? In going from *do*, to *do* there are 12 multiplications by m. We know in going from *do* to *do* that the frequency doubles. Therefore $m^{12} = 2$. What number multiplied together 12 times equals 2? It is the 12th root of 2, which is 1.059. By spacing the notes out mathematically even, we get a scale that can be repeated and that can be started at any note. However, the new set of notes does not follow the same frequency as Pythagoras' original nice sounding notes. In his spacing m = 1.066 and m^2 = 1.111 or 1.125, while in the even spacing the m = 1.059 and m^2 = 1.121. These are only approximations. Instruments like guitars and pianos are tuned to the mathematical pattern. These instruments cannot play the natural scale exactly. The violin, where players find their own notes, can play a scale that is perfectly in tune.

Materials

A guitar can be used in Activity 1 to emphasize the fractional relationships of the frequencies and a piano keyboard is useful in Activity 2 where the focus is one the black keys. Calculators are needed.

Modifications

The names of notes like C# and D# can be avoided if necessary. In music, a jump of m^2 in any frequency is called a "full tone," while a jump of m is a "half tone."

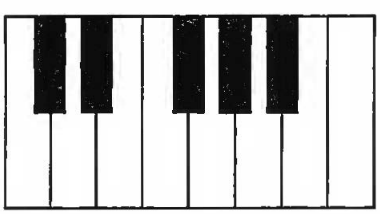
The Musical Scale Student Activities

General Question

The basis of all our music is the 8-note scale:

do, *re*, *mi*, *fa*, *sol*, *la*, *ti*, *do*.

These 8 notes get repeated several times at higher and lower frequencies. We are all familiar with the piano keyboard. We notice that, in addition to 8 white notes, we have 5 black notes.



This is the 12-note scale because the 13th note is really the first note of the next set of 12 notes. The real question is how we got the 8-note scale and how the 12-note scale works. We are going to use our knowledge of fractions and powers to find out.

Another interesting mathematical question about music is: if we can play only 12 different notes, how can we make so much different music? It is a mathematics problem to see how many different arrangements of 12 notes are possible. The number of arrangements must be very large because we have millions of tunes. In addition to the different arrangements of the 12 notes, we have the length of time each note sounds (1 beat, 2 beats, ½ beat, ¼ beat and so on). We also have the length of the interval. We also have the length of the interval between two notes that are played in sequence. The 12 different notes, the length of time the notes sound and the length of the time between the notes can be arranges in a very large number of ways to make a lot of different music.

However, this is not the question that we are dealing with today. Today we are going to investigate how we came to have 12 different notes. As you might imagine, it, like a lot of mathematics, started with Pythagoras.

Activities

1. a) A string 60 cm long gives a frequency of 256 vibrations per second. What will be the frequency of the vibrations when the string
   i) is shortened to of this length?
   ii) is shortened to ½ of this length?
   iii) is increased to 1½ times its length?

   b) What is the rule for finding the new frequency of a string when the length is changed to some fraction of its original length?
   Use F_n for the new frequency, F_o for the original frequency and f for the fraction.

   c) A string has a frequency of 152 vibrations per second. Use this rule (Question 1b) to find the frequency of the string which is 8/9 of its original length.

2. Use your calculator to fill in the table below for the frequencies of the notes.

Note	Length	Frequency Factor	Frequency
do	one unit	1	256
re	8/9	9/8	
mi	4/5	5/4	
fa	¾	4/3	
sol	2/3	3/2	
la	3/5	5/3	
ti	8/15	15/8	
do	½	2	

3. a) When we divide a string in half, we get a note of two times the frequency. This is the same note as the original at twice the frequency. Draw a diagram of a string and explain why it works this way.

   b) Find the frequencies of the C-note one octave, two octaves, three octaves and four octaves above middle C. The frequency of middle C is 256.

   c) Find the frequency of the C-notes one octave, two octaves, three octaves and four octaves below middle C.

4. a) The piano keyboard has 12 notes and then it repeats. The C note is middle C. In question 3 the frequency of C notes one, two, three and four octaves above and below middle C were calculated. The frequency for other notes can be found in the same way. Look on the chart

and find the frequency of D (next to middle C). What is the frequency of note D, an octave above?

b) We have two patterns of notes:
   i) The note to the right of any note has a frequency how many times higher?
   ii) The note an octave above any note has a frequency how many times higher?

5. Use your calculator to fill in the table below. The *multiplier* refers to the number we multiply the previous note by to obtain the next note. It is the ratio of the note to the previous note. Use your information from question 2 to fill in the column for frequency.

Note	Length	Frequency Factor	Frequency	Multiplier
do	one unit	1		----------
re	8/9	9/8		
mi	4/5	5/4		
fa	¾	4/3		
sol	2/3	3/2		
la	3/5	5/3		
ti	8/15	15/8		
do	½	2/1		

6. a) We have seen in the scale that we have developed that the relationships between the frequencies are not all the same. For example, some m^2 intervals are 1.125 and some are 1.111. Also if $m = 1.066$, then m^2 will be 1.136. Experiment with your calculator and find what m would be in order that all intervals are equal. In other words, what is m if

   $1 \times m \times m \times m$ (12 times) = 2?

   b) Pianos are made so the intervals are all the same. We can call this the mathematically perfect scale. Explain why a scale played on a piano is never exactly in tune.

   c) Violin players make their own notes, so they can always make scales that are musically exactly in tune. Does a violin player need to know the mathematics of music to play correctly? Would knowing it help the player to understand why the piano never sounds completely in tune?

The Clock Maker and the Pendulum

Intent of the Lesson

The time for one full swing of a pendulum is directly related to the square root of the length of the pendulum. The graph of the period of the swing against the length of the pendulum will produce a curved line graph. However, the graph of the square of the period against the length of the pendulum produces a linear graph. The mathematics involved is variables, graphing, linear equations and squares and square roots of numbers.

General Question

The swinging pendulum of a grandfather clock used to be a familiar sight. In designing clocks, clock makers find it convenient to have the pendulum swing in times of one second, two seconds and three seconds. When constructing a pendulum of various string lengths and a weight, it is obvious that not all pendulums swing at the same rate. How long does it take for the pendulum to make one full swing? A full swing is most easily understood as the movement from the extreme left position, across to the extreme right position and back to the extreme left position. As is the case with many crafts, experience has taught clock makers what length to make pendulums. They probably do not know why it works that way. The purpose of this activity is to examine why it works. Understanding the pendulum is largely a mathematical problem.

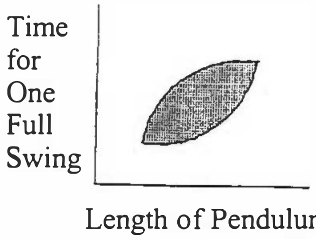
How fast a pendulum swings does not depend on how hard it is pushed. It also does not depend on the heaviness of the weight at the end of the pendulum. How fast a pendulum swings depends only on the length of the pendulum.

Discussion Questions

- Describe what a full swing of a pendulum is. (Pendulum starts at the left, swings all the way to the right and returns to the left.)
- Describe it using other starting points.
- In this lesson, which description is the most appropriate?
- Which takes longer to make one full swing, the short or the long pendulum? (Long)
- How can the approximate length of the pendulum be found that would make the time for one full swing equal to one second?
- What is the general relationship between the length of the pendulum and the time for one full swing? (Using the length as the x-axis and the time for

one full swing as the y-axis, the graph moves from the lower left to the upper right.)

- What kind of relationship would be preferred? (A straight line.)



[The shaded area shows the general region of where the points of the graph will lie. The shorter the pendulum, the shorter the time for one full swing.]

Through a teacher demonstration, have the class count the number of seconds it takes a pendulum to make 10 swings. The time for one swing is easily determined.

The general question, then, is to find the length of the pendulum which takes one second (or two seconds or three seconds) to make one full swing. Also, the *relationship* between the time for one full swing and the length of the pendulum should be examined.

The important concept being developed here is the mathematical relationship between the length of the pendulum and the time for one full swing. If the relationship is a straight line, the equation can easily be worked out between the two variables. Unfortunately, the graph of the relationship between the period and the length of the pendulum is not a straight line. This problem will be dealt with later.

Preliminary Activity

The Time for One Full Swing

To find the length for one full swing, students should find the length of time for 20 or 30 swings and then calculate the length for one full swing. Note that the timer should start when the person counting says zero and stop when the counter says 20 or 30 (whatever is agreed upon). A stop watch will be useful but not essential for this activity. The diagram below illustrates how counting is to be done. The mathematical problem of why to begin with *zero* should be discussed fully. It can be illustrated as follows. Suppose the time for a frog to make 20 jumps is to be counted. The diagram below shows the path of the jumping frog:

In counting 20 jumps, we could say "zero" at point A, "one" at point B and so on until point C is reached and numbered 20. If timing started at "zero" and stopped when 20 was reached, this would be the time for 20 jumps. The same type of thinking can be applied to full swings of a pendulum.

Have counting start when the swing is in a leftmost position. Start the pendulum swinging and start counting, beginning with zero. Again, the teacher should demonstrate this technique.

Discussion Questions

- If 20 swings take 20 seconds, what is the time for one swing? (1 second)
- If 20 swings take 33 seconds, what is the time for one swing? (1.65 seconds)
- How would the number of swings per second be found? (1/time for one swing)
- If the time for one swing was two seconds, what is the number of swings per second? (½ per second)
- If the number of swings per second was three, what is the time for one full swing? (1/3 second)
- What mathematical term describes how these two ways of talking about the swinging pendulum are related? (Inverse)

The time for one full swing is the period, while the number of swings per second is the frequency. The discussion will mainly be limited to the period. The procedure for finding the period is the time 20 or 30 full swings of the pendulum and make the calculation.

A Linear Relationship

Recall how to find a relationship between two variables:

- If I rent a car for $15 per day, what is the relationship between the cost and the number of days it is rented? What are the two variables? The graph is a straight line.
- If I make $1.25 for each box of candy I sell, what is the relationship between the number of boxes I sell and my total earnings. What are the two variables? The graph is a straight line.

In fact, if a graph turns out to be a straight line, it is very easy to find the relationship between the two variables.

Answering the General Question

The goal is to determine the relationship (equation) between the length of the pendulum and the period. Remember, the period is the "time for one swing."

Length of the Pendulum

The length of the pendulum is found by measuring the distance from the centre of the weight to the fixed end of the string: this can be determined to the nearest millimetre. The *length of the pendulum*, not the length of the string, is what needs to be found.

Time for One Full Swing

Once the pendulum is swinging freely (a few swings after it has been dropped), begin the swing at the extreme left and conclude the swing when it returns. How long will it take for 20 or 30 full cycles? One full swing can then be calculated.

Trials

The class can be divided into 10 groups. Each group is given a ruler, a piece of string and a weight and assigned a particular length of pendulum with which to work. Lengths from 0.2–2 m are recommended. At least 20 full swings should be timed. Each group should do three separate counts (of 20 swings) with the same length and find an average "time for one swing." Then 10 trials from 10 groups should be recorded, perhaps on the blackboard, as:

Trial Number Name of group)	length of pendulum	Time for one swing (period)	(Time for one swing)^2
1*	0.2		
2	0.4		
3	0.6		
4	0.8		
5			
6			
7			

* Group 1 is the teacher.

Collect the group data, make a table of values and graph them. Student in the class can draw his or her own graphs from this data which they can compare with other group members.

Teaching Suggestions

The teacher should note that the time for one swing with the pendulum of zero length is zero. In other words the graph goes through the point (0,0). Including the origin is useful in seeing that the graph is a curved line.

In noting that these points fit onto a curved line, it may be obvious that one or more of the points do not fit the general pattern. This can be used as a point of discussion with the class. A graph is a useful way of detecting patterns and noting deviations. In any case, an accurate graph will have to be drawn if a smooth curve is to be detected. The graph of the period squared should be drawn on the same graph as the initial curve.

Students should appreciate the importance of getting data that is on a straight line going through the origin and how this simplifies the mathematical problem of finding a relationship. The extra column of the table may be used for $\frac{(\text{time for one swing})^2}{\text{length of the pendulum}}$.
This should be approximately equal to the value K.

The Graph

The graph of the period against the length is a curved line. If the time for one full swing is squared, that is, the period is squared and the graph plotted, the points do fall in a straight line passing through the origin. This means that there is a linear relationship between the length and the square of the period. Some mathematical experimenting can be done (with a calculator) to find the multiplier "K" in the equation.

$L = KP^2$

Back to the Clock Maker

Returning to the original question of the clock maker, there are two ways assistance can be offered. The clock maker could be given the formula or he can be given a carefully constructed graph. The graph of the relationship between the length of the pendulum and the period (the curved line) would allow him to predict precisely the length for any desired period.

Provided the number "K" is retained, the formula itself is easily remembered. The formula is convenient to use because the clock maker knows the period he wants and through substitution can determine the corresponding length of the pendulum. For a period of one second the length is simply K. For a period of two seconds the length of pendulum is 4K.

Materials

Ruler, string and weights to which a string can be attached are needed, as well as calculators to find the coefficient in the linear relationship. As noted preciously, a stopwatch or other timing instrument would be useful.

Modifications

There is a temptation to also discuss frequency, even though this is unnecessary. Even the use of the word "period" is not essential.

Another way of writing the relationships is $P = G(L)^{½}$.

Written in this form, G is related to the constant of gravity, a very important number. Therefore, mathematical graphing has led to a scientific truth.

As a warm-up activity, each group of students could be given both a 50 g and a 500 g weight. By making two pendulums of the same length, the students should be able to determine that the weight does not influence the time for one full swing of the pendulum. This also gives the students the opportunity to practice making accurate measurements before they do the actual activity.

Answering the general question can be done in two ways: empirically (doing many trials to find the answer) or mathematically (using the graph of a few trials to find the answer). The focus of the activity could be on either or both of these methods.

The Clock Maker and the Pendulum Student Activities

General Question

The time for one full swing of a pendulum is determined by the length of the pendulum. Can you determine which length will give a time for one full swing of exactly one second? Of exactly two seconds?

Activities

1. a) The time for 30 full swings of a pendulum was 40.7 seconds. What is the time for one swing?
   b) How many swings will it make in one second?
   c) What is the mathematical relationship between the answers to question 1a) and 1b)? Why does this seem reasonable?
2. a) In counting the swings of a pendulum, the rule is to begin the stopwatch at the count of zero and stop it at the count of 30. Why not begin with 1? Explain this.
   b) Do the second thousand years after Christ was born begin in the year 2000 or in the year 2001? How does this relate to question 2a)?
   c) The same mathematics problem occurs in counting our pulse after exercise. Explain.
   d) What is the counting issue that is common to these three problems?
3. a) Mathematical relations between two variables that give a straight line as a graph are the best known relations in science. The most famous of these is the relation between the distance traveled and the time spent traveling of an object moving at a constant speed. The relationship is d(istance) = s(peed) × t(ime). What are some units for speed and time that could be used?
   b) A bullet traveled 400 metres in 2.2 seconds. What is the distance-speed-time equation for the bullet?
   c) An Arctic tern traveled 10,000 kilometres in 40 days. What is the distance-speed-time equation for the Arctic tern? If the "speed" of the tern is not constant, why does our equation still work?
4. a) In the Leaning Tower of Pisa, Galileo noticed that the time for an object to fall was related to the height at which it was dropped. Is the graph of the height against time to fall a straight line?

Time to Fall (Seconds)	Height (Metres)	Square of Height	Square Root of Height
1.8	16		
2.25	25		
2.7	36		
3.15	49		
3.6	64		

   b) Make a graph of the square of the height against the time to fall using the data in the chart provided. Is this a straight line?
   c) Make a graph of the square root of the height against the time to fall. Is this a straight line?
   d) Wherever the graphs in questions 3a, 3b and 3c were a straight line, find the value of K in the equation connecting time of fall (t) to height (h).
5. a) Here are some accurate measurements of a pendulum made by scientists on the moon. Find the square of the time for one full swing and figure out what K would be in this case? Why is K smaller on the moon?

Length	Time for One Full Swing	Square of Time for a Swing
0	0	
0.10	.32	
0.20	.45	
0.30	.55	
0.40	.63	
0.50	.7	
0.60	.75	
0.70	.84	
0.80	.9	
0.90	.95	
1	1	
1.2	1.1	
1.4	1.18	
1.6	1.25	
1.8	1.35	
2	1.4	

b) Select any eight points and graph them accurately. Make a large graph and make sure you start at (0,0). Notice the shape. Write an explanation for what the graph tells you about the relationship between length of the pendulum and time for one swing.

c) Square the time of the period and plot this. This should give you a straight line. What is the equation of this line? It is the form $y = Kx$.

d) You may have noticed that K can be figured out from just one swing. Why, then, is it necessary to make a graph to figure this out?

Note: Those readers interested in the entire volume of "A Collection of Connections" may contact Sol E. Sigurdson, Faculty of Education, Department of Secondary Education, University of Alberta, Edmonton T6G 2G5; phone 492-0753.

The authors acknowledge the financial support for this project provided by the Alberta Advisory Committee for Educational Studies (AACES) and the Central Research Fund of the University of Alberta. Reprinted with permission from the authors. Minor changes have been made to spelling and punctuation to fit ATA style.

How Many Additional Workers?

Twenty workers did ¼ of a job in 8 days. Then, because of an emergency, it became necessary to complete the job in the next 5 days. How many additional workers were added to the crew of 20 to accomplish this?