Math Projects for Science Fairs

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The Canadian Mathematical Society (CMS) (http://www.camel.math.ca) is concerned with the support and promotion of mathematics in Canada—through the teaching of mathematics, the popularization of mathematics and the creation of new mathematics (mathematics research). For many years, the CMS has sponsored the Canadian Mathematical Olympiad and in 1995 was responsible for the 36th International Mathematics Olympiad held in North York, Ontario (http://camel.math.ca/CMS/Olympiads/).

In 1996, the CMS established the CMS Awards, to be presented annually at the Canada-Wide Science Fair. At the 1997 and 1998 fairs the first prize award is $300 and a calculator. The second prize is one calculator at each of the junior, intermediate and senior levels. The awards criteria are outstanding projects in the mathematical sciences or making extensive use of mathematics in a project.

To date, there have not been many mathematics projects in science fairs; we believe that one reason for this might be that it is not clear what a mathematics project might involve. To help shed some light on this problem, we have prepared a list of possible projects and references on topics that could make exciting and interesting projects. The references are in parentheses following the ideas. But first some warning: the list is quite incomplete (as are some of the references) and not all the ideas have been fully thought out. This is intentional. After all, it is to be *your* project. Some are ideas more interesting than others, some will require more mathematics background than others and some have more scope for exploration than others. But all are related to areas of mathematics that a CMS member has found an exciting and rewarding place to explore and study.

We hope you enjoy them and that you discover wonderful and amazing things (as we did).

Numbers

1. Investigate "big" numbers. What is a big number? The following examples might guide your investigation. A bank is robbed of 1 million loonies. How long would it take to move them? How much would they weigh? How much space would they take up? How big a swimming pool do you need to contain all the blood in the world? Is 10^{100} very big? What is the biggest number anyone has ever written down (check the *Guinness Book of Records* over the last few years)? How did this number come about?
2. How do computer barcodes (the ones you see on everything you buy) work? This is an example of coding theory at work. Find others. Investigate coding theory—there are many books with titles like "an introduction to coding theory" (this is not about secret codes). (Gallian 1991, 1993, 1995)
3. Infinity comes in different "sizes." What does this mean? How can it be explained? (Kamke 1950; Huntington 1955; or refer to any book on Set Theory.)
4. It is easy to check if a number is divisible by 10 by looking to see if its last digit is a 0. How many other "tests of divisibility" can you find? Divisibility by 5 or 7 or 9? Why do they work? (Gardner 1969)
5. Most computers these days can handle sound one way or another. They store the sound as a sequence of numbers. Lots of numbers. 40,000 per second, say. What happens when you play around with those numbers? For example, add 10 to each number. Multiply each number by 10. Divide by 10. Take absolute values. Take one sound, and add it to another sound (that is, add up corresponding pairs of numbers in the sequences). Multiply them. Divide them. Take one sound, and add it to shifted copies of itself. Shuffle the numbers in the sequence. Turn them around backwards. Throw out every third number. Take the sine of the numbers. Square them. For each mathematical operation you can play the resulting sound on the computers speakers, and hear what change has occurred. A little bit of programming, and you can get some very bizarre effects. Then, try to make sense of this from some sort of theory of signal processing. You will first have to discover how sound is stored.
6. Find out all you can about the Fibonacci Numbers, 0,1,1,2,3,5,8, ... In particular, where do they arise in nature? For example, look at the spirals on a pinecone—following the pattern of

the cone, one spiral will go left, the other right. The cone will be covered by "parallelograms," the number of seeds on each side of the parallelogram will (always?) be two neighboring Fibonacci Numbers. For example, 5 and 8. It is similar for pineapples, petals and leaves on plants.

7. What is the Golden Mean? Study its appearance in art, architecture, biology and geometry and its connection with continued fractions, Fibonacci Numbers. What else can you find out?
8. Find out all you can about the Catalan Numbers, 1,1,2,5,14,42, ...
9. Investigate triangular numbers. If that's not enough, do squares, pentagonal numbers, hexagonal numbers and so on. Venture into the third and even fourth dimensions. (Conway and Guy 1996)
10. Build models to illustrate asymptotic results such as Stirling's formula or the prime number theorem.
11. There is a well-known device for illustrating the binomial distribution. Marbles are dropped through the top and encounter a number of pins before dropping into cells where they are distributed according to the binomial distribution. By changing the position of the pins one should be able to get other kinds of distributions (bimodal, skewed, approximately rectangular and so on). Explore.
12. Investigate the history of π and the many ways in which it can be approximated. Calculate new digits of π—see http://www.cecm.sfu.ca/~pborwein/ to discover what this means.
13. Use Monte Carlo methods to find areas or to estimate π. (Rather than using random numbers, throw a bunch of small objects in the required area and count the numbers of objects inside the area as a fraction of the total in the rectangular frame.)
14. Explore Egyptian fractions. In particular consider the conjectures of Erdös and Sierpinski: Every fraction of the form $4/n$ or $5/n$, $n \geq 3$ can be written in the form $1/a + 1/b + 1/c$, where $a < b < c$, and a, b and c are positive integers. See what you can discover. (Stewart 1964; Sondheimer and Rogerson 1981)
15. Look at how different bases are used in our culture and how they have been used in other cultures. Collect examples: time, date and so on. Look at how other cultures have written their number systems. Demonstrate how to add using the Mayan base 20, and maybe compare to trying to add with Roman numerals (is it even possible?) Explore the history and use of the abacus. (Bakst 1965; Ifrah 1985)
16. There are several methods of counting and calculating using your fingers and hands. Some of these methods are still in common usage. Explore the mathematics behind one of them. (Ifrah 1985)

Scheduling

1. At certain times charities call households offering to pick up used items for sale in their stores. They often do a particular geographical area at a time. Their problem, once they know where the pick-ups are, is to decide on the most efficient routes to make the collection. Find out how they do this and investigate improving their procedure. A similar question can be asked about snow plows clearing city streets, or garbage collection. (See Euclidean tours, Chinese postman problem—information can be found in most books on graph theory but one of particular interest is Behzad and Chartrand 1971.)
2. How should one locate ambulance stations, so as to best serve the needs of the community? How do major hospitals schedule the use of operating theatres? Are they doing it the best way possible so that the maximum number of operations are done each day? The reference given above may help.
3. How does the NBA work out the basketball schedule? How would you do such a schedule bearing in mind distances between locations of games, home team advantage and so on? Could you devise a good schedule for one of your local competitions? (Dinitz, Lamken and Wallis 1995)
4. How would a factory schedule the production of bicycles? Which parts are put together first? How many people are required to work at each stage of the production? (See Steen 1978, chapter by R. L. Graham.)
5. Look for new strategies for solving the traveling salesperson problem.

Games

1. What is game theory all about and where is it applied?
2. Study games and winning strategies—maybe explore a game for which the winning strategy is not known. Analyze subtraction games (nim-like games in which the two players alternately take a number of beans from a heap, the numbers being restricted to a given subtraction set). (See Berlekamp, Conway and Guy 1982—this book contains hundreds of other games for which the

complete analysis is unknown; for example, Toads and Frogs. See also, Guy 1991—pay special attention the last section; Garder 1959–61 [vol. 1].)

3. Ten frogs sit on a log—five green frogs on one side and five brown frogs on the other with an empty seat separating them. They decide to switch places. The only moves permitted are to jump over one frog of a different color into an empty space or to jump into an adjacent space. What is the minimum number of moves? What if there were 100 frogs on each side? Coming up with the answers reveals interesting patterns depending on whether you focus on color of frog, type of move or empty space. Proving it works is also interesting—it can lead to recursion. There is also a simple proof that is not immediately obvious when you start. Look for and explore the other questions like this—one of the most famous is the Tower of Hanoi.
4. Try the "Monty Hall" effect. Behind one of three doors there is a prize. You pick door #1. He shows you that the prize wasn't behind door #2 and gives you the choice of switching to door #3 or staying with #1. What should you do? Why should you switch? Make an exhibit and run trials to "show" this is so. Find the mathematical reason for the switch.
5. A graph is a mathematical structure made up of dots (called vertices) and lines joining pairs of dots (called edges). There are many games that can be played on graphs, and much mathematics involved in finding winning strategies. (See the MegaMath Web site at http://www.c3.lanl.gov/mega-math for ideas.)
6. Investigate card tricks and magic tricks based in mathematics. Some of the best in the world were designed by the mathematician/statistician Persi Diaconis. (Albers 1995; Gardner 1959–61)
7. All forms of gambling are based on probability. Investigate how much casinos anticipate winning from you when you play blackjack, roulette and so on. Study a variety of lotteries and compare them. Should one ever buy a lottery ticket? Why does three-of-a-kind beat two pairs in poker? Discover why the different types of hands are ranked as they are. (Gardner 1969; Colbourn 1995)

Geometry

1. Pool problems: if you have a rectangular table without friction and send a pool ball at an angle θ, will it return to the same spot? Investigate using a diagram in Sketchpad (or Cabri). If it does not return to the same spot, will it pass over all points on the table? Does the answer depend on the dimensions of the table? Make a sketch in which you can change the dimensions of the table and the direction of the ball, and explore the path through 10 or 20 bounces. What happens on a circular pool table? Make a dynamic geometry sketch.
2. Flatland and sphereland. If you lived in flatland (the plane) could you build a bicycle which exists in the plane and works? Could you do the same on the sphere? Explore other "machines" in a flat space. (Dewdney 1984; Hinton 1907; see also good descriptions of the problem in Gardner 1959–61, 1990.)
3. There are many aspects of spherical geometry that could be investigated. Explore congruences of triangles on a sphere. Other useful tools that are also available are a plastic sphere, with hemispherical "overhead transparencies," great circle ruler, compass and so on. One can also make very effective models with plastic spheres from a craft shop and cut-off plastic containers for rulers.

   Explore quadrilaterals and their symmetries on a sphere. Is there a family which shares most of the properties of a parallelogram? What symmetry do they have? Which two properties (for example, opposite angles equal) are sufficient to prove all the other properties?
4. What equalities of lengths and angles are sufficient to prove two sets of four points (quadrilaterals or quadrangles . . .) are congruent? (Leads directly to unsolved research problems in computer-aided design. For further references, e-mail whitely@mathstat.yorku.ca.)
5. Build models showing that parallelograms with the same base and height have the same areas. (Is there a three-dimensional analog?) This can lead to a purely visual proof of the Pythagorean theorem, using a physical model based on dissections. The formula for the area of a circle can also be presented in this way, by building an exhibit on the Pythagorean theorem but with the basis that "the area of the semicircle on the hypotenuse is equal to the sum of the areas of the semicircles on the other two sides." (Jacobs 1970)
6. Study the regular solids (Platonic and Archimidean), their properties, geometries and occurrences in nature (for example, virus shapes, fullerene molecules, crystals). Build models. (Gardner 1990, 1959–61 [vol. 2]; Jacobs 1970)
7. Consider tiling the plane using shapes of the same size. What's possible and what isn't? In

particular, it can be shown that any four-sided shape can tile the plane. What about five sides? Make sketches in a geometry program—Sketchpad, Cabri, or using Kali (available free from the Geometry Centre, or Reptiles: demo version available at the Math Forum at Swarthmore)—-these can be found at the Web sites. (See the Martin Gardner books; Grunbaum and Shephard 1987; Steinhaus 1969.)

8. Draw and list any interesting properties of various curves; evolutes, involutes, roulettes, pedal curves, conchoids, cissoids, strophoids, caustics, spirals, ovals and so on. (Coxeter et al. 1938—it has lots of other ideas too; Lockwood 1963)
9. Make a family of polyhedra, for example, the Archimidean solids, or Deltahedra (whose faces are all equilateral triangles), or equilateral zonohedra, or, for the very ambitious, the 59 Isocahedra. (Ball 1962—full of ideas; Coxeter et al. 1938; Wenninger 1971; Schattschneider and Walker 1987; Senechal and Fleck 1988)

   What polyhedral shapes make fair "dice"? What are the physical properties? What are the geometric properties? What is the root of the word "polyhedra" (and why does this fit with the use as dice)? Can you list all possible shapes? What numbers of faces can appear? What other (nonpolyhedral) shapes are actually used in games?

   What polyhedral shapes appear in crystals? List them all. Why do these appear? Why don't other shapes appear? What is the connection between the big outside shape and the inside "connections of molecules"? (Senechal 1990)
10. What is Morley's triangle? Draw a picture of the 18 Morley triangles associated with a given triangle ABC. Find 18 more triangles for each of the triangles BHC, CHA, AHB, where H is the orthocentre of ABC. Discover the relation with the nine-point circle and deltoid (envelope of the Simson or Wallace line).
11. Investigate compass and straight-edge constructions showing what's possible and discussing what's not. For example, given a line segment of length one, can you use the straight edge and compass to "construct" all the radicals? Investigate constructions using origami (paper folding). Can you construct all figures that are constructed with ruler and compass? Can you construct more figures? References can be found in articles in *Math Monthly*, *Math Magazine*.
12. The cycloid curve is the curve traced by a point on the edge of a rolling wheel. Study its tautochrone and brachistochrone properties and its history. Build models. Suppose all cars had square wheels. How would you design the road so that you always had a smooth ride? What about other wheel shapes? (Wagon 1991)
13. Find as many triangles as you can with integer sides and a simple linear relation between the angles. What about the special case when the triangle is right-angled?
14. What is a hexaflexagon? Make as many different ones as you can. What is going on? (Garner 1988, 1959–61 [vol. 1])
15. A kaleidoscope is basically two mirrors at an angle of $\pi/3$ or $\pi/4$ to each other. When an object is placed between the mirrors, it is reflected 6 or 8 times 9 (depending on the angle). Construct one. Investigate its history and the mathematics of symmetry. Make models of kaleidoscopes in a dynamic geometry program (Cabri or Geometers Sketchpad). Demonstrate why only certain angles work. (Ball 1962; Hodgson 1987)
16. You make a tangram puzzle by dividing a two- or three-dimensional object into many geometrical pieces, so that the original object can be reconstructed in more than one way. Burr puzzles are interlocking assemblies of notched sticks. For example, there are Burr puzzles that look like spheres or barrels when they are completed. (See Coffin 1990 for information on how to construct your own.)
17. Build rigid and nonrigid geometric structures. Explore them. Where are rigid structures used? Find unusual applications. This could include an illustration of the fact that the midpoints of the sides of a quadrilateral form a parallelogram (even when the quadrilateral is not planar). Are there similar things in three dimensions? Are there plane frameworks (rigid bars and flexible joints) that are rigid but contain no triangles? Are all triangulated spheres rigid (either made of sticks and joints or of hinged plastic pieces "polydron")? What is the formula for the number of bars in a triangulated sphere, in terms of the number of vertices? How does this formula relate to other rigid frameworks in three-dimensional space?

    Consider a plane "grid" composed of squares (say four squares by four squares) made of bars and joints. Which diagonals of squares will make this rigid? What is the minimum number? Can you give a recipe for deciding which diagonals will work? (There is a COMAP module related to this problem.) If the grid is composed of a trapezoid and its image after a half turn, alternating, does the same recipe work? (This is a research problem which has *not* been thoroughly worked out! E-mail whiteley@mathstat.yorku.ca.)

18. The Art Gallery problem: What is the least number of guards required to watch over all paintings in an art gallery? The guards are positioned at specific locations and collectively must have a direct line of sight to every point on the walls. (Tucker 1994; Wagon 1991)
19. The Parabolic Reflector Microphone is used at sporting events when you want to be able to hear one person in a noisy area. Investigate this, explaining the mathematics behind what is happening.

Combinatorics

1. An international food group consists of 20 couples who meet four times a year for a meal. On each occasion, four couples meet at each of five houses. The members of the group get along well together; nonetheless, there is always a bit of discontent during the year when some couples meet more than once! Is it possible to plan four evenings such that no two couples meet more than once? There are many problems like this. They are called combinatorial designs. Investigate others.
2. What is the fewest number of colors needed to color any map, if the rule is that no two countries with a common border can have the same color. Who discovered this? Why is the proof interesting? What if Mars is also divided into areas so that these areas are owned by different countries on earth? They too are colored by the same rule but the areas there must be colored by the color of the country they belong to. How many colors are now needed? (Hutchinson 1993; Ball 1962; Steen 1978)
3. Discover all 17 "different" kinds of wallpaper. (Think about how patterns on wallpaper repeat.) How is this related to the work of Escher? Discover the history of this problem. (Shephard 1976; Coxeter 1971; Conway and Coxeter 1996)
4. Investigate self-avoiding random walks and where they naturally occur. (Slade 1996)
5. Investigate the creation of secret codes (ciphers). Find out where they are used (today!) and how they are used. Look at their history. Build your own using prime numbers. (Fellows and Koblitz 1993; Ball 1962)
6. It is easy to cover a chessboard with dominoes so that no two dominoes overlap and no square on the chessboard is uncovered. What if one square is removed from the chessboard? (Impossible—why?) What if two adjacent corners are removed? What if two opposite corners are removed? (Possible or impossible?) What if any two squares are removed? What about using shapes other than dominoes (for example, 31 × 1 squares joined together)? What about chessboards of different dimensions? See the following problem as well. (Golumb 1965)
7. Polyominoes are shapes made by connecting certain numbers of equal-sized squares together. How many different ones can be made from two squares? from three, from four, from five? Investigate the shapes that polynominoes can make. Play the "choose-up" Pentomino game. (Golumb 1965; Gardner 1959–60 [vol.1], 1977)
8. Find pictures which show that $1 + 2 + ... + n = n(n+1)/2$; that $1^2 + 2^2 + ... + n^2 = n(n+1)(2n+1)/6$; and that $1^3 + 2^3 + ... + n^3 = (1 + 2 + ... + n)^2$. How many other ways can you find to prove these identities? Is any one of them "best"? (See Sondheimer and Rogerson 1981, or "Proofs Without Words," a regular feature of *Mathematics Magazine*.)

Others

1. Build a true-scale model of the solar system—but be careful because it cannot be contained within the confines of an exhibit. Illustrate how you would locate it in your town. Maybe even do so!
2. What is/are Napier's bones and what can you do with it/them?
3. Discover how to construct the Koch or "snowflake" curve. Use your computer to draw fractals based on simple equations such as Julia sets and Mandelbrot sets. (Peterson 1988; see Lauverier 1991 for example programs)

   What is fractal dimension? Investigate it by examining examples showing what happens to lines, areas, solids or the Koch curve, when you double the scale.
4. Gardner (1982) defines a paradox to be "any result that is so contrary to common sense and intuition that it invokes an immediate emotion of surprise." There are different types of paradoxes. Find examples of all of them and understand how they differ.
5. Knots. What happens when you put a knot in a strip of paper and flatten it carefully? When is what appears to be a knot really a knot? Look at methods for drawing knots. (Steinhaus 1969;

Farmer and Stanford 1996). Also check out these Web sites: KnotPlot—http://www.cs.ubc.ca/nest/imager/contributions/scharein/KnotPlot.html, Mathmania—http://www.csc.uvic.ca/~mmania/ and MegaMath—http://www.c3.lanl.gov/mega-math/.)

6. Another source of knots is the stonework and ornamentation of the Celts. Investigate Celtic knotwork and discover how these elaborate designs can be studied mathematically. (Cromwell 1993; Meehan 1991)
7. Learn about origamic architecture by making pop-up greeting cards. (Chatani 1986)
8. Is there an algorithm for getting out of two-dimensional mazes? What about three-dimensional? Look at the history of mazes (some are extraordinary). How would you go about finding someone who is lost in a maze (two- or three-dimensional) and wandering randomly? How many people would you need to find them?
9. Explore Penrose tiles and discover why they are of interest. (See Peterson 1988, and most books on tiling the plane.)
10. Investigate the Steiner problem—one application of which is concerned with the location of telephone exchanges to minimize costs.
11. Use PID (proportional-integral-differential) controllers and oscilloscopes to demonstrate the integration and differentiation of different functions.
12. Construct a double pendulum and use it to investigate chaos.
13. Investigate the mathematics of weaving. (Grunbaum and Shephard 1980; Clapham 1980)
14. What are Pick's Theorem and Euler's Theorem? Investigate them individually, or try to discover how they are related. (DeTemple and Robertson 1974)
15. Popsicle Stick Weaving: With long flat sticks, which patterns of "weaving over and under" in the plane are stable (as opposed to flying apart)? Find a pattern with four sticks. Is it unique? Does the stability change when you twist one of the sticks in the plane)? Find several patterns with six sticks whose stability depends on the particular "geometry" of where they cross (that is, the pattern becomes unstable if you twist one of the sticks in the plane). Can you give a rule for recognizing the "good geometric positions"? What kinds of "forces" and "equilibria" are being balanced here? What general rules can you give for "good" weavings? (Source of some information: whiteley@mathstat.yorku.ca.)

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Web Sites

KnotPlot:	"The KnotPlot Site," Robert Scharein, University of British Columbia, http://www.cs.ubc.ca/nest/imager/contributions/scharein/KnotPlot.html
MacTutor:	"The MacTutor History of Mathematics Archive," School of Mathematical and Computational Sciences, University of St. Andrews, St. Andrews, Scotland, http://www-groups.dcs.st-and.ac.uk/~history/
Mathmania:	"Mathmania," The Erdös for Kids Problem Sponsoring Program, http://www.csc.uvic.ca/~mmania/
MegaMath:	"This is Mega Mathematics!" Los Alamos National Laboratory, http://www.c3.lanl.gov/mega-math/
π	"Dr. Peter Borwein's Home Page," http://www.cecm.sfu.ca/~pborwein/

Many more Web sites can be found starting from the CMS Web site at http://www.camel.math.ca/. Finally, consider a subscription to a math magazine. To subscribe to *Math Horizons*, write to *Math Horizons*, MAA Service Center, PO Box 91112, Washington DC 20090-1112, USA.

To subscribe to *Crux Mathematicorum*, write to *Crux Mathematicorum*, Canadian Mathematical Society, 577 King Edward, Suite 109, PO Box Station A, Ottawa K1N 6N5; or subscribe online by visiting the Web site http://camel.math.ca/CMS/CRUX/ and view a sample issue.

Spelling Test

Each week Jill's class takes a 30-item spelling test. If she scored 20 on the first test, what is the lowest score she can make on the second test in order for the average of her first three tests to be 26?

Watermelon Weight

A giant watermelon weighed 100 lb. and was 99 percent water. While standing in the sun, some of the water evaporated, so it was only 98 percent water. How much did the watermelon then weigh?