The STEM Model

Rick Billstein

> A problem is not necessarily solved because the correct answer has been made. A problem is not truly solved unless the learner understands what he has done and knows why his actions were appropriate.
>
> —*William A. Brownell*

The Six Through Eight Mathematics (STEM) project is one of the National Science Foundation's (NSF) funded middle school mathematics curriculum projects. The project received initial funding in 1992; the finished product is expected to be available in 1998. The STEM curriculum is organized in teaching units called modules. Each module is organized around a conceptual theme, such as "flight" or "wonders of the world." The use of thematic modules promotes students' developing and applying mathematical concepts in broad contextual situations so that they do not view mathematics as a collection of separate and unrelated topics.

To raise the level of students' performance in problem solving and communication along with performance in mathematical content, STEM designed an assessment package on the basis of the following principles.

- *Assessment should have the improvement of learning as its primary goal.* The results of student assessment should inform teachers about the next steps in the teaching process. Information about students' strengths and weaknesses can prevent needless repetition and help identify which concepts have been learned.
- *Documenting students' achievement should be an integral part of the instructional process, not an add-on to it.* Assessment information can be drawn from instructional tasks, and additional instruction can be given in the context of assessment activities.
- *Assessment should include the active participation of students in open problems.* Open problems can be open-ended or open-response problems. An open-ended problem has many solutions and many ways to reach a solution (see Figure 1). An open-response problem has one answer but may have many ways to reach a solution (see Figure 2). Students should be involved with nonroutine open problems of both kinds, and alternative approaches to solutions should be encouraged and discussed.
- *Assessment should reflect real-world applications.* Students should be exposed to many real-world applications. Students should be able to specify the problem, collect and organize appropriate data, make conjectures from the data, formulate an appropriate model, validate and refine the model, and communicate the results in written and oral form.
- *Assessment should permit the full use of technology.* Assessments that truly assess mathematical power and not just computational skill should allow students to use all appropriate tools.
- *Assessment should use a variety of methods.* Assessment should use various methods and not be limited to simplified methods like multiple-choice items. Assessment should measure what is valued in the curriculum, not just what is easy to measure. The emphasis should be on assessing what students know and how they think about mathematics rather than what they do not know.

Figure 1

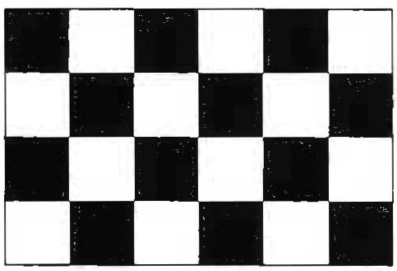
Example of an Open-Ended Problem

Design an ideal chair for students at your grade level. Be sure to tell how you obtained the data on which you based your design.

Although STEM uses several assessment techniques, including summative content activities, student journals, embedded assessments and portfolios, this article focuses on the use of open questions and their role in assessment. These types of questions involve students in activities that require interaction, communication and decision making in nonroutine situations. Open questions often reveal more about students and their thinking than do traditional questions. Open questions can be found in the project materials in two places: as part of extended explorations (E^2) and embedded within the instructional materials and practice exercises. Many of these problems have real-world applications. Alternative approaches are encouraged and discussed. The solution to these problems involves constructing a mathematical model for the situation. To find a solution, students must define the problem, devise and carry out a plan for solving it, and prepare a report that explains and interprets their solution. Full use of technology is permitted on these explorations. The presentation of the solution can take various forms: a written or oral report, a physical model or display, a dramatic presentation or a news report, or other forms that students can design.

Figure 2

Example of an Open-Response Problem



How many different squares do you see in this sports flag? Describe the method you used to find all the squares. Will your method work for any size checkered flag?

Extended explorations (E^2) are investigations or activities on which students work outside of class over a one- to two-week period. The solutions to the E^2s are evaluated with two sets of assessment criteria that are written in a language that students can understand. These criteria, or rubrics, are predetermined project standards that cover several levels of student achievement. These standards are based on NCTM's curriculum and evaluation standards (1989). One set is for student self-assessment, a process that has not been a part of a traditional program. Using the self-assessment rubric, students take an active role in assessing their own work, which is essential to improving their learning processes. Note that the self-assessment criteria in Figure 3a are written from the students' point of view, for example, "I used appropriate mathematical language. . . ."

A similar set of teachers' assessment criteria, shown in Figure 3b, is used to assess students' work from a teacher's point of view, for example, "You used appropriate mathematical language. . . ."

The combination of the student- and teacher-assessment criteria provides feedback so that students can improve as they work through the curriculum. As students become familiar with the assessment criteria, they understand what is needed to improve their problem-solving and communication skills. The assessment criteria allow the teachers to recognize a student's strengths and help the student focus on areas where growth is needed.

Students are asked to write their solutions to open problems by using mathematical language and their best representations to communicate how they solved the problem; the decisions they made as they solved the problem; and the connections they made to other problems, mathematics, or other subjects. Students' work is assessed according to the following five criteria:

- Problem solving
- Mathematical language
- Representations
- Connections
- Presentation

Through self-assessment and teacher-assessment criteria, students learn exactly what is important in good problem solving and communication and can evaluate their own progress in these areas. Students will respond if they know what is expected and if the criteria make the expectations very clear. These same assessment criteria are used in all three years of the project. They are introduced in the first module of each subsequent grade level. These assessment criteria are not add-ons to the curriculum but are an integral part of the teaching materials. The criteria clearly communicate to the students what is valued in the curriculum.

In the prepilot version of the materials, the criteria were not user-friendly and were hard to interpret. On the basis of teacher feedback, the rubrics have undergone many revisions before reaching their present forms. Notice the similarity between the criteria and the first four of the NCTM's (1989) curriculum and evaluation standards: problem solving, communication, reasoning and connections. The assessment criteria show students that these standards are valued and that they are an important part of the curriculum. Cheryl Wilson, a former middle school mathematics teacher turned principal, reports on her experience with the STEM assessment criteria:

> This year I have used drafts of this tool in my classroom and was amazed at how quickly students improved in their ability to solve complex problems and to write up their problem-solving solutions. This is a tool whose time has come. Assessment has been the focus of several recent professional journals, conferences and discussions. It is exciting to see a curriculum project develop a quality assessment instrument to accompany the curriculum materials.

Figure 3a
STEM Student Self-Assessment Criteria

Name ______________________________ Problem ______________________________

Student | **Self-Assessment**

☐ If your score is in the shaded area, explain why on the back of this sheet and stop.

☆ The star indicates that you excelled in some way.

Problem Solving

1 2 3 4 5 ☆

- **1:** I did not understand the problem well enough to get started, or I did not show any work.
- **3:** I understood the problem well enough to make a plan and to work toward a solution.
- **5:** I made a plan, I used it to solve the problem, and I verified my solution.

Mathematical Language

1 2 3 4 5 ☆

- **1:** I did not use any mathematical vocabulary or symbols, or I did not use them correctly, or my use was not appropriate.
- **3:** I used appropriate mathematical language, but the way it was used was not always correct, or other terms and symbols were needed.
- **5:** I used mathematical language that was correct and appropriate to make my meaning clear.

Representations

1 2 3 4 5 ☆

- **1:** I did not use any representations, such as equations, tables, graphs or diagrams, to help solve the problem or explain my solution.
- **3:** I made appropriate representations to help solve the problem or help me explain my solution, but they were not always correct or other representations were needed.
- **5:** I used appropriate and correct representations to solve the problem or explain my solution.

Connections

1 2 3 4 5 ☆

- **1:** I attempted or solved the problem and then stopped.
- **3:** I found patterns and used them to extend the solution to other cases, or I recognized that this problem relates to other problems, mathematical ideas, or applications.
- **5:** I extended the ideas in the solution to the general case, or I showed how this problem relates to other problems, mathematical ideas, or applications.

Presentation

1 2 3 4 5 ☆

- **1:** The presentation of my solution and reasoning is unclear to others.
- **3:** The presentation of my solution and reasoning is clear in most places, but others may have trouble understanding parts of it.
- **5:** The presentation of my solution and reasoning is clear and can be understood by others.

Figure 3b

STEM Teacher Assessment Criteria

Name ______________________________ Problem ______________________________

Teacher | **Assessment Criteria**

☆ The star indicates that you excelled in some way.

Problem Solving

1 — 2 — 3 — 4 — 5 — ☆

You did not understand the problem well enough to get started, or you did not show any work.	You understood the problem well enough to make a plan and to work toward a solution.	You made a plan, you used it to solve the problem, and you verified your solution.

Mathematical Language

1 — 2 — 3 — 4 — 5 — ☆

You did not use any mathematical vocabulary or symbols, or you did not use them correctly, or your use was not appropriate.	You used appropriate mathematical language, but the way it was used was not always correct, or other terms and symbols were needed.	You used mathematical language that was correct and appropriate to make your meaning clear.

Representations

1 — 2 — 3 — 4 — 5 — ☆

You did not use any representations, such as equations, tables, graphs or diagrams, to help solve the problem or explain your solution.	You made appropriate representations to help solve the problem or help you explain your solution, but they were not always correct or other representations were needed.	You used appropriate and correct representations to solve the problem or explain your solution.

Connections

1 — 2 — 3 — 4 — 5 — ☆

You attempted or solved the problem and then stopped.	You found patterns and used them to extend the solution to other cases, or you recognized that this problem relates to other problems, mathematical ideas or applications.	You extended the ideas in the solution to the general case, or you showed how this problem relates to other problems, mathematical ideas or applications.

Presentation

1 — 2 — 3 — 4 — 5 — ☆

The presentation of your solution and reasoning is unclear to others.	The presentation of your solution and reasoning is clear in most places, but others may have trouble understanding parts of it.	The presentation of your solution and reasoning is clear and can be understood by others.

Content used: ______________________________ Computational errors Yes _____ No _____

Notes on errors: ______________________________

The assessment criteria are not mastered immediately by either the teacher or the students. Mastery takes time and is part of the learning process. Peer assessment is also used to help students become comfortable with the criteria. The more that teachers and students work with the criteria, the more comfortable they become using them. Learning to work with these criteria can be thought of in the same way as learning a new language. At the beginning, communicating is very hard, but before long, teachers and students alike are using the same vocabulary and looking for the same things as they assess solutions to problems.

The assessment criteria are normally completed with a marking pen, and the score on each scale is marked along the ray below each heading. Notice the star after the 5 on each criterion. This feature was added to the original rubric so that if a student really excelled in some way, the effort could be rewarded. Students strive all year to score at this highest level, but it is reserved for exceptional performances.

To better illustrate how the assessment criteria work, a sample response on the E^2 in Figure 4 from a student named Marge is given in Figure 5. A discussion of Marge's scores follows.

Problem solving. Marge gave a level-5 response here. She used charts and tables to find a pattern and worked toward finding a general rule. Her explanations about her decisions as she solved the problem were clear, and her reasoning was correct. Her approach worked well, and she solved the problem.

Mathematical language. Marge correctly used some mathematical language, such as *probabilities, patterns* and *general rule,* and she used correct notations for fractions, percents and exponents. However, she should have referred to the "bottom number" as the *denominator.* Also, 2^n is not *n* raised to a power of 2, but rather 2 is a *base* and *n* is an *exponent,* or *power.* Because she could have used more mathematical language and some language was not used correctly, this response is at level 3.

Representations. Marge's representations, a table, charts and an equation, are accurate and appropriate for the problem, for a level-5 response.

Connections. Marge arrived at an algebraic generalization that showed how to find the probability for any number of children, which is a level-5 response. If she had extended her solution to more cases than called for, such as for 10 children, this response would have been at level 4.

Presentation. Marge's presentation is clear and can be understood by others—a level-5 response. Note that she did not have to type the material or produce computer graphics to score at level 5.

Figure 4

E^2

Russian Peasant Sets World Record in 1700s

The greatest officially recorded number of children born to one mother is 69. In 27 different pregnancies she gave birth to 16 pairs of twins, 7 sets of triplets, and 4 sets of quadruplets. What is the probability of having a family of one sex if the family consists of

a. one child?
b. two children?
c. three children?
d. four children?
e. five children?

Marge would receive a teacher-assessment criteria that looks like the one in Figure 6. If a student's self-assessment and the teacher's assessment do not agree, a great opportunity arises for a discussion between the student and the teacher. These discussions can lead to improved problem-solving and communication skills. If students assess themselves on a regular basis, they will become more accustomed to the criteria. Students know from the beginning what is important and what they have to do to score well. If one or more areas need work, a glance at the returned rubric tells the student immediately where effort is needed to improve performance.

Not all problems elicit all the criteria. Sometimes students are not scored on a particular criterion because it is not appropriate for the problem. In this situation, the criterion that is ignored is not counted when determining a grade on the activity. Profiles are available for those teachers who want to translate the rubric to a letter grade or a score to record in their grade book. A sample of a "B" profile is given in Figure 7.

Teachers and students found that it took time and work to become comfortable using the criteria. Some sample comments from field-test teachers follow:

- For my students, I think that this gives them real honor. It gives them real pride. It worked really well for me. It probably took me a good third of the year before they understood how to do it. When they left, everybody was a better evaluator.
- I think it is great. The kids really loved it. They liked the self-assessment. Amazingly, they were honest with their opinions about what they had done.

Figure 5

Marge's Work

Probabilities of the Same Sex

I wanted to find the probabilities of families having all girls or all boys if the families had 1 child, 2 children, 3 children, 4 children and 5 children. Maybe I could find out the probability of the Russian peasant's having all boys or all girls.

I decided to look at a family of 1 child first, then 2 children and so on, and see if I could find a pattern. I made the charts below to find the probabilities. The charts show all the ways the births could happen. I only made a chart for 1, 2 and 3 children because I found a pattern.

#	1	1
B	✓	
G		✓

1 child

	1		2		3		4	
#	1	2	1	2	1	2	1	2
B	✓	✓	✓					✓
G				✓	✓	✓	✓	

2 children

	1			2			3			4			5			6			7			8		
#	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
B	✓	✓	✓	✓	✓		✓						✓		✓		✓			✓	✓			✓
G						✓		✓	✓	✓	✓	✓		✓		✓		✓	✓			✓	✓	

3 children

I made a table showing the probabilities of 1 sex for families with different numbers of children.

# of children	Probabilities of 1 sex
1	$\frac{2}{2}$, or 100%
2	$\frac{2}{4}$, or 50%
3	$\frac{2}{8}$, or 25%
4	$\frac{2}{16}$, or 12.5%
5	$\frac{2}{32}$, or 6.25%

The pattern is that the probabilities get smaller by half for every extra kid. This pattern gave me the data, and it was much quicker. I could get a general rule:

If n = # of children, then the probability of the same sex is $\frac{2}{2^n}$.

I figured out the formula because I saw that the bottom number doubled with each extra kid. I figured that if the number of children was raised to the power of 2, the probability would always be given by $\frac{2}{2^n}$.

If there were 3 kids, then the probability of the same sex is $\frac{2}{2^3} = \frac{2}{8}$. I checked that this is what I got in my chart. I checked it for all the other cases.

I think that the probability that all the children of the Russian woman being the same sex is $\frac{2}{2^{69}}$, which I think is a very small number, so it probably would not happen.

- This can be very positive for the kids because they don't have to do it right. If they have graphs and tables and are communicating to express things using mathematical terms, hey, that's very good. It's good at building self-esteem at this point.
- It's excellent. The criteria that are written underneath each of the numbers are very specific but able to encompass lots of different problems. . . . It was not easy for them [students] at the beginning. It was my lack of self-assurance. I am used to a red answer key. As I got more confident, students were better able to tell what I expected.

Some sample student comments about what they thought of using rubrics are given next.

- I like having the rubrics because they help me a lot. When I just receive a grade, I don't know why. With the rubric I know why I got the grade and what I can do about it next time.
- I like having a rubric before I write something so I know what is expected of me. Sometimes I don't like rubrics because they make me feel a lot is expected of me, but that helps me set higher goals.
- Almost every time a rubric has helped me because when I get a grade that I don't like, I can either make it right or I can improve on the next problem.

Figure 6

Teacher Assessment of Marge's Work

Name *Marge* Problem *Russian Peasant*

Teacher | **Assessment Criteria**

☆ The star indicates that you excelled in some way.

Problem Solving

1	2 – 3	4 – 5
You did not understand the problem well enough to get started, or you did not show any work.	You understood the problem well enough to make a plan and to work toward a solution.	You made a plan, you used it to solve the problem, and you verified your solution.

Mathematical Language

1	2 – 3	4 – 5
You did not use any mathematical vocabulary or symbols, or you did not use them correctly, or your use was not appropriate.	You used appropriate mathematical language, but the way it was used was not always correct, or other terms and symbols were needed.	You used mathematical language that was correct and appropriate to make your meaning clear.

Representations

1	2 – 3	4 – 5
You did not use any representations, such as equations, tables, graphs or diagrams, to help solve the problem or explain your solution.	You made appropriate representations to help solve the problem or help you explain your solution, but they were not always correct or other representations were needed.	You used appropriate and correct representations to solve the problem or explain your solution.

Connections

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You attempted or solved the problem and then stopped.	You found patterns and used them to extend the solution to other cases, or you recognized that this problem relates to other problems, mathematical ideas or applications.	You extended the ideas in the solution to the general case, or you showed how this problem relates to other problems, mathematical ideas or applications.

Presentation

1	2 – 3	4 – 5
The presentation of your solution and reasoning is unclear to others.	The presentation of your solution and reasoning is clear in most places, but others may have trouble understanding parts of it.	The presentation of your solution and reasoning is clear and can be understood by others.

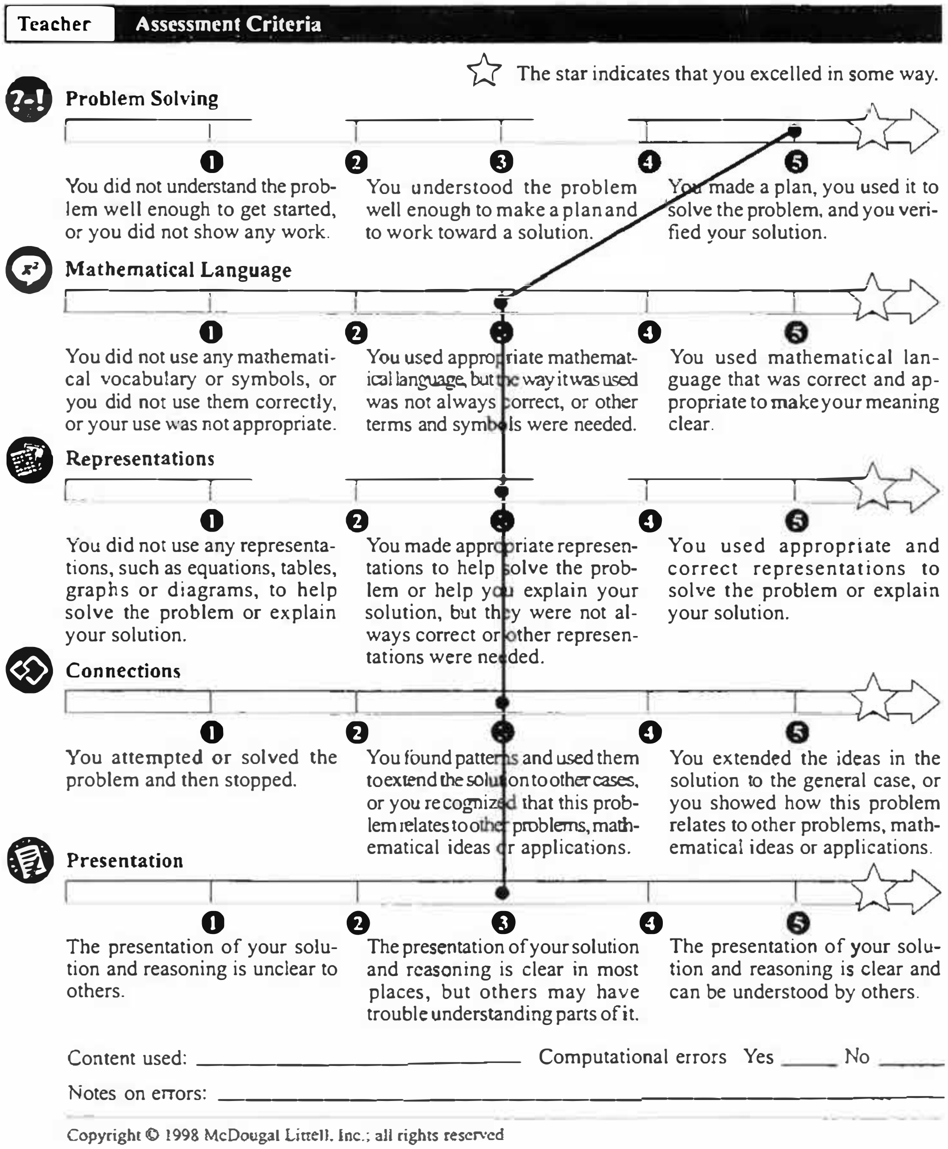
Content used: *probability, fractions, percents* Computational errors Yes ____ No *✓*

Notes on errors: *In 2^n, "n" is called the "exponent" and 2 is the "base." In a fraction, the "bottom number" is called the denominator."* *Great Job!*

In the STEM curriculum, we have asked students to become involved in real mathematics—to make conjectures, to solve problems and to present their solutions in a clear manner using convincing arguments. We have tried to design our curriculum around the ideas stated in *Measuring What Counts* (MSEB 1993):

> Important mathematics must shape and define the content of assessment. Appropriate tasks emphasize connections with mathematics, embed mathematics in relevant external contexts, require students to communicate clearly their mathematical thinking and promote facility in solving nonroutine problems. Considerations of connections, communication and nonroutine problems raise many thorny issues that testmakers and teachers are only

Figure 7
Possible "B" Profile



Name *B Profile* Problem ______

Teacher | **Assessment Criteria**

☆ The star indicates that you excelled in some way.

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Content used: ______ Computational errors Yes ____ No ____

Notes on errors: ______

beginning to explore. However, these considerations are essential if students are to meet the new expectations of mathematics education standards.

The use of extended problems and projects and their scoring with generalized multidimensional scoring rubrics are important components in the STEM project. When assessment is improved, learning is improved. The assessment techniques that are used not only indicate to students what is valued but also give them the criteria for judging their achievement. The rubrics engage students in thinking about performance expectations. Students' solutions give valuable information about their conceptual understanding or lack of understanding of mathematical concepts—teachable moments. A good assessment program can provide a mechanism for creating more teachable moments.

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