NCTM Standards in Action

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Learning to reason mathematically is fundamental to doing mathematics. The curriculum standards for school mathematics for Kindergarten to Grade 12 include mathematical reasoning and proof as one of five standards that describe mathematical processes through which students should acquire and use their mathematical knowledge. At each level, students should study mathematics in ways that include opportunities for mathematical reasoning and the construction of proofs.

At the primary level, the curriculum and evaluation standards suggest that the study of mathematics should emphasize reasoning with a focus on

- drawing logical conclusions about mathematics;
- using models, known facts, properties and relationships to explain students' thinking;
- justifying students' answers and solution processes;
- using patterns and relationships to analyze mathematical situations; and
- making students believe that mathematics makes sense. (NCTM 1989)

However, the standards do not suggest that formal reasoning strategies be taught at the elementary level. Instead, at this level, mathematical reasoning should centre on informal thinking, conjecturing and validating that help students to see that mathematics makes sense. It typically involves questions such as, Why do you think that is the correct answer? or Do you think that you would get the same answer if you added the parts in a different order? Enhancing students' confidence in their ability to reason and justify their thinking is critical, because as they move through the grades, students will begin to see that mathematics is not simply memorizing rules and procedures but that it makes sense, is logical and is enjoyable. It is also critical that the teacher nurture a climate in which the students have a genuine respect and support for one another's ideas. Statements made by both teacher and students should be open to question, reaction and elaboration from others. Students should be constantly challenged and encouraged to justify their solutions, thinking processes and conjectures in many ways. The use of manipulatives and models is an effective way of engaging students as active participants in the learning process. Models and manipulatives also provide students with concrete objects to gain a better understanding of the mathematical ideas and concepts.

At the middle school level, the curriculum and evaluation standards suggest that the study of mathematics should emphasize reasoning with a focus on

- recognizing and applying deductive and inductive reasoning;
- understanding and applying reasoning processes, with special attention to spatial reasoning and reasoning with proportions and graphs;
- making and evaluating mathematical conjectures and arguments;
- validating their own thinking; and
- appreciating the pervasive use and power of reasoning as a part of mathematics. (NCTM 1989)

Students should be provided with opportunities to explain their own reasoning and such explanations should be followed with questions: Why? What if...? Can you give me a counterexample? Can you give me an example of...? Do you see a pattern? Is it always true? Sometimes true? Never true? How do you know? Such questions prompt students to validate and value their own thinking. Having students identify patterns provides them with a powerful problem-solving strategy. It is also the essence of inductive reasoning. These patterns, in turn, can lead to conjectures about the problem. Students at these grade levels should be exposed to problem situations that are challenging but within reach. This may also lead to the use of computers for specific problems.

Students should be introduced to many kinds of mathematical reasoning. They can use reasoning to illustrate when something always, sometimes or never works. Situations involving counterexamples are also useful and important. Throughout the grades at this level, students should also develop the ability to reason proportionally, a process that requires a great deal of time for its development.

Reasoning must pervade all mathematical activities if students are to develop the ability to conjecture and to demonstrate the logical validity of conjectures. Teachers need to be mindful that at this level students still need many concrete materials to support their reasoning; this is especially true for spatial reasoning. Whether it is the use of technology or the presentation of challenging mathematical situations, students need the freedom to explore, conjecture, validate and convince others if they are to develop the ability to mathematically reason.

At the Grades 9–12 level, the mathematics curriculum should include numerous and varied experiences that reinforce and extend logical reasoning skills with an emphasis on

- making and testing conjectures;
- formulating counterexamples;
- following logical arguments;
- judging the validity of arguments;
- constructing simple valid arguments;

and, for students intending to go on to postsecondary studies,

- constructing proof of mathematical assertions, including indirect proofs and proofs by mathematical induction. (NCTM 1989)

For students at this level, inductive and deductive reasoning are required individually and in concert in all areas of mathematics. The goal is for students to experience both forms of reasoning in mathematics and in situations outside mathematics. For example, conjecturing by generalizing from a pattern of observations made in particular cases (inductive reasoning) and then testing the conjecture by constructing either a logical verification or a counterexample (deductive reasoning) are important mathematical experiences for students.

A second goal of this standard is to expand the role of reasoning, which is currently addressed in geometry only but which needs to be emphasized in all mathematics courses for allstudents. In addition, students planning postsecondary studies need to learn more formal methods of proofs.

The third goal of this standard is to give increased attention to proof by mathematical induction, the most prominent proof technique in discrete mathematics. In Grades 9–12, as the depth and complexity of content is increased, this emphasis on the interplay between conjecturing and inductive reasoning and the importance of deductive verification should be maintained. Furthermore, it is most appropriate that students see the application of various forms of reasoning to areas outside mathematics. The potential for transfer between mathematical reasoning and the logic needed to resolve issues in everyday life can be enhanced by explicitly subjecting assertions about daily affairs to analysis in terms of the underlying principles of reasoning.

Assessment of students' ability to reason mathematically has a critical part in ensuring that students have actually understood the different types of reasoning. To determine students' understanding and use of different types of reasoning, assessment must be focused on how students use all types of reasoning appropriate for their grade level. Such assessments of students' ability to reason mathematically should provide evidence that they can

- use inductive reasoning to recognize patterns and form conjectures;
- use reasoning to develop plausible arguments for mathematical statements;
- use proportional and spatial reasoning to solve problems;
- use deductive reasoning to verify conclusions, judge the validity of arguments and construct valid arguments;
- analyze situations to determine common properties and structures; and
- appreciate the axiomatic nature of mathematics. (NCTM 1989)

Teachers need to be aware that while all aspects of reasoning can be used at any grade level, some aspects of reasoning might be more appropriate than others at a given grade level. "Reasoning" and "proof" should not be taught in isolation, but rather, reasoning and proof must be a consistent part of students' mathematical experiences from Kindergarten to Grade 12.

The three articles that follow provide excellent examples of how reasoning can be developed through consistent use in the classroom.

Bibliography

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