A Collection of Connections for Junior High Western Canadian Protocol Mathematics

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We have put together "A Collection of Connections" that consists of 12 uses of junior high school mathematics. These activities support the communication and connections strands of the Western Canadian Protocol mathematics curriculum. In using them, teachers may adapt them extensively. They can serve as a basis of one to three mathematics periods. We have also found that teachers need to plan if they intend to incorporate them into their teaching units. They can be used as end-of-unit activities or as focal activities in the development of a unit. We would encourage teachers to use the activities as a means of bringing mathematical skills to life. These contexts provide an opportunity to enhance a student's view of mathematics. As one Grade 9 girl said at the conclusion of one activity: "That just proves that mathematics is everywhere."

The following are samples from the number and algebra strand.

Number (Ratios and Scale Drawings)
Church Windows
Church Windows Student Activities
Algebra
Rating the Bouncing of Balls
Rating the Bouncing of Balls Student Activities

Church Windows

Intent of the Lesson

This practical problem can be solved using only scale drawings and simple ratios. Students studying trigonometry should compare the scale drawing solution to the trigonometric solution. Solving the problem in two ways can be an important learning experience.

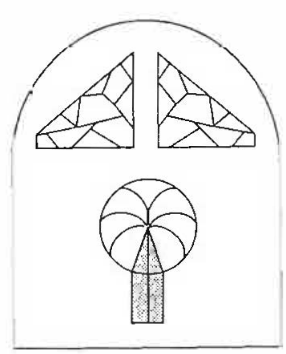
General Question

Architects are working on designing a church, whose main feature is a 9 m-wide "window wall." The windows in the wall are going to be stained glass. Because this wall faces east, the design will cast interesting patterns of light on the altar during morning service. Stained glass is an old tradition in many religions. However, sold by the square metre it is very expensive. The best price that the architects have been able to find is $3,000 per square metre. The total glass budget for the church is $270,000 and no more than a fifth of this amount can be used for stained glass. Architects want to know if the design below falls within the budget. The design consists of two windows that make a Gothic design at the top of the wall and a stylized candle with a glow in the lower part. In addition to the total area and the cost of the windows, the carpenter will have to know their dimensions, namely, how long each side is and the size of any angles.

The design is given below with the additional information:

The two windows in the upper part of the wall have a top angle of 49°. The length of the base of these windows is 3 m and the short vertical side is 0.4 m.

In the candle window the glow circle has a diameter of 3 m. The candle is 1 m wide, and the distance from the centre of the glow to the bottom of the candle is the same as the diameter of the glow.



Discussion Questions

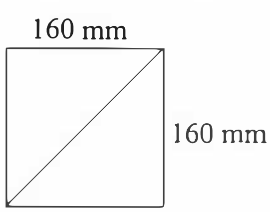
- Why is stained glass so expensive? (Because it is custom-made with lead filling.)
- What does *Gothic* mean in architecture? (*Gothic* is a style of architecture, featuring such things as pointed arches, common in Europe in the 12th–16th centuries.)
- Why do candles figure so prominently in many religions?
- Why are places of worship often oriented toward the east?

Preliminary Activity

Activity 1

Although building problems of this type can be solved using advanced mathematics, namely, trigonometry, they can also be solved by drawing to scale on grid paper. Most grid paper does not measure exactly to the millimetre. Because of this, a ruler can be used which measures to the nearest millimetre. The grid paper is, therefore, only useful in keeping the drawing at right angles.

Suppose we had a square window, each side measuring 1.6 m. What would be the length of the lead strip forming the diagonal? Our knowledge of the Pythagorean relationship would tell us that the length is given by $(1.6)^2 + (1.6)^2 = L^2$, or L = 2.27 m (approximately). Using the procedure of a scale drawing, a convenient scale should be chosen and the square can then be drawn. if we choose 1 mm = 10 mm (for a scale of 1:10) then 160 mm represents 1,600 mm (1.6 m).



By drawing the square as carefully as possible, the diagonal measures 226 mm. Because our scale was 1 mm = 10 mm, the answer for the length of the diagonal is 2,260 mm (2.26 m). This result is very close to the result achieved using the Pythagorean relationship (2.27 m). This activity shows that this scale measurement system for finding lengths is fairly accurate. In making the drawing, the thickness of even a pencil mark can make a difference of a millimetre or two. (When cutting a board, a carpenter must take into account the width of the saw blade.)

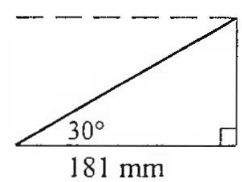
Discussion Questions

- What should be considered in choosing a scale? (It should be large enough for accuracy, but should still fit on the page.)
- Why do we make the drawing as large as possible?
- Why does the thickness of the pencil line lead to inaccuracies? (It is 1 mm wide.)
- What might be done to minimize this? (Use the procedure of measuring to the inside of the line.)
- When we solve problems by scale drawing, what mathematical concept are we using? (Ratios.)

Activity 2

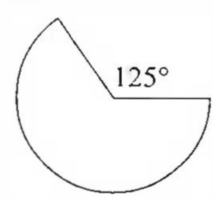
What are the dimensions of a rectangle whose longer side is 3.62 m and whose diagonal makes a 30° angle with the longer side? Choosing an appropriate scale when making the drawing is important. For one side to be roughly 180 mm long, the scale of choice will be 1 mm = 20 mm. Our rectangle will be 3,620 ÷ 20 =181 mm long. Draw this on the square paper first, and then construct an angle of 30° to this line.

The shorter side measures 106 mm. Converting back to the original scale, the side of the rectangle will be 106 mm × 20 which is 2,120 mm (2.12 m). Using trigonometry the answer is 2.12 m. Again our scale drawing method gives very close answers.

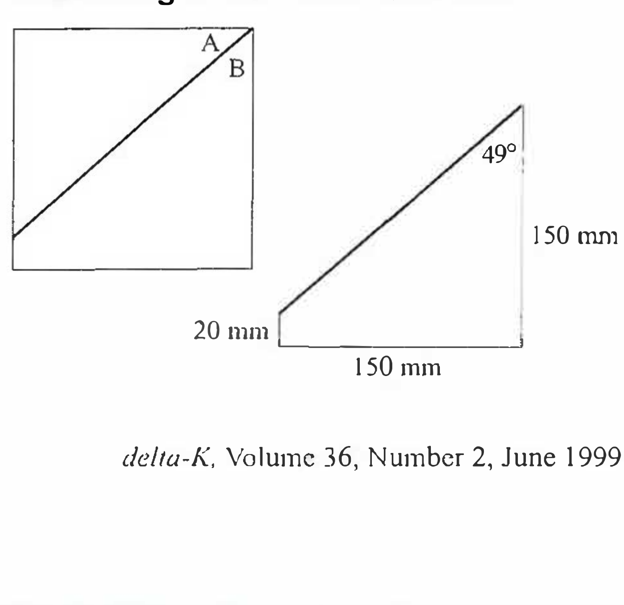


Activity 3

The stained glass pattern on the door can be thought of as a fraction of a circle. For example, the area of a quarter circle can be determined by $\frac{1}{4}\pi r^2$. If the angle in the pattern is 125°, the area of the pattern is $235/360\ \pi r^2$.



Answering the General Question



The Trapezoidal Windows

The student is asked to solve the problem by drawing a fairly large rectangle on the grid paper. The figure can be drawn starting with the 3-m side of the rectangle and angle A equal to 41°.

A convenient scale is 1 mm = 20 mm. The 3-m side becomes 150 mm. The angle at A is drawn. The 0.4-m side is 20 mm. The remainder of the rectangle can now be drawn.

The remaining side is measured as 150 mm in the representative drawing or as 3,000 mm on the window. Using these values, the area of both trapezoidal windows can be found. To calculate the area, it is necessary to divide the window into two parts: a rectangle and a triangle. The base of the triangle would be 3.0 m and the height would be 2.6 m (since 3.0 m − 0.4 m = 2.6 m). The following calculations give the values of the areas for these sections:

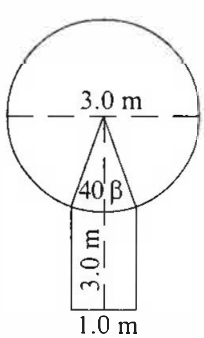
Area of Rectangle = 0.4 m × 3.0 m
= $1.2\ m^2$

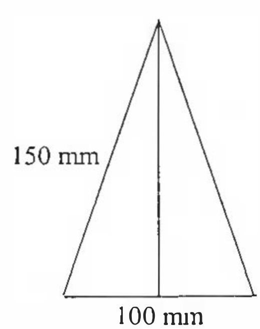
Area of Triangle = 0.5(3.0 m)(2.6 m)
= $3.9\ m^2$

Therefore, the area of both trapezoidal windows is $2(1.2 + 3.9)\ m^2 = 10.2\ m^2$

The Candle

Again the problem is solved with a scale drawing. Using a scale of 1:10, the radius of the glow is 3,000 ÷ 10 = 300 mm. The distance from the centre to the bottom of the candle will be 300 mm and the width of the candle will be 100 mm. Once the drawing is complete, measurements can be made. The angular measurement of the glow is 320°. Therefore the area of the glow is $320/360\ \pi r^2$, where r = 1.5 m. The area of the glow is $2\pi\ m^2$ or $6.28\ m^2$.





The area of the candle can be determined as a rectangle with a triangle on top. The dimensions of these figures can both be determined through measurement. Ruler measurement yields a rectangle of 100 mm by 155 mm while the triangle has a base of 100 mm and a height of 300 − 155 = 145 mm.

In using scale drawings for this activity, students do not need to reproduce the complete window designs; they only need to make those scale drawings that help in the calculations. Without using trigonometry, scale drawing is the only means of solving this problem.

The areas of these figures are given:

Area of Triangle = 0.5 (1.0 m)(1.45 m)
= $0.725\ m^2$

Area of Rectangle = (1.55 m)(1.0 m)
= $1.55\ m^2$

The total area of the candle is then 0.725 + 1.55 = $2.275\ m^2$.

In calculating areas, students can be reminded that they can calculate the area in the scale figure or convert to actual dimensions of the window and find the area. The latter is less confusing.

The students can use the calculated areas to solve for the overall cost of the stained glass window. They will find that they will be over budget if all the designs are made from stained glass. Perhaps suggest that the candle be made of wood instead. Calculations of the cost of the stained glass are as follows:

Cost of Candle = $(2.275\ m^2)(\$3,000/m^2)$
= \$6,825

Cost of Glow = $(6.28\ m^2)(\$3,000/m^2)$
= \$18,840

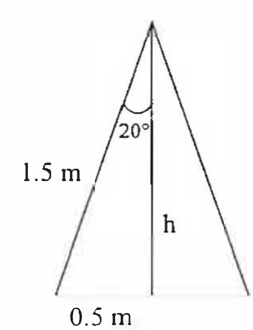
Cost of Trapezoidal Window = $(10.2\ m^2)(\$3,000/m^2)$
= \$30,600

The total cost of the windows is then \$56,265.

Materials

Grid paper (paper with 0.5-cm markings is useful), a ruler with millimetre markings and a protractor will all be very useful.

Modifications



This lesson can also be taught as a review or extension of trigonometry rather than by scale drawings. If trigonometry is to be used, the height of the triangle can be calculated using the Pythagorean theorem: $h^2 = (1.5\ m)^2 - (0.5\ m)^2$

$h = 1.4\ m$

Trigonometric functions can also be used to solve for the value of h as follows:

$$\cos 20° = \frac{h}{1.5\text{ m}} \qquad \text{or} \qquad \tan 20° = \frac{0.5\text{ m}}{h}$$

$$h = 1.4\text{ m} \qquad\qquad h = 1.4\text{ m}$$

Church Windows Student Activities

General Question

Architects are working on designing a church whose main feature is a 9 m-wide "window wall." The windows in the wall are going to be stained glass. As this is an east facing wall, the design will cast interesting patterns of light on the altar during morning service. Stained glass is an old tradition in many religions. However, sold by the square metre it is very expensive. The best price that the architects have been able to find is $3,000 per square metre. The total glass budget for the church is $270,000 and no more than a fifth of this amount can be used for stained glass. Architects want to know if the design below falls within the budget. The design consists of two windows that make a Gothic design at the top of the wall and a stylized candle with a glow in the lower part. In addition to the total area and the cost of the windows, the carpenter will have to know their dimensions, namely, how long each side is and the size of any angles.

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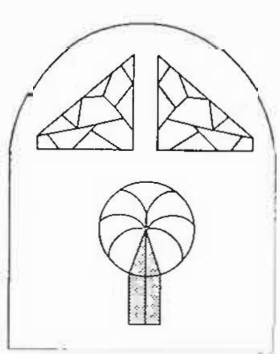
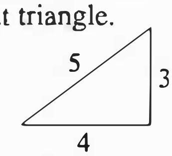
> The two windows in the upper part of the wall have a top angle of 49°. The length of the base of these windows is 3 m and the short vertical side is 0.4 m.
>
> In the candle window, below, the glow circle has a diameter of 3 m. The candle is 1 m wide, and the distance from the centre of the glow to the bottom of the candle is the same as the diameter of the glow.



Activities

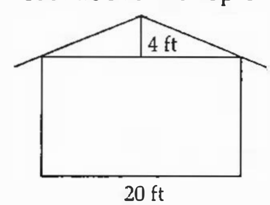
1. (a) Carpenters make a right angle by drawing a 3, 4, 5 right triangle.



Draw the triangle to scale and determine the measure of each angle.

(b) What are the angle measures of the 6, 8, 10 right triangle? Explain.

2. (a) A building is 20 feet wide and the peak of the roof is 4 feet above the top of the sides.

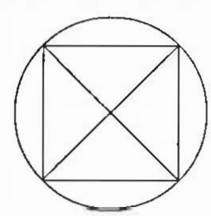


How long must the rafters be if they need to stick out 2 feet from the building?

(b) What is the angle of the peak of the roof?

(c) The building code requires that any roof rise has a ratio of 1:4. This means the roof must go 1 foot up for every 4 feet across. Is this roof steep enough? What is its ratio?

3. (a) What is the area of a square inside a circle 1 m in diameter?
(A square can be drawn to scale by making the diagonals at the centre at right angles.)

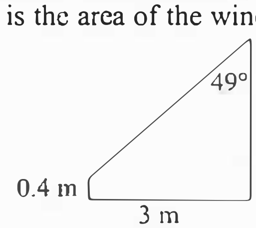


(b) What is the ratio of the area of the square compared to the area of the circle? Does this answer seem reasonable?

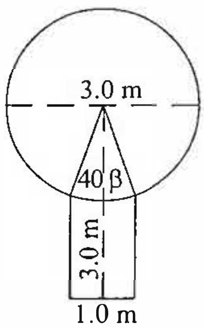
(c) Find the area of the square using the Pythagorean relationship. Is this answer similar to the answer in 3a?

4. (a) The angle at the top of this window is 49°. The base is 3 m and the short vertical side is 0.4 m. What are the dimensions of the window?

(b) What is the area of the window?

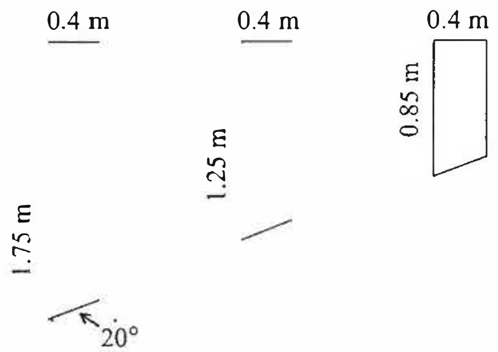


5. (a) In the candle window below, the circle, or the glow, has a diameter of 3 m. The candle is 1 m wide, and the distance from the centre of the glow to the bottom of the candle is the same as the diameter of the glow. What is the area of the glow? (Do not include the area of the glow that the candle takes up.)



(b) What is the area of the candle?

6. Suppose that instead of a candle, the following design was used in the window. Find the area of each of the strips, given the following diagram:



Rating the Bounce of Balls

Intent of the Lesson

Regardless of the height from which a ball is dropped, it will always bounce the same fraction of the original height. A series of linear equations is developed based on the bounce of several balls. The mathematics involved is fractions, variables, graphs of linear equations, the slope of a line and the line of best fit.

General Question

Everyone knows that different balls bounce differently. How high will a ball bounce? The question is asked by a consumer rating group that wants to rate a variety of balls to tell its consumers how much bounce any given ball will have. In this activity, a system for rating the "bounce" of a variety of the common balls currently sold will need to be developed. The use of the unscientific word "bounce" is informal and intuitive. Bounce will turn out to be the coefficient of elasticity, a scientific concept defining a property of elastic materials. A much more informal method of rating of the bounce in balls will be examined in this lesson.

Initially, the teacher can take a volleyball and bounce it. Be sure to hold it high and simply drop it. The class can guess what fraction of its original height it bounces to. It can then be held at a lower height, and the fraction guessed again. The question is how to make a solid estimate of the bounce of any ball. A rating system will definitely be needed that will describe the bounce from a whole range of heights.

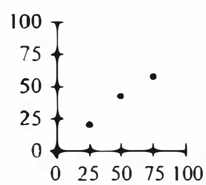
There are 10 balls that should be rated. Each will be tested and a way of labeling the balls according to their bounce will need to be developed.

Discussion Questions

- On what does the bounce of a ball depend? (On how it is made: hard rubber, full of air, less air; on the surface of the ball: completely spherical or with tiny indentations, like a golf ball; on the surface of the floor: soft or hard.)
- How high will a ball bounce? (Depends on how hard it is thrown or on its fall height.)
- In comparing balls, how can conditions be kept the same? (Drop them on the same floor or from the same height.)
- Does the speed of the fall depend on the weight of the ball? (A large ball will encounter air resistance but heavy balls fall at the same speed as light balls.)
- Does the ball bounce the same from different heights? (This will be one thing to find out in the trials. Do some balls not bounce very well from low heights? The "perfect ball" will bounce the same fraction at different heights.)

Preliminary Activity

Given three points on a graph which are connected by a straight line, how can the equation for the line be deciphered? First of all, it is very easy because the points lie in a straight line. Second, it becomes even easier if the line goes through the origin. Example of points: (25, 20), (50, 40), (75, 60).



These points lie in a straight line that goes through the origin. If each x value is multiplied by 0.8, the result is the y value. For example, $0.8 \times 25 = 20$; this shows the relationship between the y and x values is

$y = 0.8x$

Another set of points is (25, 15), (50, 30), (75, 45). By experimenting, the multiplier is seen to be 0.6. Therefore, the relationship is

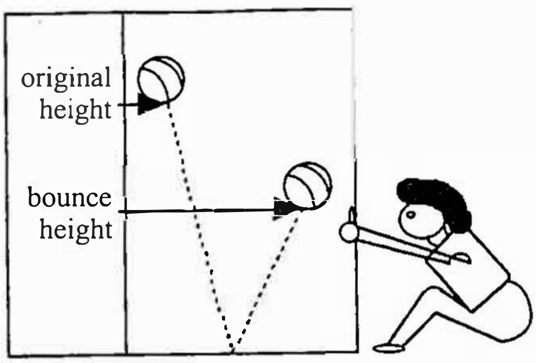
$y = 0.6x$

Whenever there is a set of points in a straight line going through the origin, there is always a simple relationship where y is some number times x.

Answering the General Question

Making the Measurements

Have a look at a volleyball. One way of investigating this is to drop the ball and take measurements of the *original height* and the *bounce height*. Measurements should be taken from the bottom of the ball. A discussion of this point will demonstrate that when the original height is zero, the desired bounce height should also be zero. In the case of the volleyball, if measured from the top, our smallest original height would then be 25 cm.



The Trials

An investigation of the bounce for different heights will be conducted because the consumer rating service wants to give the reader a complete picture of how each ball bounces. In these trials, students should take five different original heights in addition to the zero height. Experimenting with the volleyball makes it clear that measurements are not very accurate for small heights. The teacher should suggest five trials with heights ranging from the lowest of 0.6 m to the highest at 2 m. For each original height, there is a corresponding bounce height. The students can record their data in a table such as the one below, which is a typical record of one ball's trials.

Each trial will result in two values; these can be thought of as variables. The values can then be plotted on a graph. Now the mathematical problem starts. The problem for each group is to do the trials, make the graph and see if they can tell from the graph what the relationship between the original height and the bounce height is.

The Graph

Two suggestions for making a graph are

1. Put the original height on the x-axis and the bounce height on the y-axis.
2. Use grid paper but make the y-axis a scale of 2 larger than the x-axis.

Teaching Suggestions for the Trials

Student groups can work at different stations around the classroom. A hard floor is essential because most balls do not bounce very well. Each group needs a different ball, as well as adding machine tape on which to mark original heights and bounce heights. They should mark the initial bounce height, then repeat the trial to make sure the mark is correct. Once the marks are made accurately, the paper may be laid on the floor and measured precisely. Students might use a system of marking the original height with one colored pen and bounce height with another color (or a pencil). They will need to experiment with a few drops of the ball to get their markings more accurate. One person should hold the ball, a second should make sure it is at the right original height and the third should be ready to note its bounce height.

Trial number	Original height	Bounce height	Bounce height based on the line of best fit	Line of best fit equation
1	0	0	0	
2	0.6	0.2	0.2	$y = 0.3x$
3	0.9	0.3	0.3	
4	1.2	0.4	0.4	
5	1.6	0.5	0.53	
6	2.0	0.7	0.66	

After each trial has been completed at the different heights, the students can begin making the graph. Each student should make his or her own graph to ensure that each has a record of the group's findings for future reference.

The teacher may want to specify the original heights. This is not necessary but adds a measure of control to the trials. We have noted that, unless the teacher gives firm guidelines, students are not good at carrying out accurate measurements. Accuracy is not essential for the purposes of this lesson, but it is still important.

Observations from the Graph

When the graphs are completed, a discussion can reveal that the graph is described as linear. Students should talk about error in measurements; they should then make a "line of best fit" in accordance with these measurements. This, of course, is an approximation though it still must go through the origin (0, 0); if the original height is zero, the bounce height will be zero. Once they have a line, students should rename the points of the bounce height (the fourth column of our table). The students can now use their calculators to find the multiplier of the x-value (original height) to give the y-value (bounce height). This may require some teacher guidance because it is really a trial-and-error process. Following the discovery of the multiplier, every bounce height should be checked against its original height. A two-decimal place number (designated as B) should be used. This will result in the equation $y = Bx$, where y is bounce height and x is original height for the line of best fit. A discussion of the equation will reveal that the numerical coeffecient, B, is the bounce of the ball.

After students have the graphs of their points, straight lines and B values, all 10 lines can be plotted on the same graph on the blackboard, overhead projector or computer, if available. All lines should go through the origin, and they should all have a different slope.

Discussion Questions

- What does mean if the line is straight? (The ball bounces the same fraction for all heights.)
- Which graph has the points closest to the line of best fit? What does this mean? (A "perfect" ball situation and careful measurement.)
- What does it mean when a point falls far from the line? (It probably means an error in measurement.)
- In comparing all of these lines, which one represents the ball with the most bounce? (The line with the steepest slope.)
- Why are none of the B values greater than one? (A ball cannot bounce higher than its original height.)
- What number can be used for labeling the balls? (Use rounded B value.)

Lesson Conclusion

Each group can choose one of its graphs to be displayed. The original points and the line of best fit should be indicated in addition to the B value for their ball. If the points are close to the line, it means that the measurements were taken accurately. The fact that all these points lie on a straight line shows that the ball's bounce is the same fraction regardless of the height it is dropped from. For the consumer group, each ball could be labeled with a two-digit number, given by B × 100. Therefore, a ball with a label of 24 will bounce 0.24 of its original height. Question 5 in the student activities deals with consumer labels.

The discussion should also include balls of special note such as volleyballs and basketballs. These balls should have a standard bounce. In fact, if two basketballs and two volleyballs are in the trial, their bounce can be compared. Would a ball that had more bounce be better in either sport? In baseball, for example, the "superball" favors the batter.

Students may also be interested to know that the numerical coefficient which is being called "bounce" is actually called the "coefficient of elasticity" in science. Mathematically speaking, however, this coefficient is the slope of the line. It is evident that for every metre by which the original height increases, the bounce height increases B metres. It should also be noted that none of the lines of best fit has a larger slope than B = 1. If B were greater than one, it would mean that the ball was bouncing higher than its original height, which is not possible. If the x and y-axes have the same scale, then no line is at an angle greater than 45°.

Students may be asked to attach their adding machine tape to their graph. This is a kind of trial record to show an observer how accurately the trial was done. It is noteworthy that the science involved in this activity is in making nice drops and accurate measurements. Scientists would also be interested in explaining why the graph of the ball's bounce height falls in a straight line. Mathematicians, on the other hand, would be curious mainly about the relationship being developed.

Materials

Several differing balls, adding machine tape and grid paper are essential items for this activity. In preparation for this class, the teacher should either borrow some balls from the gymnasium or have the students bring 10 different balls. Some students will have super-bounce balls, while others will bring volleyballs, basketballs, tennis balls or racket balls. Students should also be encouraged to bring any balls that are especially bouncy. Perhaps, if more than 10 balls are brought, the teacher can select balls that have some range of bounce.

Modifications

If students are not familiar with finding linear relationships or if they are not familiar with graphing, the fraction for the five different heights can be averaged. This average will be the bounce of the ball.

Rating the Bounce of Balls Student Activities

General Question

Everyone knows that different balls bounce differently. How high will a ball bounce? A consumer group wants to rate how high a variety of balls will bounce. Can you develop a system for rating the bounce of a variety of balls? The bounce from different heights will have to be considered.

Activities

1. Use your calculator and experiment to find the multiplier (the multiplier times the first number will give you the second number) for these pairs of numbers.
   (a) (2, 5), (4, 10), (6,15)
   (b) (20, 1.2), (40, 2.4), (60, 3.6)
2. In the following sets of numbers, one of the sets does not quite fit the pattern. What is the pattern (that is, what is the multiplier)? Which set does not fit the pattern?
   (a) (3, 0.75), (5, 1.25), (6, 1.45), (8, 2)
   (b) (0.2, 0.3), (0.3, 0.5), (0.5, 0.75), (0.6, 0.9)
3. Write the four different relations from questions 1 and 2 as
   second number (S) = multiplier (M) × first number (F),
   or S = M × F
   For example, the relation for question 1a is
   S = 2.5 × F
4. Given below are the measurements for a trial of bounces:
   (a) For each trial, find the multiplier that the original height must be multiplied by to get the bounce height.
   (b) Examine the column of multipliers and choose a reasonable average multiplier. Using this average multiplier, fill in new bounce heights in the appropriate column. What is the relation now between the new bounce height and the original height?
   (c) What is the difference between the new bounce height and the experimental bounce height for each trial? Fill in the final column of the table with the differences? Why is there a difference?
   (d) Plot the original heights and bounce heights on a graph. Then, plot the new bounce heights on the same graph. Indicate any discrepancies.
5. (a) A consumer rating group has given you a ball that has a bounce rating of 45. Make a graph using similar original heights as in your trial and the expected bounce heights.
   (b) Using the graph and your knowledge of bounce heights, determine how much higher this ball will bounce for each increase of 1 m in the original height.

Authors' Note: Those readers interested in the entire volume of "A Collection of Connections," may contact Sol E. Sigurdson, Faculty of Education, Department of Secondary Education, University of Alberta, Edmonton T6G 2G5; phone (780) 492-0753.

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Trial	Original height	Bounce height	Multipliers	New bounce height based on average multiplier	Differences bounce ht. and new bounce ht.
1	0	0			
2	0.6	0.24			
3	0.9	0.36			
4	1.2	0.50			
5	1.6	0.65			
6	2.0	0.78			