Did You Know That? Against All Odds, Lotto 6/49!

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The growth of lotteries in Canada has provided a source of great math connections and interesting problems involving probability and combinations that are well worth sharing with students.

Did you ever wonder how they calculate the odds of winning any of the five prizes in the Lotto 6/49? I spent many an evening with my calculator trying to make sense of the many combinations. I chanced upon an article in the November 4, 1996, *Maclean's* that discussed the probability of winning the Lotto and which in turn led me to an Internet site compiled by Fred Hoppe, a professor of mathematics and statistics at McMaster University in Ontario: http://www.the-web.net/Lotto.

I could easily calculate the odds of winning the top prize, the jackpot, but I must admit that I was baffled as to how to calculate the odds of winning the lesser prizes. So what follows was gleaned from Professor Hoppe's explanations.

Jackpot (All Six Winning Numbers Selected from 49 Possibilities)

There are a total of 13,983,816 different groups of six numbers that could be drawn from the set of numbers 1, 2, ..., 49. How do we get that? There are 49 possibilities for the first number drawn, following which there are 48 possibilities for the second number, 47 for the third, 46 for the fourth, 45 for the fifth, and 44 for the sixth. Multiply (49)(48)...(44) to get 10,068,347,520.

Each possible group of six numbers (combination) can be drawn in different ways depending on which number in the group was drawn first, which was drawn second and so on. There are 6 choices for the first, 5 for the second, 4 for the third, 3 for the fourth, 2 for the fifth, and 1 for the sixth. Multiply these numbers out to get 720. We then divide 10,068,347,520 by 720 to arrive at 13,983,816 as the number of groups of six numbers (different picks).

This is the same as calculating the number of combinations of 49 numbers taken six at a time, ${}_{49}C_6$. Since all numbers are assumed to be equally likely and since the probability of some number being drawn must be one, it follows that each pick of six numbers has a probability of $1 \div 13,983,816 = 0.00000007151$. That is the probability that you will win the jackpot in the Lotto 6/49!

To use a concrete example, this can be considered to be roughly the same probability as obtaining 24 heads (or tails) in succession when flipping a fair coin, 2^{24}.

Second Prize (Five Winning Numbers + Bonus)

To win the second prize in the 6/49, the pick of six must include five winning numbers plus the bonus. Because five of the six winning numbers must be picked, this means that one of the winning numbers must be excluded. There are six possibilities for the choice of the excluded number, ${}_6C_1$; therefore, there are six ways for a pick of six to win the second place prize. Therefore the probability is $6 \div 13,983,816 = 0.0000004291$. Invert this number to get the odds of 2,330,459:1. This is roughly the same probability as obtaining 21 heads (or tails) in succession when flipping a fair coin! That is about 2^{21}.

Third Prize (Five Winning Numbers Only Selected)

As with the second prize, there are six ways for a pick of six to include exactly five of the six drawn numbers, ${}_6C_5$. The remaining number must be one of the 42 numbers left after the six winning numbers and the bonus number have been excluded. Therefore there are a total of $6 \times 42 = 252$ ways for a pick of six to win the third prize. To get this probability divide $252 \div 13,983,816$ to get 0.00001802, which when inverted gives odds of about 55,493:1. Again, this is about the same as the probability of obtaining 16 heads (or tails) in succession when flipping a fair coin. That is about 2^{16}.

Fourth Prize (Four Winning Numbers Selected)

There are 15 ways to include four of the six winning numbers, ${}_6C_4$, and 903 ways to include two of the 43 nonwinning numbers, ${}_{43}C_2$, for a total of 15 × 903 = 13,545 ways for a pick of six to win the third prize, which works out to a probability of 13,545 ÷ 13,983,816 = 0.0009686, which is odds against of about 1,032:1 or about the same as tossing 10 heads in a row with a coin.

Fifth Prize (Three Winning Numbers Selected)

There are 20 ways to include three of the six winning numbers, ${}_6C_3$, and 12,341 ways to include three of the 43 nonwinning numbers, ${}_{43}C_3$, for a total of 20 × 12,341 = 246,820 ways for a pick of six to win the fourth prize. This is a probability of 246,820 ÷ 13,983,816 = 0.01765, which gives odds against of about 57:1. Again, this is about the same as tossing 6 heads (or tails) in a row, 2^6.

I also discovered that a British actuary, Jim Roberts, used the same odds as Lotto 6/49 to compute the life expectancy of men and women. Here is what he calculated:

> A 20-year-old man is as likely to die in the next 40 minutes as to win the jackpot. A 40-year-old man is as likely to die in the next 20 minutes. A 60-year-old man is as likely to die in the next 2 minutes and an 80-year-old man in the next 20 seconds!

It seems that things are somewhat brighter for females:

> A 20-year-old woman has an equal chance of dying in the next 107 minutes as winning the jackpot, while an 80-year-old could last 30 seconds!

So now you know!

The Brooches of the Princess

Princess Astrid has a number of valuable brooches in her safe. Every brooch has the same number of diamonds on it. If one knew the number of diamonds in the safe, one would be able to determine the number of brooches and the number of diamonds on each brooch. Let me tell you that there are between 200 and 300 diamonds in the safe. How many brooches with how many diamonds on each brooch does the princess own?