Tiling Pattern Possibilities: A Geometric Classroom Activity

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The NCTM curriculum standards call for integrating geometry at all levels of the curriculum. Teachers are always looking for interesting settings which can be used to aid in this integration. We shall present a situation involving floor tiles.

We recently saw two floor tile patterns in two unrelated buildings. In each case, identical copies of an original 8 × 8 square were used to cover the entire floor. These patterns are displayed in Figures 1 and 2. An obvious question is, Are these all the same pattern? A cursory investigation suggests that they are not. For instance, the upper left-hand corners are contained in, respectively, 2 × 1 and 1 × 2 rectangles. However, a count of the number of different sized rectangles in each arrangement yields the results shown in Table 1 for each of Figure 1 and Figure 2.

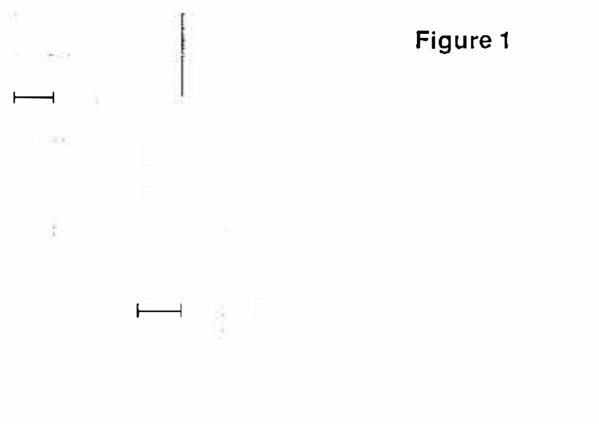


Figure 1

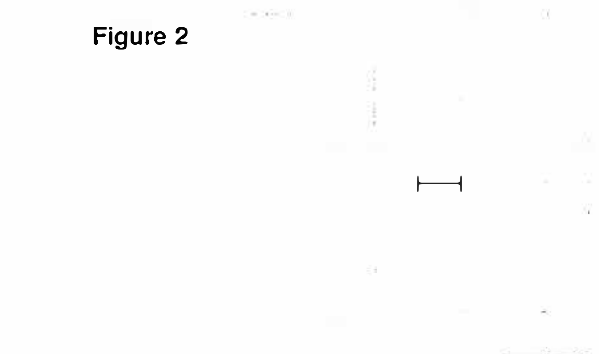


Figure 2

Table 1

Size of Rectangle	Number in Each Figure
1 × 1	14
2 × 2	6
2 × 1 or 1 × 2	13

The number of square units resulting from this distribution of rectangles is

14(1) + 6(4) + 13(2) = 64. This result led us to consider whether or not a rigid transformation can be applied to Figure 1 to yield Figure 2.

In fact, such a rigid transformation does exist. By trial and error, we found that the following steps will transform the pattern of Figure 1 into the pattern of Figure 2.

1. Rotate Figure 1 clockwise 90° about its centre.
2. Reflect the results of step 1 about the horizontal axis which passes through the midpoints of the opposite sides of the square.

The results of these two steps yield a pattern identical to that of Figure 2. Since a rotation is the combination of two flips about intersecting lines, the total transformation can be accomplished by three flips. Draw them.

You may note that a single reflection, about the lower-left to upper-right diagonal of Figure 1, will also transform the pattern of Figure 1 into the pattern of Figure 2. How can this result be reconciled with the previous rotation-reflection sequence?

Some students may note that although the transformation of Figure 1 into Figure 2 works mathematically, it would yield an upside-down tile. Would your students feel that this is an acceptable transformation?

In how many other ways could Figure 1 be transformed into other patterns by a series of rotations and/or flips? Recall that there are four possible rotations (0°, 90°, 180°, 270°); four possible flips (vertical at the midpoints of the opposite sides, horizontal at the midpoints of the opposite sides, and through the two diagonals); and two possible orderings of rotations and flips.

This is a nice group activity. Give each student a copy of Figures 1 and 2 and let the class experiment.

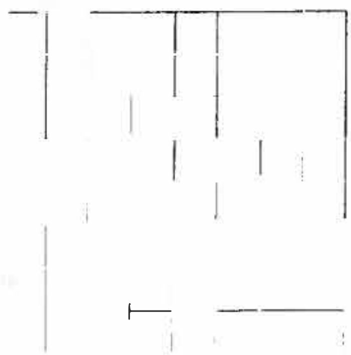
Extensions

1. Arrange the rectangle of Figure 1 to achieve patterns that are not the result of applying a rigid transformation.

   For example, Figure 3 is achieved by interchanging the 2 × 1 rectangle in the upper left-hand corner with the two 1 × 1 squares directly beneath it. In how many other distinct ways can the 33 original rectangles of Figure 1 be arranged into an 8 × 8 square?
2. Use combinations of other numbers of the original three types of rectangles to make new patterns on the 8 × 8 square.
3. Use rectangles of other sizes to make the 8 × 8 square.
4. Change the size of the 8 × 8 square.

We believe that mathematical situations constantly occur in nonconventional places. Teachers and students must be alert for them and share them in their classrooms.

Figure 3



Two Drivers

At 8:00 a.m., one driver starts his 782-km journey from A to B. He drives his car at 125 km/h. At 8:40 a.m., a second driver starts at B and drives toward A at 115 km/h. How far apart are the two cars 15 minutes before they meet on the highway?