

The Problem with Factorials!

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One Saturday, my friend was preparing a presentation for an important conference when his daughter, Susan, stepped into his office. She appeared bored, began to play with his old typewriter, touched everything else in sight and muttered some incomprehensible gobbledygook. My friend asked her what she had done in school the day before. She replied that she had learned something about “factorial of a number.” “What is that?” he asked. Susan replied that “the factorial of a positive integer is the product of all positive integers from 1 up to and including the given number.” She added, “For example, the symbol for factorial is !, so that 4!, which is read ‘4 factorial,’ means $1 \times 2 \times 3 \times 4 = 24$.”

At this point, my friend presented Susan with a challenging problem in the hope that it would keep her occupied for a long time, so that he could complete his presentation. My friend made Susan a proposition: “When you add the factorials of all numbers from

1 to 100, that sum is a very large number. If you can find the last digit of that sum I will give you \$5.” His hope to be free from disruption by Susan was short-lived. In less than two minutes, Susan shouted out a number and claimed that this was the last digit of that sum. My friend, not knowing the answer himself, thought that Susan simply made up her answer. Well, my friend was not prepared to accept this answer without checking it, nor was he going to make it easy for Susan. He put his presentation aside and began to use his calculator. He soon realized that the calculator was not much help. Suddenly, he remembered a shortcut, which his daughter had probably used, and determined that her answer was correct.

At this point, I invite you, the reader, to participate. What is the last digit of the sum of all factorials from 1 to 100? Please submit your solution and solution process to the editor (see the executive list on the back inside cover for address).

The Sum of the Digits

Take the sum of the digits for each number from 1 to 1,000,000 and then find the sum of these sums.
