The Geometry of Soap Bubbles and Soap Films

Sandra M. Pulver

Soap bubbles and films incite a certain special fascination at any age. They exist in perfect geometrical form and have a simplicity that instantly appeals to the mathematical mind. Soap films minimize area and soap bubbles in three dimensions try to minimize enclosed volumes.

Soap bubbles and films contain soap molecules and water molecules. Soap molecules consist of sodium stearate, a long-chain fatty acid. When immersed in water, the molecules become ionized, thus, creating a pure soap solution.

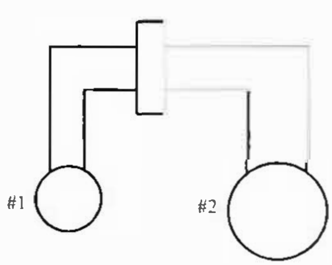
Pure soap bubbles and films are very sensitive to impurities such as dust particles and components such as excess fat. In the case of the pure soap solution, glycerine is usually added to increase the lifetime of the bubbles.

Molecules near the surface of a pure liquid do not undergo the same force as those within the interior of the liquid. Molecules in the gaseous region outside the liquid experience a weaker force than those in the interior. Thus, the molecules experience an overall force pulling them inward and this results in the minimizing of volume (Lovett 1994).

Soap bubbles and films are very elastic and permeable. If you blow a soap bubble, you can easily see its elasticity by poking a pencil through the bubble and noting that it will not burst. Soap films can withstand punctures and outside pressures.

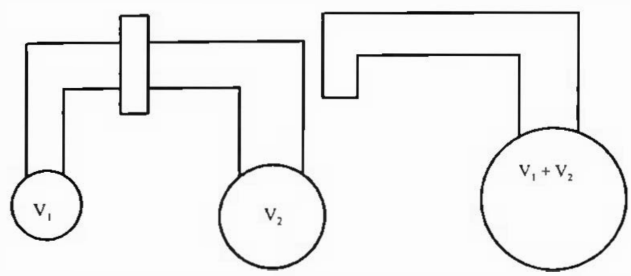
The smaller the bubble, the greater the pressure it exerts upon the soap film; the larger the bubble, the less pressure it exerts.

Figure 1



In Figure 2, bubble #1 is smaller than bubble #2, and the outlet between them is closed. When the outlet opens, #1 disintegrates and #2 enlarges. When the latch is opened, the pressure and air diffuse from a higher concentration of pressure to a lower one as found in bubble #2.

Figure 2



In a large soap bubble, the curvature and the pressure are small and, in a soap bubble of less surface area, the curvature and pressure are high.

The colors of soap bubbles and film span the visible light spectrum ranging from blue to green and yellow to purple.

All of these colors can be readily observed when viewed in sunlight. The soap film acts as a prism when sunlight passes through and the colors are reflected. You will note that if you look at a soap bubble in dim light, you won't see many colors.

One can see why soap bubbles and films can exist in certain configurations but not in others. A single soap bubble, such as the one in Figure 3-a, is spherical for a simple reason: a sphere is the surface of least area which encloses the fixed volume of air trapped inside. The soapy surface of the bubble wants to be small, wants to contract as much as possible and wants to minimize the energy of surface tension.

The soap films pictured in Figures 3-b and c are the surfaces of minimal area with the given boundaries. The double bubble of Figure 3-e is the bubble of least area which encloses the two fixed, separate volumes of air. This compound bubble consists of three pieces of surface: the larger bubble, the smaller bubble and the surface between them.

The loci of points in the circle that is formed when these surfaces meet are referred to as singularities. The triple bubble shown in Figure 3-d has not only curves where three surfaces join but also two distinct singularities where four curves (see diagram) and six pieces of surface (all three bubbles and all three separating surfaces) meet at a vertex.

Figure 3a

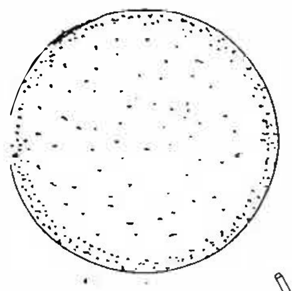


Figure 3b

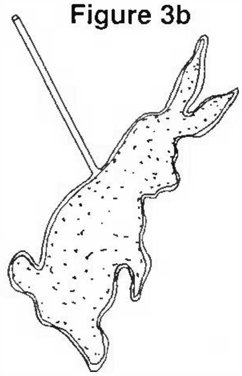


Figure 3c

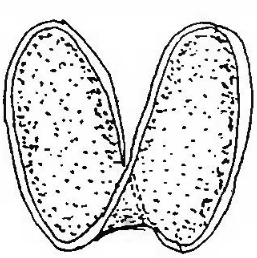


Figure 3d

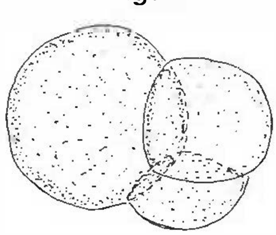
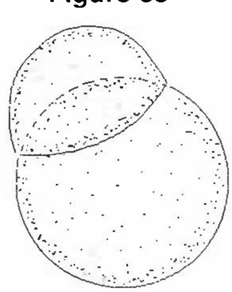


Figure 3e



The first person who studied the geometry of soap bubbles and films and established the general principles above was a Belgian scientist, Joseph A. F. Plateau (1801–1883). Plateau's results are summarized below.

Soaps bubbles can meet in only two ways:

1. Three surfaces meet along a smooth curve and form equal angles of 120 degrees.
2. Six such curves converge at a vertex and form equal angles of approximately 109 degrees.

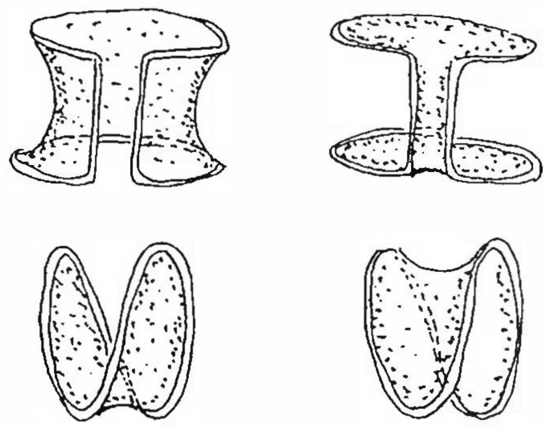
Soap bubbles obey the above rules and a simple principle of area minimization. The basic principle concerning surface tension and area minimization states that a physical system, such as a soap film or bubble, will remain in that configuration only if it cannot readily transform to a configuration of less energy. A soap film or bubble wants to minimize its energy of surface tension in order to become more stable; therefore, it attempts to contract to minimize its area.

Suppose C is a closed curve in space. What is the surface S of smallest area having boundary C? Plateau performed extensive research into this question. The medium he used to help solve the question was soap films and bubbles. Now any question dealing with soap bubbles and soap film is known as "Plateau's Problem."

There are five parts to this problem: existence, uniqueness, regularity, singularity and construction. The existence problem was explored by Jesse Douglas (1897–1965) who won the first Fields medal for his work. The existence problem asks whether there is a surface S having boundary C whose geometric properties are similar to those of a soap film. The uniqueness problem was studied by Frank Morgan of Massachusetts Institute of Technology. The uniqueness finds the answer to "Can there be two such configurations given all the same conditions?" In the regularity problem, Frederick J. Almgren defines how smooth a surface is and how much is smooth. This property is also called differentiability. Almgren's aim in studying soap films was to construct a mathematical model because of the beauty and geometric appeal of the surfaces formed by soap films. In developing a model, Almgren had to find an appropriate mathematical object for the surfaces formed by soap film. He used integral varifolds. With this medium of study, Almgren was able to devise solutions to these three parts of Plateau's Problem.

The problem of singularity was taken up by Almgren's wife, Jean Taylor. The singularity asks what the surface would look like when it is not smooth. The construction problem is open to interpretation and many different versions of the solution have been found.

There are a few boundaries which support two different films with exactly the same areas.



Immersing these boundaries in soap solution sometimes yields one film; at other times, it yields different film, depending on how the wire is removed from the solution. More complex boundaries can be constructed that share the same characteristic of non-uniqueness merely by interconnecting the two above boundaries in different patterns.

Can a boundary, as it shrinks progressively in a certain direction, support infinitely many soap films? Evidence to answer such questions has yet to be found, but experiments with real soap films sometimes lead to answers. The answers are accessible to anyone with a solution of soapy water and wire frames. If available, glycerine can be added to common dishwashing detergent. It then becomes but a simple pleasure to send soap bubbles dancing in the breezes of an afternoon.

Since soap bubbles and the patterns they create are in perfect geometrical form, they can be very useful in the field of architecture. Soap films and soap bubbles deal with minimum area and volume. Therefore, structures can be built with the sphere in mind and less material can be used. Consider, for example, the German Pavilion at Expo '67 in Montreal designed by Frei Otto and Rolf Gutbrod (Lovett 1994). Otto and Gutbrod actually used soap-film models to develop the designs.

Circular tents and roofs also demonstrate the usefulness of minimum area—using the least amount of material to enclose the maximum volume. This way of constructing tents is similar to the way minimum area is shown by a metal ring with a loop of thread in the middle. When dipped in soap solution and a hole is punctured in the loop of string, the string begins to take the form of a perfect circle-minimizing area.

This is only the beginning of a topic that deserves much more time and explanation. Soap bubbles have thrilled people of all ages for centuries. For the young, they are intricate items of play and wonder. For the young at heart, soap bubbles offer a world of complexity in the simplicity of shape. Whatever the age, whenever the century, soap bubbles make their way into our architecture, science and everyday life experiences.

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