Using a Graphing Calculator to Compute Interest Rates

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Teachers are always alert to real-world situations in which interest rates are involved. It is advantageous if a graphing calculator can be employed.

Suppose that Joel is considering purchasing a yard tractor with a cash price of \$23,650. A financing plan is available: a down payment of \$5,500 and five subsequent yearly payments of \$5,500. If Joel takes advantage of this financing opportunity, what annual interest rate would he be paying?

To analyze this problem let

P = the original price after the down payment. In this case P = \$18,150.
R = the yearly payment. In this case R = \$5,500.
r = the yearly interest rate.

Next we must express all of the money at the same point in time; that is, we must move each yearly payment to the time of purchase immediately after the down payment. The present values of the five yearly payments are, respectively,

$$\frac{R}{(1+r)};\ \frac{R}{(1+r)^2};\ \frac{R}{(1+r)^3};\ \frac{R}{(1+r)^4};\ \frac{R}{(1+r)^5}.$$

The original price after the down payment then is equal to the sum of the five present values; that is,

$$P = \frac{R}{(1+r)} + \frac{R}{(1+r)^2} + \frac{R}{(1+r)^3} + \frac{R}{(1+r)^4} + \frac{R}{(1+r)^5}.$$

Call this equation 1.

Multiplying both sides of equation 1 by (1+r) yields:

$$P(1+r) = R + \frac{R}{(1+r)} + \frac{R}{(1+r)^2} + \frac{R}{(1+r)^3} + \frac{R}{(1+r)^4}.$$

Call this equation 2.

Subtracting equation 2 from equation 1 yields:

$$P[1-(1+r)] = \frac{R}{(1+r)^5} - R$$

or $$P(-r) = \frac{R}{(1+r)^5} - R.$$

Then: $$\frac{R}{(1+r)^5} - R + P(r) = 0$$

And: $$R - R(1+r)^5 + P(r)(1+r)^5 = 0$$

Thus: $$R[1-(1+r)^5] + P(r)(1+r)^5 = 0.$$

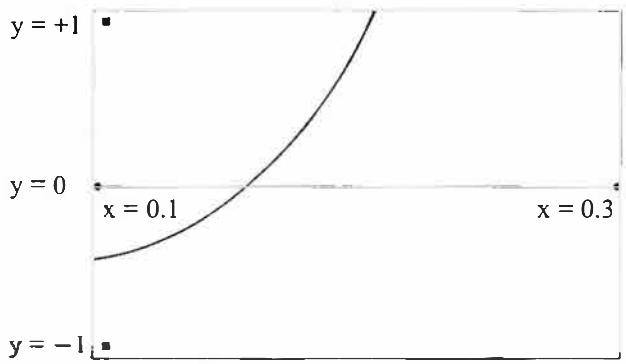
Since P and R are known, it remains only to solve this equation for r. Since this is a 6th degree equation in r, we used the TI-85 to graph:

$y = 5500[1-(1+x)^5] + 18150\,x(1+x)^5$.

To reduce the range, we actually graphed

$y = 5.5[1-(1+x)^5] + 18.15\,x(1+x)^5$.

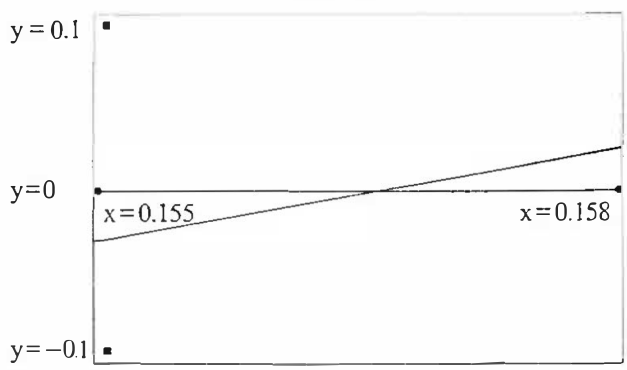
Graph 1



Window $0.1 \le x \le 0.3$
$-1 \le y \le 1$

The x-intercept of this graph will be the interest rate in decimal form. Using the trace key, x is between 0.1556 and 0.1571. To determine a closer approximation, redefine the window. Let $0.155 \le x \le 0.158$
$-0.1 \le y \le 0.1$

Graph 2



The x-intercept is then between .15664 and .15667. The interest rate is thus approximately 15 ²/₃ percent.

To illustrate the operation of this periodic interest process, let us build an amortization table for Joel. The first year proceeds as follows:

1. At the beginning of the first year, the principal owed is $18,150.
2. At the end of the first year, the annual payment of $5,500 is made.
3. At the end of the first year, the interest due is 15 $^2/_3$ percent of $18,150 or $2,843.50. This part of the first annual payment is used to pay interest and does not reduce the principal.
4. The difference between the annual payment ($5,500) and the interest due ($2,843.50) is then used to reduce the principal. This difference is $2,656.50.
5. Reducing the original balance by $2,656.50 leaves a closing balance after the first year of $15,493.50.
6. This closing balance from year 1 becomes the original balance for year 2. These steps are then repeated for years 2, 3, 4 and 5.

Challenges

1. Is the 15 $^2/_3$ percent a good interest rate for Joel to pay?
2. Apply the same process to home mortgages. If you know the initial price of a home and the monthly payment of a friend's mortgage, calculate the interest rate that your friend is paying.

Table 1
The Entire Amortization Process

Year	Original Balance	Total Annual Payment	Interest Paid	Principal Paid	Closing Balance
1	$18,150.00	$5,500.00	$2,843.50	$2,656.50	$15,493.50
2	15,493.50	5,500.00	2,427.32	3,072.68	12,420.82
3	12,420.82	5,500.00	1,945.93	3,554.07	8,866.75
4	8,866.75	5,500.00	1,389.12	4,110.88	4,755.87
5	4,755.87	5,500.00	745.09	4,754.91	0.96*

* Note the rounding errors which result from the rounded value of r.

A Strange Sequence

What is the rule by which the following sequence of numbers has been created? What are the next three elements of this sequence?

1, 11, 21, 1211, 111221, 312211, 13112221, 1113213211, ...