NCTM Standards in Action
Content Standard: Geometry

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The *Principles and Standards for School Mathematics* (NCTM 2000) identifies 10 content standards that represent what should be valued in school mathematics education. The standards represent a connected body of mathematical understandings and competencies, and they specify the understanding, knowledge and skills that students should acquire from pre-Kindergarten through Grade 12. Geometry, like the other content standards, applies to all grade levels, albeit with varying emphasis both within and between grade levels.

In particular, the geometry standard identifies the following areas of emphasis that enable all students to

- analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships;
- specify locations and describe spatial relationships using coordinate geometry and other representational systems;
- apply transformations and use symmetry to analyze mathematical situations; and
- use visualization, spatial reasoning and geometric modeling to solve problems.

Geometric shapes and representations surround us, thus allowing the easy transfer of what students learn to other areas of mathematics and to real-world situations. Geometric representations can help students to make sense of area and fractions, histograms and scatterplots can give insight about data, and coordinate graphs can serve to connect geometry and algebra.

Spatial reasoning is helpful in reading maps, designing plans or creating artworks. Geometry is more than definitions—it is about relationships and reasoning, which help students to learn and see the axiomatic structure of mathematics. Therefore, this content standard has a strong focus on the development of careful reasoning and proof. Technology also plays an important role in the teaching and learning of geometry. The use of technology allows students to generate many examples as a way of forming and exploring conjectures.

When students begin the study of geometry, the initial activities consist primarily of observing and describing shapes and noticing properties. These are essential activities because they form a strong foundation for further study. As the students proceed in their study, the learning includes exploration of properties, concepts of similarity and congruence, deductive reasoning and formal proof techniques. Eventually students are able to construct their own proofs.

The standards also require students to specify locations and describe spatial relationships using coordinate geometry and other representational systems. As students focus on this standard, the concepts initially deal with relative position, such as *above, behind, near* and *between*. They then proceed to locating points in a rectangular grid system, which allows them to discover and analyze properties and shapes. In the middle grades, students find distances between points in the plane, using the Pythagorean relationship. Here, a fundamental connection between algebra and geometry is established as well. By the time students reach high school, they will have become proficient in using the Cartesian coordinate plane to solve a variety of problems.

Applying transformations and using symmetry to analyze mathematical situations begin by capitalizing on students' intuitive understanding of how shapes can be moved. In particular, students learn about motions such as slides, flips and turns through the use of mirrors, paper folding and tracing. Understanding the effects of transformations occurs in the upper

elementary grades, and by the time students have entered the middle grades, they know what it means for a transformation to preserve distance. At the high school level, students learn multiple ways of expressing transformations, including using matrices to show how figures are transformed on the coordinate plane, as well as function notation. The concept of symmetry is learned at all grade levels providing insight into mathematics, the arts and esthetics.

Using visualization, spatial reasoning and geometric modeling to solve problems is an important content standard that applies across the grades. The early experiences are hands-on and involve a variety of geometric objects. Through the use of technology, students learn to turn, shrink and deform two- and three-dimensional objects. In later years, students learn to infer attributes that cannot be seen. Students are challenged to physically and mentally change the position, orientation and size of objects, as they develop their understanding about congruence, similarity and transformation. Moving between two- and three-dimensional shapes and their representations is important content acquired at the elementary level. This leads to wrapping blocks into nets. By the time students reach the middle grades, they should be able to interpret and create top or side views of objects.

This skill is further developed so that students learn to build structures given only side and front views. By the time students are at the secondary level, they are able to find the minimum number of blocks needed to build given structures. Visualizing and drawing cross-sections of structures and a range of geometric solids is a content skill required at the high school level.

Geometry gives students a different view of mathematics. As they explore patterns and relationships with models, blocks, geoboards and graph paper, students learn about the properties of shapes and sharpen their intuition and awareness of spatial concepts. One of the most important connections in all of mathematics is the one between geometry and algebra, thus making geometry an important content standard.

The three articles that follow relate to the geometry standard. The first presents four related problems that are examples of ways to include justification of interesting mathematics prior to a geometry course. The reasoning used in solving these problems involves both algebra and geometry and encourages the meaningful use of technology. The second article suggests ways that interactive geometry software, with classic problems as context, can be used to connect students to a richer experience of what mathematics is and what it means to know and do mathematics. The third article describes how mathematics teachers can structure classroom activities so that students will be intellectually challenged. It also offers techniques for developing tasks for group investigations.

Bibliography

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> What is best in mathematics deserves not merely to be learned as a task but to be assimilated as a part of daily thought, and brought again and again before the mind with ever-renewed encouragement.
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> *Bertrand Russell*