Aspects of Numeracy in the Primary Years (K–3): Selected Challenges for Teachers

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Information about numeracy is available for teachers, as well as parents. It is referred to in the revised British Columbia Ministry of Education documents, *Supporting Learning: Understanding and Assessing the Progress of Children in the Primary Program* (2000) and *The Primary Program: A Framework for Teaching* (2000). Numeracy is the subject and title of a 1998 pamphlet prepared by the British Columbia Association of Mathematics Teachers (BCAMT). This pamphlet includes the notable statement that numeracy is as important as literacy.

What is numeracy? What are some reasons for the suggested importance of numeracy? Why do teachers play such a key role in fostering the development and growth of numeracy?

Numeracy

According to the BCAMT (1998) pamphlet, numeracy is much more than knowing about the numbers and number operations. It relates to a person's abilities to confidently apply mathematical knowledge in various, even unfamiliar, situations. These abilities include flexible thinking, willingness to take risks and connecting new ideas to what is known. The mathematical knowledge includes the important aspects of number sense, as well as spatial sense, statistical sense and sense of relationship. These components clearly illustrate and support the statement from the pamphlet that numeracy is important because people need this skill to function in everyday life, in the home, the workplace and the community.

Contributions by Teachers

Teachers and parents play key roles in fostering numeracy development (Leder 1992). Students will become flexible thinkers and acquire the willingness to take risks if teachers of mathematics value and nurture these characteristics in their classrooms. This may not be an easy task, however, because the majority of teachers likely experienced a closed or heavy-handed approach to mathematics learning. In such a setting, the focus is on a teacher who shares ways of thinking or even prescribed steps of solving different problems. Students memorize these given or prescribed procedures and strategies. Assessment is based on ability to recall what has been memorized, frequently in a timed setting, rather than on understanding and flexible thinking.

In an open-ended setting, students learn that different procedures, strategies and/or answers may exist for given problems. Creating such an awareness can, over time, contribute to the fostering of students' self-confidence, risk-taking and flexible thinking (Spungin 1996; Liedtke, Kallio and O'Brien 1998).

The following examples illustrate a possible difference between closed and open-ended approaches. In a closed setting, one answer or strategy would be considered correct during class discussions, for activity sheets or for assessment tasks such as the following:

A. Which comes next? □ ○ □ ○
   1 2 3 ____
B. Which does not belong? 2 8 9 10
C. How do you find the answer for 8 + 7?

In an open-ended approach, the questions would be

A. Which one *do you* think comes next? Why?
B. Which one *do you* think does not belong? Why?
C. What are some different ways to find the answer for 8 + 7?

After an answer is elicited, students might be encouraged to respond to the question, What is another possible answer/way? Evidence of flexible thinking becomes apparent when, for example A, students extend the pattern beyond any shape that has been chosen, that is, a square, and when they consider ways of creating repeating or growing patterns for 1, 2, 3, ___.

In one Grade 3 classroom I visited, a type of weekly challenge task consisted of examining five displayed numerals and trying to identify the one that did not belong. One numeral was considered to be the correct answer. To encourage flexible thinking, this task can easily be changed to challenging students to think of questions that would make each displayed numeral a correct answer to the request. This approach is possible for examples of the type shown in B.

An open-ended approach does not imply that students will not learn the basic facts. On the contrary, students will learn these and much more. The different strategies they learn in such settings to re-invent forgotten facts will transfer to other mathematical ideas (Isaacs and Carroll 1999). This would be true for students who know several ways, other than counting, of convincing someone that the answer for 8 + 7 is 15.

These examples illustrate that the desirable goals that are part of numeracy are unlikely to be reached without a skillful teacher who is able to create an appropriate classroom atmosphere and orchestrate discussions that provide opportunities for students to think, to think about thinking and to explain and compare thinking or thinking strategies.

Aspects of Numeracy—Goals

The BCAMT (1998) pamphlet identifies the following aspects of numeracy: number sense, spatial sense, statistical sense and sense of relationship. A detailed discussion of each aspect is beyond the scope of this article. My goal is to identify relevant outcomes from *Mathematics, K to 7: Integrated Resource Package* (British Columbia Ministry of Education 1995) for each and to indicate possible goals that might be considered characteristic of a numerate student. Because it can be argued that "a sense of number" is of prime importance, and this is reinforced by the fact that this skill appears at the top of every grade level in the integrated resource package, the greatest attention is given to this aspect of numeracy.

Prenumber and Number Sense

Table 1 identifies thinking strategies that are part of the prenumber sense. These strategies are necessary prerequisites for understanding number or acquiring a sense of number and being able to count rationally. Because activities with patterns involve strategies similar to those for ordering, the aspect of numeracy labeled sense of relationship has been included in Table 1. Although the goals included under the heading "fluent" may appear to be rather specific whenever possible, many do include reminders that flexible thinking is an integral part of fluency or being numerate.

Outcomes for the important benchmarks (to five; to ten) of early number sense are included in Table 2. The activities with numbers to five should lead students to recognize these numbers without having to count (subitizing). Finger-flash activities can be used to assist with reaching this visualization goal (Liedtke 1992–93). The part-part whole understanding of number, or being able to assign different names to a number (for example, 5 and 2 *or* 3 and 4 for 7), will transfer to the development of thinking strategies for the basic facts. Rational counting implies that students can tell why certain numbers come next in ordered sequences (for example, 2, 4, 6, _; 1, 3, 5, _; 1, 1, 2, 3, 5, _). When counting a set of objects, students should learn to realize that counting is independent of direction. Counting can be carried out in any possible way as long as names are appropriately matched with objects.

The goals of a sense of number to 999 are included in Table 3. Two key goals for two-digit numbers are related to visualization and realizing that each number can have two or more names. The visualization process can be enhanced, for example, by having students think of the fewest number of students it would take to show a given number on fingers or with base-ten blocks (that is, for 47—five students or four "tens" and seven "ones").

Activities that involve estimation can contribute to the development of number sense. Students need to know what is meant when they are asked to *guess* and when to *estimate*. Care should be taken not to use these terms carelessly or interchangeably. Care also needs to be taken when acknowledging students' responses to requests for guesses and estimates. Judging or labeling the response as good or even excellent can easily have other students consider their guesses or estimates as inappropriate. As a result they may lose some of their willingness to take risks. It is advantageous to consider all guesses and estimates as good and to let students know that that is the case. (If, by chance, criteria for judging appropriateness of estimates are used, students should know how these are determined.)

Properties of numbers can be discovered and visualized as students move counters, shade in squares on paper or use pegs and rubber bands on geoboards.

Table 1
Goals for Prenumber Sense Thinking Strategies

	Prenumber	Emerging	Early	Fluent
Sorting	• free play • response to "Which one does not belong?" cannot be explained	• sorts by color or shape or size • response to "Which one does not belong?" cannot be explained	• can sort using two characteristics • responses to "Which one does not belong?" can be explained	• is flexible, can sort beyond color, shape and size • is able to defend response to "Which one does not belong?" with several different answers • recognizes number as a common characteristic of groups of different looking objects
Ordering three objects such as toys, dolls, sticks...	• random	• selects biggest or smallest object	• can order from "smallest" to "biggest"	• can order from "biggest" to "smallest" and vice-versa
Ordering more than three objects such as toys, dolls, sticks...	• random	• can copy an ordered sequence with a set of objects of the same type	• can copy an ordered sequence with a set of objects of a different type	• can order from "biggest" to "smallest" and vice-versa and use appropriate language to describe adjacent numbers • can extend in both directions • is able to insert objects into a given sequence • is able to construct own ordered sequences for variety of characteristics • is able to apply the ordering strategies to number
Matching: find as many or show the same number (without counting)	• random, no understanding of the request "Find as many" or "Show the same number"	• begins matching but fails to complete the one-to-one correspondence and may revert to free play	• matches but counts in order to verify the result	• recognizes that matching results in equivalent sets even if the objects differ according to color, shape, or size • recognizes equivalence even after the objects are re-arranged.
Pattern	• uses term incorrectly	• able to copy a given pattern	• able to extend simple repeating patterns • selects correctly when given a choice to extend a pattern	• recognizes predictability of repeating and growing patterns • aware that patterns can be extended in more than one way • able to extend and describe hidden members of patterns and justify responses • constructs own patterns • identifies patterns in different mathematical settings • looks for patterns as a possible problem solving strategy

Table 2
Goals for a Sense of Numbers—to 5, 10 and 20

	Prenumber	Emerging	Early	Fluent
Early number (zero to five): ideas of few, many, some	• does not use this language (prefers big)	• uses terms like *many*, *lots* or *most*	• uses *few, many* and *some*	• realizes the relativity of the terms (for example), six fish is a lot in a fish bowl; not many in the pond or lake)
Early number (zero to five: **subitizing** (recognition of number)	• may not recognize one for similar objects (without generalizing about oneness)	• recognizes one and two for dissimilar objects, checks by counting	• recognizes up to three for dissimilar objects, checks by counting	• recognizes up to five similar objects without counting • recognizes number for a variety of arrangement of similar objects
Early number (zero to five): **part-part-whole** (different ways to show a number)	• no recognition that one and four is the same as five	• shows number in one different way	• can show two ways, uses counting to justify response	• can show all the possible ways and uses matching to justify
Early number (six to ten): **subitizing**	• makes guesses	• uses rote counting	• uses rote counting but sometimes counts on from five	• names all familiar arrangements for two to ten objects and can re-organize for easy recognition • for unfamiliar arrangements uses a part-part-whole strategy
Counting	• makes errors (misses)	• may make a few errors • counts up to a certain number	• error free rote counting	• error free and is aware of the patterns: one more, two more, etc.
Matching ordered numbers with numerals (such as 5 or five)	• incorrect matches	• correct matches for one and two	• occasional errors	• error free
Ordering the numerals to ten	• some correct parts of the sequence; may miss	• most of the sequence correct	• recites whole sequence	• is a rational counter: can start anywhere, count on, count back, count by twos and justify responses
Counting to twenty	• some correct parts of the sequence • may skip or miss	• major parts of the sequence are correct	• recites whole sequence using standard names	• counts rationally (see above) • can give two names for every number: standard; also knows that thirteen is one ten and three ones

Table 3
Goals for a Sense of Number—99 to 999

	Prenumber	Emerging	Early	Fluent
Two-digit numbers and numerals **to 99**	• no apparent understanding—reverses digits, misses numbers, no meaning, counts by rote to a specific number	• standard names • rote recital in one direction	• knows two different names • can count in two directions	• knows many names for a given number (37 is 30 and 7, 20 and 17, 10 and 27 and so on) • can explain how to construct a number using the least number of base ten blocks or students (showing their fingers in tens and ones) • skips counts in different directions • fills in between two given numbers
Three digit numbers **up to 999**	• considers each digit as ones • makes errors when rote counting • unable to read numbers, may read 126 as "one, two, six"	• rote recital of standard names • knows what comes after a given number • reads most numbers correctly	• can represent the number in one way • can count in two directions	• states many different names for a given number • can visualize in terms of base ten blocks or money denominations (ones, tens, hundreds) • skip counts in several ways • given numbers, knows the number before, next and in between
Estimation up to ten (about, how many?)	• guesses	• recognizes more than three	• recognizes more than five • may use the word *about* incorrectly by giving an exact answer	• uses a referent of five (one hand) or ten (two hands) to make a visual comparison • uses the word *about* correctly
Estimation up to 99	• guesses	• guesses for numbers greater than ten	• estimates but sometimes reports exact answers ("That is about seventy-nine.")	• uses ten as a referent, gives answers that end in zero • does not give exact answers when using *about*
Estimation up to 999	• guesses	• can identify extreme incorrect choices	• correctly chooses best answer from a list of choices	• can connect to experience by selecting the best answer for a given setting (Number of students in your school: 20, 200 or 900?) • combines measurement sense with number sense (height of the door: 200 cm, 500 cm or 900 cm?)
Properties of Numbers: odd even, triangular, rectangular, square	• no recognition	• given objects, inconsistent but may be able to understand even numbers to ten	• can identify even and odd for numbers up to twenty	• visualizes numbers for number names and is able to label numbers as *odd, even, triangular, rectangular, square*

Spatial and Measurement Sense

Goals for spatial sense are shown in Table 4. Some mathematics programs tend to suggest and emphasize that students learn (memorize) the names for blocks. Just as a sense of number is independent of knowing how to print numerals (or even reversing them) and number names, visualization or spatial sense is not in any way enhanced by knowing the names for blocks (my experiences in classrooms have shown that many times young students use incorrect names; for example, *square* for *cube*).

In the BCAMT (1998) pamphlet, measurement is referred to under the heading of spatial sense. Important ideas related to measurement sense are included in Table 5. One key idea for all topics of measurement is related to the fact that "our eyes may deceive us." Things that in some way appear to be different (longer; bigger in area, capacity or volume; heavier; "takes longer" and so on) may in fact be the same, and things that are similar in appearance may be quite different. Another important idea is related to appropriateness of selecting units of measurement. Students need to learn that this appropriateness depends on the type of problem to be solved and not on "speed of obtaining results" (that is, is a door to be covered with a metal frame or with newsprint?).

Statistical Sense

Goals for statistical sense are included in Table 6. Activities that involve the collecting and organizing of data are part of every grade. As students are taught how to interpret the graphs they have constructed or

Table 4
Goals for Developing Spatial Sense

	Prenumber	Emerging	Early	Fluent
Examination of 3-D figures	• free play, no connections	• fails to look at the whole block, talks about parts	• uses one label for a given block	• relates a block to many familiar objects by concluding "it depends on how you look at it"
Examining diagrams or photographs of 3-D figures ("Which block goes with this picture?")	• random matches	• selects the block that may have one or a few similar characteristics	• matches one block by placing it on top of the picture	• matches with appropriate block, describes part not visible, recognizes when more than one covered answer is possible • given a picture can hold a block in the same position
Special 2-D shapes (triangles, rectangles, circles)	• no distinction between open and closed curves	• recognizes difference between open and closed curves	• recognizes one type of triangle • recognizes most types of rectangles • recognizes circles	• correctly labels all parts of triangles (rectangles, circles) • recognizes how triangles (rectangles, circles) are the same and how they can differ
Transformation 3-D and 2-D	• not meaningful	• identifies difference, unable to explain	• identifies the results of slides • predicts the outcomes of slides and connects to experience	• predicts the outcome flips, slides, turns and connects these to events/actions from experience (printing, physical education, art)
Symmetry	• unable to verbalize ideas about symmetry	• recognizes simple symmetry and may use the word "same"	• can identify/locate one line of symmetry	• identifies/locates multiple lines of symmetry • relates to experience (printing, nature, art) • constructs symmetrical figures (3-D, 2-D)

Table 5
Goals for Developing a Measurement Sense

	Prenumber	Emerging	Early	Fluent
Length	• relies on eyesight, sees no need to measure	• considers both endpoints, but neglects what lies in between	• inconsistent use of ruler • no estimation strategies, just guesses	• able to measure to nearest unit • knows when to estimate and when to measure • knows several estimation strategies • knows that appropriateness of selecting a unit depends on the type of problem to be solved • connects to events from experience (art, science, social studies, physical education)
Area	• long means big, longer means bigger	• compares by fitting similar shapes on top of one another	• modifies dissimilar shapes to make comparisons	• realizes that eyes can "deceive me"—shapes that look different can have the same area • knows more than one estimation strategy • connects to ideas from events/experience
Capacity/ Volume	• classifies as big or small	• orders similar containers from smallest to biggest	• uses nonstandard units (handful) to check a given guess	• realizes that different-looking containers/buildings can have the same capacity/volume • has at least one estimation strategy • connects to events/actions from experience
Mass	• bigger means heavier	• realizes similar objects could differ • uses hands to make comparisons	• willing to make predictions about two objects, inconsistent	• realizes that different looking objects could be the "same"; objects of the same size and shape could be "different"; and "bigger" looking object could be "lighter" • connects to events/actions from experience
Temperature	• understands hot and cold	• understands hot, cold, warm	• relates to experience (not to the Celsius scale)	• knows that hot, cold and warm could be relative • identifies important numbers on the Celsius scale (0, 20, 37, 100)
Time	• no understanding	• understands "takes a long time" versus "takes a little time"	• rote reading of analog clock to the half hour • reads digital clock without understanding	• knows that time can be relative • uses units of time appropriately • can explain the meanings of markings and parts of a clock • connects time measurement to experience (parts of a day, days, weeks, months, years, seasons)

Table 6 Goals for Developing Statistical Sense				
	Prenumber	**Emerging**	**Early**	**Fluent**
Examination of Collected Data	• unable to make logical interpretations of data	• detects only results obvious to visual inspections	• can make simple arithmetical interpretations (how much more, less) • can make true/false interpretations	• can interpret and create statements (talk/write) that involve *true, false, could be true, likely, unlikely, certain* and *uncertain* • uses appropriate language to talk about outcomes of events from experience • applies probabilistic thinking to data, as simple interpolations and extrapolations are made

drawn, they should be challenged to respond to many types of questions and statements. Some of these should be answerable by examination of the graphs, while others could deal with the displayed topic or idea without the data directly enabling students to prepare responses. Statements about graphs should be created that students are required to classify as *true, false* or *could be true* (but the graph does not tell us). Students should be challenged to create different types of questions and statements of their own.

As data from graphs and reports from daily events (news, weather reports, sports and so on) are discussed, ask students to make/write statements that use the terms *likely, unlikely, certain* or *uncertain*. Graphs that have been constructed to find and show the answer to a problem (Liedtke 1992)—Which color do we prefer? What is our heartbeat before and after an exercise?—could be related to other populations or events—Do you think other groups would have the same preferences? Do you think other types of exercises would show the same results? Questions such as these provide many opportunities for students to explain and justify their thinking. They can be given a chance to pursue their own ideas and problems.

Conclusion

The importance of the role of a teacher has been mentioned. Major outcomes for different aspects of numeracy have been identified. To successfully foster development and growth of numeracy, teachers must provide opportunities for students to use their own language as they justify their thinking and provide reasons for their responses. Students need to become confident as they explain their thinking. As Lappan (1999) suggests, students need to learn to persevere. A high level of confidence and a willingness to try things in mathematics are "gift[s] that will carry them forth to future mathematics and life success [and] help them learn to produce their own ideas" (p. 3).

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