Mathematics as communication is an important curriculum standard; hence, the mathematics curriculum emphasizes the continued development of language and symbolism to communicate mathematical ideas. Communication includes regular opportunities to discuss mathematical ideas and explain strategies and solutions using words, mathematical symbols, diagrams and graphs. While all students need extensive experience to express mathematical ideas orally and in writing, some students may have the desire—or should be encouraged by teachers—to publish their work in journals.

delta-K *invites students to share their work with others beyond their classroom. Submissions could include, for example, papers on a particular mathematical topic, an elegant solution to a mathematical problem, an interesting problem, an interesting discovery, a mathematical proof, a mathematical challenge, an alternative solution to a familiar problem, poetry about mathematics, a poster or anything that is deemed to be of mathematical interest.*

Teachers are encouraged to review students' work prior to submission. Please attach a dated statement that permission is granted to the Mathematics Council of the Alberta Teachers' Association to publish the work in delta-K. *The student author (or the parents if the student is under 18 years of age) must sign this statement, indicate the student's grade level, and provide an address and telephone number.*

The following article, "An Example of an Error-Correcting Code," was written by Mark Rabenstein when he was a Grade 8 student at McKernan School in Edmonton. The article was originally published in Mathematics Magazine *and is reprinted here with the kind permission of the author and the Mathematical Association of America.*

Mark earned a Ph.D. in chemistry in 1996 and is now doing pharmaceutical formulations research for Bend Research in Bend, Oregon. While in Edmonton, he attended the Saturday Mathematical Activities, Recreations & Tutorials Club (SMART Club) under the tutelage of Dr. Andy Liu, University of Alberta. He feels that the opportunity to participate in the SMART Club and work under the guidance of Dr. Liu contributed immensely to his intellectual development.

An Example of an Error-Correcting Code

Mark Rabenstein

I am a student in Grade 8. Recently, I went to an enrichment program run by Andy Liu of the University of Alberta. The topic we studied is called "error-correcting codes."

The problem goes like this. A secret agent has to send a message back to headquarters. He uses a transmitter which sends a string of 0s and 1s. Unfortunately, from time to time, a 1 gets changed into a 0 while the message is on its way, or vice versa. So he has to send some extra digits to make sure there is no misunderstanding. Fortunately, no more than k digits in the expanded or encoded message are changed at one time.

We studied many interesting schemes for encoding a message. The first one is really simple. Just repeat the message $2k + 1$ times, and the copy that appears at least $k + 1$ times is the correct one. However, this requires lots of digits, and this is not good for a secret agent.

When I thought things over, I did not see why it was necessary to repeat the message $2k + 1$ times. If there are no more than k mistakes and the message is repeated $k + 1$ times, one of the copies must be correct! The only problem is: How can we tell which is the correct one?

Well, there is a simple way to tell whether a copy has one mistake (or any odd number of mistakes). Add a 1 to the original message if it has an odd number of 1s, and add a 0 otherwise. This way, the number of 1s in the encoded message is always even.

This extra digit is called a "parity-check digit," parity meaning odd or even. If an odd number of mistakes is made in the encoded message, then the number of 1s in it will be odd and not even as it is supposed to be, and in this case we can tell something is wrong. Of course, the method fails for an even number of mistakes.

Let us go back two paragraphs and see how parity-check digits can be of help there. As I said, repeat the message $k + 1$ times. Now add a parity-check digit to each of the last k copies. I will show that this works.

Check each of the last k copies of the received message to see if anything is wrong. Suppose we find a copy with an odd number of mistakes. Well, throw it out! With each such copy goes at least one mistake. In an extreme case, everything is gone except the first copy. It must be correct because all the mistakes have been thrown out.

Suppose we are left with $l + 1$ copies (the last l copies having parity-check digits). We know that there are at most l mistakes, and they come in pairs in the last l copies.

What does this mean? This means that at least half of these l copies contain no mistakes. We should be able to tell what the correct message is, unless there is a two-way tie. In that case, we still have the first copy, which must be correct because all the mistakes have been used up.

So my scheme does work. Of course, it still needs lots of digits, unlike some of the really clever schemes I learned in the enrichment program.

Remarks by A. Liu

The code presented in this note is apparently new. The reader may supply a more formal proof. The "apology" in the last paragraph is really not necessary in that the ease of encoding and decoding for this scheme offsets its lack of sophistication. Its rate of information is asymptotically nearly twice that of the repetition codes (Alt 1948).

Codes with higher rates (the "really clever schemes") were known early on in the history of error-correcting codes (see, for example, Golay 1949, Hamming 1950 and Shannon 1948). A recent publication (Thompson 1983) gives an interesting historical account and shows the interrelationship of error-correcting codes with other areas of mathematics. A definitive treatise (MacWilliams and Sloane 1977) details the state of the art as well as listing over 1,000 references.

References

Alt, F. L. "Bell Telephone Laboratories' Computing Machine." *Mathematical Tables and Other Aids to Computation* 3, no. 21 (January 1948): 1–13.

Golay, M. J. E. "Notes on Digital Coding." *Proceedings of the Institute of Electrical and Electronic Engineers* 37 (1949): 657.

Hamming, R. W. "Error Detecting and Error Correcting Codes." *Bell System Technical Journal* 29, no. 2 (April 1950): 147–60.

MacWilliams, F. J., and N. J. A. Sloane. *The Theory of Error-Correcting Codes.* Amsterdam: North-Holland, 1977.

Shannon, C. E. "A Mathematical Theory of Communication." *Bell System Technical Journal* 27 (July and October 1948): 379–423, 623–56.

Thompson, T. M. *From Error-Correcting Codes Through Sphere Packings to Simple Groups.* Washington, D.C.: Mathematical Association of America, 1983.

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