Is There a Worldwide Mathematics Curriculum?

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We in mathematics have many names for numbers, among them square numbers, prime numbers, rational numbers, transfinite numbers, Fibonacci numbers, complex numbers, amicable numbers, and on and on. We are number people. We have many words for numbers just as Eskimos have many words for ice and Arabs have many words for camels.

As we analyze curriculum, we have also developed many names for curriculum. We have become curriculum people. In A Study of Schooling, John Goodlad identified five different curricula: the ideal curriculum (beliefs of scholars), the formal curriculum (expectations of what should be done in the class as seen in syllabi, guidelines, textbooks and so on), the instructional curriculum (what teachers report they do), the operational curriculum (what actually goes on in the classroom) and the experiential curriculum (what students report learning and what they actually learn) (Klein, Tye and Wright 1979; Goodlad 1979). Three of these were chosen, though with different names, to constitute one of the main organizing structures in the design of the Second International Mathematics Study (SIMS): the intended (ideal) curriculum, the implemented (operational) curriculum and the attained (experiential) curriculum. In the Third International Mathematics and Science Study (TIMSS), a fourth curriculum was added: the potentially implemented curriculum, a name chosen to represent the curriculum of textbooks and other available materials.

Distinguishing these various types of curricula was important in those international studies, for the various categories are used in curricular analyses that occupied a volume apiece. Yet, these curricular analyses would be purely academic exercises—and, in fact, the lack of media attention given to them suggests that they are purely academic exercises—were it not for the natural interest in comparing not *what* but *how much* is learned by students in different countries.

The existence of TIMSS and other international comparisons of performance in mathematics is founded on the premise that there exists enough of a commonality in the mathematics curriculum worldwide that a test over that commonality represents some sort of fair test of the entire curriculum. And so the question of the title of this presentation is already seen to require some clarification. If we ask, "Is there a worldwide mathematics curriculum?", to which of these curricula are we referring?

At a conference like this one, we can be a little more relaxed: Does there exist enough commonality in the curricula of different countries that when we use such content descriptors as geometry or algebra or functions or linear equations or statistics, or when we speak of the use of calculator or computer technology, we are talking about the same things? I find it useful to examine this with a type of analysis of curriculum different from the intended, implemented or attained curriculum. It is an analysis using *sizes of curriculum.*

The Sizes of Curriculum

There are at least six sizes of the mathematics curriculum, each differing from the previous by roughly one order of magnitude: (1) the individual problem or episode, (2) the problem set or lesson, (3) the unit or chapter, (4) the semester or year-long course, (5) the mathematics curriculum as a whole and (6) the entire school experience. Proceeding from the smallest to the largest, we see that the ratios of sizes are quite appropriate for a difference in orders of magnitude. There are perhaps 5–20 episodes or problems in a typical day in a mathematics classroom, 10–20 days in a typical unit, 7–15 units in a school year, 13 years of schooling from K–12, and perhaps 6–8 other subjects vying with mathematics for space in the curriculum. The fundamental property of differences in order of magnitude asserts that a strategy, practice or policy that is appropriate for one of these sizes of curriculum may not be appropriate for another.

We often see people oversimplifying educational policy by taking something that is appropriate for a small size of curriculum and then recommending it for a larger size. A pretty concept, appropriate for a unit, may be taken as the main idea behind an entire course. The major recommendation of the National Council of Teachers of Mathematics' *Agenda for Action* report issued in 1980 was that "problem solving be the focus of school mathematics in the 1980s" (p. 1), by which it was meant that the curriculum should be centred around problem solving. Here the recommenders were taking something that was hard to disagree with at the individual problem level or lesson level, namely the presentation and solving of interesting problems, and recommending that the idea be carried out three or four orders of magnitude higher.

At the time of the *Agenda for Action* recommendation, there did exist many examples of good problems and good problem-solving lessons, and a few problem-centred units, but to my knowledge there did not exist one example of a problem-centred course, and certainly there was no example of an entire curriculum of this type. What would be the place of skill work in such a curriculum? Where would mathematical systems and structure be discussed? A full curriculum requires balancing a variety of priorities, whereas a lesson, unit or even course does not require the same sort of balance, and balancing an individual problem is like balancing an individual person on a seesaw.

For the most part, a student's experience with curriculum is the union of his experiences with individual tasks, problems or episodes. The curriculum developer tries to find interesting tasks and sequence them in a way that is clear to the student and teacher. A particular problem may be there to motivate the student, or to emphasize a particular idea, or to review an idea or to set the stage for another problem that will come later. Episodes in teaching serve similar purposes. The items that are selected for testing reflect the priorities of the teacher, and when tests are analyzed by performance on individual items, one obtains a picture at this size of curriculum.

The next larger size in the order of magnitude hierarchy—the lesson—should be more than a collection of episodes or a set of problems. A good lesson is built around a concept, which for understanding requires a variety of activities. In a lesson there is always a fundamental decision to be made regarding the balance between what is explained to the student and what is expected to be learned by the student himself. For all these reasons, a good lesson needs coherence, and the best lessons have particular ideas that they emphasize.

Similarly, a good unit is more than a set of lessons. It has a sequence of related concepts that carry it from its beginning to its end. A good unit brings together these concepts in an attempt to show their power. The student, too, is asked to demonstrate power of a different sort, for one of the fundamental properties of most units in school mathematics is that they end with a performance test.

The course is normally the largest chunk of curriculum that the student encounters with a single teacher, and it is usually the only size of curriculum for which there is a grade on record. Because the course is associated with a teacher, a course has a personality. Its personality is interwoven with that of the teacher, and it is difficult to separate student opinions about a course from student views of the teacher. Problems, lessons and units tend not to be of long enough duration to develop a personality. Only in a course is there time to develop a mathematical system of any complexity; only in a course is there time to cultivate a method of thinking.

The mathematics curriculum as a whole is the sum of courses. It has properties different from those of a single course. We might not want every course to deal with mathematical proof, but the curriculum as a whole should. The study of "curriculum coverage" found in the TIMSS analysis (Schmidt et al. 1996, 52) and the earlier analysis of review in U.S. elementary textbooks by Jim Flanders (1987), each of which involves multiyear looks at the curriculum, provide pictures that no one course could provide. And seldom are tests over the entire curriculum created by individual teachers; we need teams of writers for such tests.

Some ideas work at a variety of sizes. For instance, it is often desirable and sometimes obligatory that consecutive problems, lessons, units and courses incorporate a sense of flow, of connectivity, of growth.

Analyzing the Question by Size of Curriculum

Returning to the question, "Is there a worldwide mathematics curriculum?", I would like now to interpret this question for each of the various sizes of curriculum. I will start from the largest size.

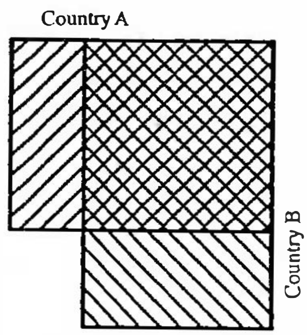
Entire Curriculum

If a student takes mathematics through secondary school in different countries, will that student cover the same mathematics? If not, then it is rather silly to speak about comparing performance in different countries, for we are comparing apples to oranges.

Obviously, we are not looking for 100 percent agreement. But it is not clear how much agreement is sufficient. The situation comparing two countries, A and B, can be represented by a diagram somewhat like a Venn diagram. In the case pictured in Figure 1, 3/4 of the topics taught in Country A are also taught in Country B, and 2/3 of the topics taught in Country B are also taught in Country A.

Figure 1

A Hypothetical Example of Overlapping Curricula



We think that there is a great deal of commonality, but in this made-up situation a full 45 percent of the total number of topics in the two countries are not common topics; that is, almost as many topics are not common as are common! Obviously, commonalities are less frequent if there are more countries.

The current trends in the mathematics curriculum that we have made themes of this conference serve to *decrease* the overlap in curricula. The movement toward mathematics for all has generally led to curricula with greater numbers of applications and data. Appropriate applications for one country may be inappropriate for another, and familiar data in one part of the world may be quite abstract in another. The use of technology in some places and not in others also creates obvious differences in what is expected of students even when the problems may be the same. For all these reasons, it is my guess that, in our quest to make it possible for mathematics to be tested worldwide, we have deemphasized the differences in total curricula.

For instance, I don't think it has been publicized that the TIMSS Grade 12 advanced mathematics test contains more geometry than algebra (see Table 1). To any person in the United States, that would seem odd, because far more time in Grades 9–12 is spent on algebra than on geometry.

Table 1

Distribution of Advanced Mathematics Items by Content Category, from the Third International Mathematics and Science Study

(taken from Mullis et al. 1998, p. B-9, Table B-2)

Category	% of items*	Number of points
Numbers and Equations	26	22
Calculus	23	19
Geometry	35	29
Probability and Statistics	11	8
Validation and Structure	5	4
Totals	100	82

*There were a total of 65 items.

Individual Courses

From the standpoint of individual courses, I think it has been demonstrated rather clearly by the TIMSS researchers that there is no worldwide curriculum. Examining Table 2, which summarizes four mainstream topics in six countries, we find that the course treatment of all four topics varies from country to country. In fact, no two of these countries treats any of these topics in the same ways over the years! The significance of this is that no one can expect to export even one or two years of a curriculum from one country to another. Individual courses simply differ by too much.

Table 3 shows the numbers of years of coverage and the numbers of years of emphasis for these four topics in these six countries.

In these tables the "mile wide, inch deep" characterization of the U.S. curriculum does not appear particularly valid, and I could not find any relation between the years of coverage or emphasis and student performance. The maximum years of coverage for the topics is shared among three countries, and the maximum years of emphasis for the topics is shared among four, and they have vastly different performance profiles.

Table 2

Curriculum Coverage for Selected Mathematics Topics Across Student Ages

(taken from Schmidt et al. 1996, p. 52, Figure 2-7)

Example 1: Properties of Whole Number Operations

Student Age

Country	6	7	8	9	10	11	12	13	14	15	16	17	18
France	–	–	–	–	–	–	–	–	–	–	–		
Japan		•	•	•	–								
Norway			–	–	–	–	–	–	–	–			
Spain			–	–	–	–	–						
Switzerland		–	•	•	•	–	–	•	•				
USA	–	–	–	•	•	–							

Example 2: Relation of Common and Decimal Fractions

Student Age

Country	6	7	8	9	10	11	12	13	14	15	16	17	18
France				–	–	–	–	•					
Japan			–	–	•	•							
Norway					–	•	–	–	–	–			
Spain					•	•	–	•	–				
Switzerland							•	–	–	–			
USA			–	–	–	•	•	–					

Example 3: Exponents, Roots and Radicals

Student Age

Country	6	7	8	9	10	11	12	13	14	15	16	17	18
France						–	–	•	–	–	–	•	
Japan							•		•	•	–		
Norway								–	–	–	•	–	–
Spain						–	–	–	•				
Switzerland								–	•	•	–	–	
USA						–	–	•	•	–	–	•	

Example 4: Properties of Whole Number Operations

Student Age

Country	6	7	8	9	10	11	12	13	14	15	16	17	18	
France									–	–	–	•	–	
Japan				•	•	•	•	•	•	•	•	•	•	
Norway			–	–	–	–	–	–	–	–	–	–	–	–
Spain								–	–	–	–	–	–	
Switzerland			–	–	–	–	–	–	•	•	–	•	•	
USA			–	–	–	–	–	•	•	•	–	•	–	

Note: Ages 9 and 13 are TIMSS Student Populations 1 and 2
– *topic covered in curriculum* • *topic emphasized in curriculum*

In analyzing curricula in the United States, the difficulty for us is the diversity that exists within our country. The most recent report we have links our own National Assessment Grade 8 scores to those of TIMSS (Mullis et al. 1998). The differences in the 41 reporting states are striking. Examine Table 4. Compared to Mississippi, 19 of the 21 countries with samples that met the international guidelines, including the U.S. as a whole, score significantly higher, and none score significantly lower. In contrast, only 6 countries score significantly higher than North Dakota, and 8, including the U.S. as a whole, score significantly lower. We must conclude that the taught curriculum is not the same in these states. But every report coming out of Washington treats our entire country as if the curriculum were the same everywhere. Why not do the obvious: find out what is done in North Dakota, Iowa, Maine and other high-performing states, and emulate it. Find out what is done in Mississippi, Louisiana and the District of Columbia and work hard to change.

It is true that the United States, despite the lack of a national curriculum, does have a common algebra curriculum, if one looks at textbooks. Here are what I believe to have been the five most used first-year algebra textbooks (counting all editions as one) in the last school year in the United States, though together they only constitute, at most, 60 percent of the first-year algebra texts in use:

Merrill Algebra 1, by Foster et al.
Algebra, by Brown, Dolciani et al.
Heath Algebra 1, by Larson, Kanold et al.
Prentice Hall Algebra, by Bellman et al.
UCSMP Algebra, by McConnell et al.

In major ways, all five books are very much alike. They have 10–13 chapters. They all begin with algebraic expressions. They have 2–5 chapters on linear equations and inequalities (here UCSMP [University of Chicago School Mathematics Project] spends more time than the others). They graph lines and then they graph and solve systems. There is work with the laws of exponents and one or two chapters on polynomials. There is a chapter on quadratics, and thus some work with radicals. All solve quadratics by the Quadratic Formula and by factoring, and all but the Brown, Dolciani et al. do this graphically. All but UCSMP have a chapter dealing with rational expressions and rational equations. All have some geometry, including area formulas and the Pythagorean Theorem. In this sense, there is very much an algebra curriculum in the United States.

There are many other algebra texts in use in the United States: the texts of Smith, Charles et al. and of Foerster published by Addison-Wesley before the merger with Scott Foresman; of Saxon published by Grassdale: of Benson et al. published by McDougal Littell; of Coxford et al. published by Harcourt Brace; and so on. These books cover the same content as the five most used books and, except for Saxon, do it in pretty much the same way.

And there are the project algebras, none used very much at this point in time: the CORD algebra, the CMP algebra out of the University of California at Davis, the computer-intensive algebra of Fey and Heid published under the title *Concepts in Algebra—A Technological Approach.*

But there are also major differences even among the books in most use. The more recent copyrights give strong attention to graphing calculators. The more recent texts have large numbers of applications and real data. The data differ significantly from book to book so that students learn different things from one book than from another. UCSMP and the recent Prentice Hall give more attention to geometry. All give some attention to functions, but some of the recent texts use function language from the beginning, whereas others do it toward the end, where most students would not even see it. The picture one receives from these books is of an algebra curriculum that is reasonably fixed, but in flux.

Table 3

Years of Coverage and Years of Emphasis of Certain Topics

(from Schmidt et al. 1996, p. 52)

Years of Coverage

Country	Whole Numbers	Fractions, Decimals	Exponents, Roots	Equations, Formulas	Average
France	11	5	7	5	7
Japan	4	4	4	10	5.5
Norway	8	6	6	12	8
Spain	5	5	4	6	5
Switzerland	8	4	5	11	7
United States	6	6	7	11	7.5

Years of Emphasis

Country	Whole Numbers	Fractions, Decimals	Exponents, Roots	Equations, Formulas	Average
France	0	1	2	1	1
Japan	3	2	3	10	4.5
Norway	0	1	1	0	0.5
Spain	0	3	1	0	1
Switzerland	5	1	2	4	3
United States	2	2	3	4	3.75

And I have not mentioned the NSF projects that are exhibiting here. in which the traditional first-year algebra topics mentioned above are dispersed over two or three years. These integrated curricula are quite different from those mentioned above, and also quite different from each other. If we gave all available curricula equal weight, then we would have to conclude that there is no standard U.S. curriculum. What percent of students need to be enrolled in similar curricula in order for there to be considered to be a standard curriculum for the entire country? It is the same question we ask for the world, but in an individual country the more appropriate size of curriculum for the question is not the entire curriculum, but the course level.

Units

At the unit level, the mathematical approach taken to a topic becomes important. How are the various ideas related? So we ask: Are the approaches taken to large chunks of content the same worldwide?

We do not have a standard way for measuring different approaches to topics. In fact, except for broad approaches to geometry, with names such as "vector approach," "transformation approach" or "synthetic approach," different approaches to mathematics have seldom been discussed. There is no universal way to decide when two approaches differ.

But I will give some examples to indicate that there are differences. Consider the approach to systems of linear equations taken in the Japanese books UCSMP translated some years ago. In the chapter entitled "Simultaneous Equations" in the Grade 8 book (Kodaira 1984, 1992), there is not one graph. The reason is that students have not yet graphed lines with equations of the form $y = ax + b$. Yet in every algebra book in the United States, the study of systems begins with graphical solutions.

The Japanese text defines slope as the number a in $y = ax + b$. All the U.S. books define slope as $\frac{y_2 - y_1}{x_2 - x_1}$. UCSMP texts and Japanese books discuss rate of change *before* they discuss slope, an approach which we have found to be very successful in enhancing student understanding of the idea of slope. Yet we use applications and the Japanese do not. Is this enough to be different? UCSMP texts and the Japanese text describe the slope as the increase in y when x increases by 1. Is this enough to be different from other texts?

The Japanese text defines figures to be congruent if one figure can be laid on top of the other by combining translations, rotations and reflections. We do the same in UCSMP texts and spend some time over a period of years developing competence in these transformations. This is not done in most United States texts. Freudenthal (1983) pointed out that the way in which a term is defined automatically constrains it for future discussion. I believe these differences in the way congruence is approached cause differences in the ways in which students think about figures and their relationships to each other, and later, in the study of functions, in the ways in which students think about their graphs. I think it's a significant difference.

However, in general, the unit level is a difficult level at which to analyze curriculum. Over the years, we have developed very little language to describe different approaches to systems of equations or quadratics or congruence or similarity. A comprehensive study of curriculum at the unit level might prove quite enlightening.

Lessons

Turning now to the lesson level, the TIMSS videotape work of Stigler suggests that there are great differences in the ways that lessons are taught in Japan, Germany and the United States. A Japanese algebra class is shown spending 27 minutes on one problem, 15 minutes on another. A Japanese geometry class is shown spending 22 minutes on one problem and then 27 minutes on an extension of the same problem. In contrast, the U.S. algebra class has students working on all sorts of problems at once—in a cooperative learning situation—and the teacher spends no more than 2 minutes discussing any one problem in front of the entire class. The U.S. geometry class is more traditional in its setup but again there are a large number of questions with not much time spent on any one of them (Seago 1997).

In reports on these lessons, Stigler criticizes the ways in which United States teachers conduct their lessons (Beatty 1997, 11–12). There is an underlying assumption that lessons in Japan, Germany and the United States could be transported from one country to another. In fact, when one looks at the classrooms and at the content, it seems that the lessons could easily be transported. But one U.S. teacher, upon viewing these lessons, said to me that there is no way that her students would tolerate spending the amount of time on one problem that the Japanese do.

Are our societies enough different to make lessons that are viable in one country not viable in another? There are people who think so even for different groups within the United States. A call for "culturally relevant" pedagogy has been made by members of groups historically underrepresented in mathematics (Ladson-Billings 1995). This call rests

Table 4

Performance of NAEP Jurisdictions Compared to 20 TIMSS Countries at Grade 8

(from Mullis et al. 1998)

Jurisdiction	# of Countries Higher	# of Countries Lower	% of Students in Top 10%	% of Students in Top 50%
North Dakota	6	8	5	64
Iowa	6	7	4	63
Maine	6	7	6	62
Minnesota (est.)	6	7	6	62
Montana	6	7	6	62
Nebraska	6	7	5	61
Wisconsin	6	7	6	61
Minnesota (actual)	6	7	7	57
Vermont	7	5	4	57
Connecticut	7	5	5	56
Massachusetts	7	5	5	55
Alaska	7	4	7	55
Michigan	7	4	5	54
Utah	9	5	3	54
Oregon	9	4	4	53
Washington	9	4	4	53
Colorado	9	4	4	52
Indiana	9	4	3	52
Wyoming	10	3	3	52
Missouri	10	3	3	49
Texas	12	3	3	46
New York	12	2	3	47
Maryland	12	2	6	45
Virginia	12	2	3	45
Rhode Island	13	2	3	46
Arizona	13	2	2	43
North Carolina	13	2	3	42
Delaware	13	2	3	41
Florida	13	2	2	40
Kentucky	13	2	2	40
West Virginia	13	2	2	38
Tennessee	15	2	2	38
Hawaii	15	2	3	37
New Mexico	15	2	2	36
California	15	1	3	38
Georgia	15	1	2	38
Arkansas	15	1	2	37
South Carolina	15	1	2	34
Alabama	16	1	1	32
Louisiana	18	0	1	25
Mississippi	19	0	1	23
Dist. of Columbia	21	0	1	13

on the assumption that mathematics is taught in the U.S. from a Eurocentric lens that works against the performance of Hispanic, African-American and Native American students. It is closely related to the ethnomathematics movement to recognize not only the contributions to mathematics of non-European cultures but also the unique ways in which mathematics is informally used every day in one's own native culture.

And yet, in viewing middle school classrooms in Shanghai some years ago, I was struck by the similarities in the mathematics far more than the differences. In more than one class studying geometry proofs, doing problems exactly like those found in Japanese or American texts, I saw, in the midst of Chinese characters, the abbreviation SAS for the Side-Angle-Side congruence theorem. I mentioned my surprise to my hosts, who reminded me that Chinese characters do not represent sounds in the way that Latin characters do, so there is no Chinese character for the first letters of words. I was still astonished that the English first letters would be used. But it definitely seems to indicate that, at least with certain content, some lessons are quite exportable from one country to another.

Problems

The smallest size of curriculum—the individual problem or task—is not the least important size. The TIMSS and other international tests of comparison are based on the premise that there is a commonality of problems or other short tasks that can be used worldwide.

For example, the publication *What Students Abroad Are Expected to Know About Mathematics* (American Federation of Teachers 1997) displays examinations that top students in France, Germany and Japan have taken. and compares these with the SAT and Advanced Placement BC Calculus exams in the U.S. The published *Baccalauréat* Exam in Mathematics from the Aix province of France, taken by students at the end of their lycée experience in Grade 12, is an exam in vector analytic geometry, calculus and algebraic descriptions of geometric transformations. The *Abitur* Exam in Mathematics from the state of Baden-Württemberg in Germany is evenly split among calculus, solid analytic geometry and stochastics. The Tokyo University Entrance Exam in Mathematics resembles one of the American Invitational Mathematics Exams we use to select students for the U.S. Olympiad team. And of course our BC Calculus exam is all calculus. If these exams cover the curriculum in their respective locales, it is rather clear that there is no worldwide mathematics curriculum for the best students. The content differs markedly from country to country.

Recognizing these differences in content, in the TIMSS Grade 12 Advanced Mathematics study, a Test-Curriculum Matching Analysis was done. An expert in each country determined whether the items were in the intended curriculum of at least half of the students in the population. The idea was "to show how student performance in individual countries varied when based only on the test questions that were judged to be relevant to their own curriculum" (Mullis et al. 1998, C-1). The expert for the United States judged that 100 percent of the items were in the intended curriculum for the U.S. students. I have never seen the entire test, but 19 of the 82 items (see Table 1) and 2 of the 6 released items required calculus, and the highest estimates are that 6 percent of U.S. students take calculus. Since 14 percent of U.S. students were in this population, these items were in the intended curriculum of less than half of the U.S. students. They should not have been considered as relevant. Curiously, the analysis of only those items identified as appropriate had no major effect on the relationships among countries on either the mathematics or the physics tests. I have no logical explanation for this. Perhaps all of the experts tried to be as ecumenical as possible in including items.

As I mentioned earlier, selecting what is appropriate is only one part of the picture, however. One must ask whether there are things the students have learned that are not being tested. At all levels, would students in other countries perform as well as U.S. on items requiring measurements in feet and inches, or in pounds and ounces? I doubt it.

A quarter-century ago, I wrote a course called "Algebra Through Applications with Probability and Statistics." At the time, we had a student in a master's program from Colombia in South America. She was very impressed by the materials and took upon herself the task of translating them into Spanish. But she said she had to change a few problems. Not the data on baseball—they play baseball in Colombia. She needed to alter those problems in which we had people going on diets and losing weight at some constant rate. She said, "In Colombia, it's not considered advantageous to be thin. People don't diet."

Answering the Question

Now, for the last time, let me state the question that I have been trying to answer with these remarks: Is there a worldwide mathematics curriculum? I did not have an answer when I first thought of the talk. For most of the time that I prepared the talk, my feeling was the usual professor's response to such a question, Yes and No. But after working through the analysis, I have a different answer.

Mathematics is a worldwide language. Beyond the writing of numerals, schools, colleges and universities use virtually the same written language for algebra, geometry, analysis and statistics. Computers worldwide use the same programming languages. Multinational companies can hire mathematicians from virtually any country in the world. The problems tackled by mathematics are universal not only in place but also in time. We are able to hold conferences like this one because we recognize those characteristics of our subject.

But in our natural desire to show off the universality of our subject, I think we may have gone too far to think that *mathematics education* is the same worldwide. From arithmetic to beyond calculus, mathematics is vast. In our different cultures, different choices are made from all the mathematics available, and different aspects of this vastness are emphasized. In France, the mathematics is more theoretical, still reflecting the influence of Bourbaki. In the U.S., the mathematics is becoming more applied. In most countries, advanced mathematics students are using calculators, but this is not true in all countries. At the broadest level, we are all teaching very much the same ideas, reflecting the commonalities of mathematics. But we do so in different course structures, with the subject matter organized sometimes in quite different units, with lessons that may be appropriate for one country but not another, and often with problems that do not transfer from one site to another.

Thus beyond the teaching of arithmetic, there are common goals but there is not today a worldwide mathematics curriculum, and let us not delude ourselves into thinking that there is. But let us not be disappointed by this. We are able to have much richer conversations because of the differences.

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