Patterns and Patterning: Myths and Not Myths

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You may be having a few troublesome thoughts about the patterns and relations strand in the early and middle years curriculum documents. Because it is a new strand, there are likely to be good reasons for those thoughts. The strand covers two related processes: sorting and patterning. This article discusses both, with the focus on patterning.

In the current era of reform in mathematics education, it is important to be honest about what is mathematics and what is not. The rhetoric and pressures of reform can too easily lead us to be charmed by such clichés as "Mathematics is in everything" and "Problem solving is mathematics." Such clichés are suspect in designing instruction that involves worthwhile mathematics.

What does this have to do with sorting and patterning? Unfortunately, myths are attached to the two processes. Sorting and patterning are not mathematical processes in and of themselves. Mathematics is a cultural invention—an organized set of concepts, symbols, relationships and procedures created by people. Sorting and patterning are fundamental thinking processes that we are born with; the processes are hard-wired in us, if you will. We use them in our daily lives: when parenting, when reading a book, when shopping, when learning a language and so on. As a parent, I detected a pattern when my baby son was hungry: he cried. Is detecting that regularity doing mathematics? My wife sorts the laundry according to color; I sort it according to aroma. Are we doing mathematics when we sort the laundry?

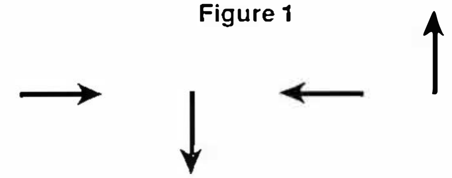
When we sort and look for patterns in ways that involve mathematical objects (for example, numbers or shapes) and/or attributes (for example, length or thickness), we are working in the domain of mathematics with the help of the processes of sorting and patterning. When we sort and look for patterns in ways that involve science concepts such as atoms and plant species, we are working in the domain of science with the help of these processes.

Patterning involves a kind of thinking generally referred to as inductive reasoning—searching for a consistent feature in something and having faith that it will continue. We are able to think in that way from birth. And patterning is not unique to humans: animals look for patterns as well, as anyone who has a pet can attest to. The ability to identify patterns is one of our mental survival tools, but it is not necessarily mathematical thinking.

What is a pattern from the perspective of mathematics? First, a mathematical pattern is not something that must repeat three times. Nor is it equivalent to a pretty visual design or a template such as a dress pattern. Mathematically speaking, a pattern involves something that remains constant about a collection of numbers, shapes or mathematical symbols, concepts or attributes. The critical matter is it *remains constant.*

Consider the series of arrows shown in Figure 1. What is the pattern? There is a regularity in the way the arrows point. The direction of each subsequent arrow involves a constant rotation or turn. One way to describe the pattern is the change in direction is always 1/4 of a turn clockwise.

Figure 1



Patterns do not shrink or grow. The elements of a pattern may shrink or grow, but the pattern does not. Consider this series of numbers: 23, 19, 15, 11, 7. The numbers (the elements) in the series decrease (shrink), but the pattern does not. One way to describe the pattern is "Each successive number is 4 less than the number before it." This pattern does not shrink for the series of numbers. It remains the same.

Consider the arrangements of dots in Figure 2. The number of dots increases or grows, but the pattern remains constant. It can be described in a variety of ways. From a geometry perspective, the pattern can be described as "The number of rows and columns of successive diagrams increases by 1." This increase of 1 in each dimension remains constant.

We could create a data table (see Table 1) from the dot diagrams and describe number patterns in it.

The table contains more than one type of numerical pattern. There is a vertical pattern that can be expressed as "The increase in the number of dots increases by 2 each time." There is a horizontal pattern that can be expressed as "The number of dots equals the square of the diagram number." Both of these patterns remain constant as the number of dots in the diagrams increases.

Figure 2

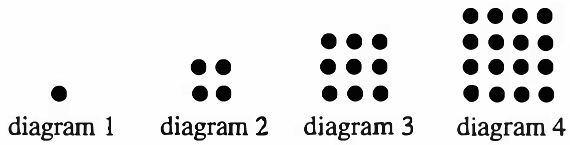


Table 1

Diagram Number	Number of Dots in Each Diagram
1	1
2	4
3	9
4	16

Teachers should not assume that, just because students are looking for and identifying patterns, this necessarily means that they are doing significant mathematics or mathematics at all. For example, determining what comes next in the color sequence Red Yellow Red Yellow . . . involves, at best, trivial mathematics (counting to one). At worst, it does not involve mathematics at all. A child does not need to count or use any other mathematical skill to identify the pattern of alternating colors.

With respect to early years curricula, identifying pattern types should not be an important goal in teaching patterning. For example, consider the following sequences: Red Yellow Yellow Red Red Yellow Yellow Red Red Yellow Yellow Red . . ., and 1331133113311331 . . . . An underlying pattern can be identified in the two sequences; some call it the ABBA pattern (not to be confused with the 1970s Swedish pop group). The underlying pattern can certainly be viewed in an ABBA way, but that is not really the point of doing patterning activities. Being able to recognize the ABBA pattern is of dubious benefit to the student or society. What can be of benefit is the student becoming comfortable with the processes of making and testing hypotheses when searching for mathematical patterns.

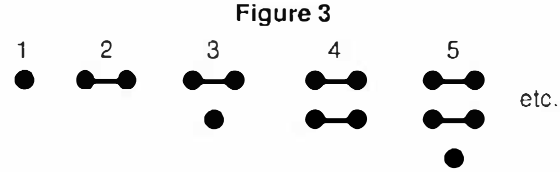
Having said this, nevertheless, some identification of underlying patterns should be pursued to assist students in generalizing. For example, it is useful to a point for students to recognize that the sequences Red Yellow Yellow Red Red Yellow Yellow Red . . . and 133113311331 . . . have something in common—even though one involves color and the other involves number. One way to describe what they have in common is to call it the ABBA pattern.

Although the mental process of searching for and identifying patterns is hard-wired in us, the language associated with that process is not. Students need to learn language descriptors for a process they can do naturally. For this reason, it is good pedagogy for teachers to make use of things that are familiar to students to help them understand the language descriptors. Nonmathematical items such as shoes and teddy bears are appropriate contexts for developing Kindergarten and Grade 1 students' understanding of language. However, once they understand the meaning of such words as *pattern*, patterning activities should involve reasonable to significant mathematics and serve two important purposes: (1) helping students learn mathematics and (2) helping stimulate the fundamental thinking process of patterning so that it can grow in depth and scope.

Patterning activities should be integrated with other strands of the mathematics curriculum. This can be done in two ways: (1) using patterns to learn concepts and skills from other mathematics strands and (2) solving patterning problems that involve concepts and skills from other strands. Using patterns to help students learn concepts and skills from other strands of the curriculum can be considered an authentic use of patterning. Three examples follow.

Early years children learn about odd and even numbers. Patterning can be used to help them understand these kinds of numbers in an activity that involves using actual objects and the symbols for numbers. For each represented number, children could be asked to connect all of the objects two at a time (make pairs) in some way (line segments are used in the diagram).

Figure 3



Children then could be asked to look for a pattern. With discussion, they should come to the conclusion that the numbers 2, 4, 6, . . . always can be paired while the numbers 1, 3, 5, . . . always have an unpaired object left over. All that remains is for the teacher to attach the word *even* to the numbers 2, 4, 6, . . . and the word *odd* to the numbers 1, 3, 5, ...

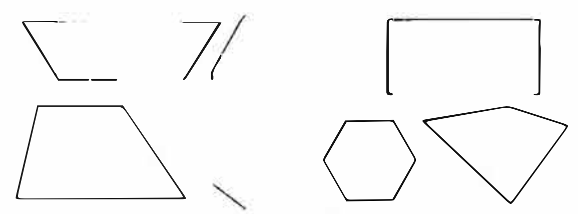
Middle years students learn about integer multiplication. Patterning can be used to help them understand that a negative number multiplied by a negative number equals a positive number. To use patterning for this goal, students must first understand that a positive number multiplied by a negative number equals a negative number. This understanding can be developed by viewing the matter in terms of "I owe . . ." (for example, 2×-3 can be interpreted as "I owe each of 2 friends 3 dollars. How much do I owe in all?"). This approach is inappropriate in the case of a negative number multiplied by a negative number because there is no such thing as a negative group in mathematics (though there can be groups of people who are negative toward something, but that is an entirely different thing). Once students understand that a positive number multiplied by a negative number equals a negative number, a situation such as the one shown in Figure 4 can be used for the case of a negative number multiplied by a negative number. Students would need to look for a pattern in the results (for this example, the answer to the multiplication gets bigger by 3 each time).

Figure 4

$3 \times -3 = -9$
$2 \times -3 = -6$
$1 \times -3 = -3$
$0 \times -3 = 0$
$-1 \times 3 = ?$

Patterning can be used to help students understand geometry terminology. For example, a teacher can display two collections of shapes (as shown in Figure 5), ask students to look for a pattern and then provide a definition of a trapezoid. One pattern is that trapezoids have four sides, one pair of opposite sides is parallel and the other pair is not parallel.

Figure 5



These are trapezoids — These are not trapezoids

Having students solve problems that involve patterning and concepts and skills from other strands is the other way to integrate patterning within the mathematics curriculum. For some of these problems, teachers can pay attention to additional matters such as constructing tables and using systematic ways to look for patterns in numbers. Many problems are possible. Mathematics curriculum documents contain good examples of such problems.

The problem presented below is different from the typical ones. The mathematics content involved in the pattern is around the Kindergarten/Grade 1 level, but the problem is likely to challenge the reader. The answer is not included here. The reader is invited to e-mail the author at j.ameis@uwinnipeg.ca after thinking about the problem for a while.

The numbers in each row are derived from the numbers in the row above in the same way. What is the next row of numbers (the eighth row)?

Figure 6

1
1 1
2 1
1 2 1 1
1 1 1 2 2 1
3 1 2 2 1 1
1 3 1 1 2 2 2 1